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TWO WAREHOUSES PRODUCTION INVENTORY MODEL FOR DETERIORATING ITEMS WITH STOCK-DEPENDENT DEMAND UNDER PRESERVATION TECHNOLOGY USING MODIFIED GENETIC ALGORITHM

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Abstract

This paper considers two warehouses production inventory model for deteriorating items with stock-dependent demand and completely backlogged shortages under preservation technology. For display and storage of inventory, one warehouse of finite capacity is located at the main market, called primary warehouse (PW) and another warehouse with large capacity at a small distance from the main market, called secondary warehouse (SW). Here we consider items are transported from SW to PW in continuous release pattern and the transportation cost is negligible. The aim of this study is to obtain the optimal cycle length for maximum average profit through a modified genetic algorithm (MGA). Finally the model is illustrated using a numerical example. A sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

Keywords:

Inventory, Primary Warehouse, Secondary Warehouse, Preservation Technology, Modified Genetic Algorithm

1. INTRODUCTION

The production base economic quantity model deals with different parameters such as production rate, demand rate, shortages, deterioration etc. In EPQ model, the rate of production is so vital. In this paper we consider that the production is going on a constant production rate. When the storage capacity attains its maximum level, then the production will stop and it will again start when inventory level reaches the level of maximum allowed shortages.

It is obvious that, in production base economic quantity management, the system deals with demand and supply chain, i.e; the business is totally depending on demand and supply of goods. So demand is one of the important parameter. A good number of models are developed with different types of demand such as stock dependent, time dependent, etc. In this paper we consider a stock dependent demand, as huge stock of an item reflects the customer's mind to buy it. So it is relevant that demand of an item depends on the stock level. To fulfill the demand of consumer, it is necessary to store enormous amount of items. For this purpose, the sufficient space is required to store the goods to fulfill the demands. The space used to store the goods is termed as warehouse. But in the field of production management, when a production of large amounts of units of items cannot be stored in the existing limited storage (known as Primary Warehouse) as in the busy markets like super markets, municipality markets, corporation markets etc. the space constraints is quite relevant. In this situation for storing the excess items, one additional warehouse (known as Secondary Warehouse) is hired on rental basis which may be located little away from the market place. Here we consider that the produced items are stored first in PW and then excess stock is stored in SW, which are emptied first by transporting the stocks from SW to PW in a continuous releasing pattern to neglect the transportation cost. The demand of items is met up at PW only.

Deterioration of items is a general phenomenon in real life. The assumption that the produced items preserve their physical characteristics forever is not true. Therefore while determining the optimal policy of such type of products (like medicine, fresh fruits, volatile liquids, foods etc.) the loss due to deterioration must be considered. In general deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of original usefulness. So deteriorating items inventory model have been studied by many authors in past. Generally, there is no initial deterioration of such goods and after certain time period deterioration starts. Naturally, deterioration is a process that diminish the original quality of an item, so the production house, owner/retailer can reduce the rate of deterioration of items by using the preservation technologies, including chemical treatment, physical methods and biotechnology. Preservation technologies have been applied to maintain storage quality and to extend the storage life of fresh item. Developing advanced preservation techniques to prolong the storage life of items is of importance for improving social and economic benefits.

So, in this paper we try to develop a production base economic model with stock dependent demand of non-instantaneous deteriorating items using preservation technology with two warehouses facilities having different deterioration rates due to the difference in the environmental conditions. Here also we allow shortages which is fully backlogged. We assume that the production rate is constant. The objective of this model is to find the optimal cycle length to maximize the average profit.

2. LITERATURE REVIEW

In the present competitive market, the production house, owner/retailer /supplier influence the customers in many different ways to capture the market. Teng et al. [1] developed an EOQ model for stock-dependent demand under supplier's trade credit offer with a progressive payment scheme. Das et al. [2] presented an EPQ model with stock-dependent demand rate under inventory control. A group of researchers have considered inventory control systems with stock-dependent demand in their research such as Giri et al. [3], Hou [4], Roy et al. [5], kar et al. [6] and others.

Hartley [7] is the first who considered an additional storage facility with additional holding cost and called it Secondary warehouse. Pakkala and Achary [8] extended the two warehouses inventory model for deteriorating items with finite replenishment rate. Bhunia and Maiti [9] developed a two warehouses inventory model for a linear trend in demand. There are several related papers presented in the field of inventory management such as Pakhala and Achary [10], Kar et al. [11], Das et al. [12] and others.

Goyal and Gunasekaran [13] have developed an integrated production-inventory-marketing model involving deteriorating items for a multi-stage EPLS and EOQ system. Ghare and Schrader [14] first focus on the effect of decay in the inventory analysis. Sachan [15] extended the model of Dave and Patel [16] by allowing shortages. Assuming the deterioration in both warehouse Sarma [17] extended his earlier model to the case of infinite replenishment rate with shortages. Then Benkherouf [18] developed a deterministic order level inventory model for deteriorating items with two storage facilities. Two-warehouse inventory models for deteriorating items with shortages under inflation were developed by Yang [19]. Lee and Dye [20] invented an inventory model for deteriorating item under stock dependent demand and controllable deterioration rate. Singh and Pattnayak [21] discuss on the deterioration of inventory with variable deterioration and partial backlogging. Das et al. [12] presented EPQ models with deteriorating item with finite and random product life cycle. To reduce the deterioration rate, the retailer use the preservation technology. Hsu et al. [22] invented a deteriorating model using the preservation technology. Again Dye and Hsieh [23] extended the model of Hsu et al. by assuming the preservation technology cost is a function of the length of the replenishment cycle. Dye [24], Das and Jana [25], Das [26] focus on the effect of preservation technology on a non-instantaneous deteriorating inventory.

3. NOTATIONS AND ASSUMPTIONS

To formulate the mathematical model we used the following notations and assumptions in this paper as below:

3.1 NOTATIONS

P = Constant production rate per unit time.

$S(t)$ = On hand inventory of the item at any time $t(>0)$, when shortages are allowed.

$I_1(t) =$ On hand inventory of the item at any time $t(>0)$ in PW.

$I_2(t) =$ On hand inventory of the item at any time $t(>0)$ in SW.

$k =$ Production cost per unit item.

$W_1 =$ Maximum shortages level.

$W =$ Maximum inventory level in PW.

$V =$ Maximum inventory level in SW.

$\theta_1 =$ Constant deterioration rate in PW, where $0< \theta_1 <1$.

$\theta_2 =$ The constant deterioration rate in SW, where $0< \theta_2 <1$.

$t_1 =$ Time at which shortages reach its maximum level and production starts.

$t_2 =$ Time at which shortages are met.

$t_3 =$ Time at which inventory level reaches its maximum level in PW.

$t_4 =$ Time at which inventory level reaches its maximum level in SW.

$t_5 =$ Time at which inventory level vanishes in SW.

$T =$ Duration of complete cycle where inventory level vanishes.

$t_p =$ Time period during which no deterioration.

$C_p =$ Total production cost.

$c_{11} =$ Holding cost per unit item at PW.

$c_{12} =$ Holding cost per unit item at SW.

$C_h =$ Total holding cost.

$c_2 =$ Shortages cost per unit item.

$C_s =$ Total shortages cost.

$C_{pr} =$ Total Preservation cost.

$D_1 =$ Total deteriorated items throughout the process.

$P_i =$ Total amount of produced item.

s = Sales revenue per unit item.

$S_t =$ Total selling price.

$P_T =$ Total profit.

ξ = The preservation technology cost per unit time for reducing deterioration rate in order to preserve the products in the PW. Preservation technology is used and the reduced deterioration rate $m(\xi)$ is an increasing function of the preservation technology cost ξ, where $0 \le m(\xi) \le 1$. According to our assumption the corresponding deterioration rate is $[\theta_1 - m(\xi)]$.

λ = preservation technology cost per unit time for reducing deterioration rate in order to preserve the products in the SW. Preservation technology is used and the reduced deterioration rate $n(\lambda)$ is an increasing function of the preservation technology cost λ, where $0 \le n(\lambda) \le 1$. According to our assumption the corresponding deterioration rate is $[\theta_2 - n(\lambda)]$.

AP = average profit.

3.2 ASSUMPTIONS

- Production rate is known and constant.
- The time horizon of the inventory system is infinite.
- Shortages are allowed and fully backlogged.
- Demand is stock dependent.
- Deterioration is allowed on the both warehouses.
- There is no repair or replacement of deteriorated units.
- There is no deterioration during the time $[t_1, t_2]$.

4. MODEL FORMULATION:

In this model, we have considered a manufacturing system in which the demand rate $D(t)$ is assumed to vary with stock level at *PW* and is of the form:

$$
\begin{aligned}
D(t) &= \alpha,\ 0 \le t \le t_2 \\
&= \alpha + \beta I_1(t),\ t_2 \le t \le t_3 \\
&= \alpha + \beta W,\ t_3 \le t \le t_4 \\
&= \alpha + \beta W,\ t_4 \le t \le t_5
\end{aligned}
$$

where $\alpha,\beta > 0$ are constants.

In the development of the two warehouses production model with preservation technology, here we assume that the shortages reaches its maximum level W_1 at time $t=t_1$ and to make up shortages, production starts at $t=t_1$. When production and demand occur simultaneously, backorders are made up to $t=t_2$. Depending on the position of the parameter t_p there exists four cases: case-I: $t_2\leq t_p\leq t_3$, case-II: $t_3\leq t_p\leq t_4$, case-III: $t_4\leq t_p\leq t_5$, case-IV: $t_5\leq t_p\leq T$.

Case I: $t_2\leq t_p\leq t_3$

In case-I, inventory items in PW begin to accumulate up to W units with deterioration under preservation technology. After $t=t_3$ the produced quantity exceeding W must be stored in SW and production continuous up to $t=t_4$ (cf. Fig. 1) and inventory level of SW reaches its maximum level V. At the end of production, the inventory in SW would be depleted due to demand and deterioration and it vanishes at $t=t_5$. During the time interval $[t_3,t_5]$, inventory in PW are also lowered at a level below W due to deterioration only and in the time interval $[t_5,T]$ the remaining stock in PW are then fully depleted at $t = T$ due to both demand and deterioration.

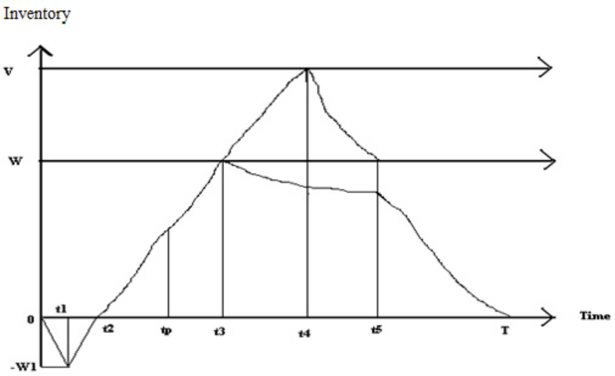

Fig.1. Graphical representation of a two-warehouse production system with stock-dependent demand

The differential equations describing the inventory level are given as follows:

$$\frac{dS(t)}{dt} = -\alpha,\ 0\leq t\leq t_1 \qquad (1)$$
$$= P-\alpha,\ t_1\leq t\leq t_2$$

With the boundary conditions, $S(t_1)=-W_1$, $S(t_2)=0$

And

$$\frac{dI_1(t)}{dt} = P-\alpha-\beta I_1(t), t_2\leq t\leq t_p$$
$$= P-\alpha-\beta I_1(t)-\{\theta_1-m(\xi)\}I_1(t), t_p\leq t\leq t_3 \qquad (2)$$
$$= -\{\theta_1-m(\xi)\}I_1(t), t_3\leq t\leq t_5$$
$$= -\alpha-\beta I_1(t)-\{\theta_1-m(\xi)\}I_1(t), t_5\leq t\leq T$$

With the boundary conditions $I_1(t_2)=0$, $I_1(t_3)=W$, $I_1(T)=0$ and

$$\frac{dI_2(t)}{dt} = P-\alpha-\beta W-\{\theta_2-n(\lambda)\}I_2(t), t_3\leq t\leq t_4 \qquad (3)$$
$$= -\alpha-\beta W-\{\theta_2-n(\lambda)\}I_2(t), t_4\leq t\leq t_5$$

With the boundary conditions,

$$S(t)=\alpha(t_1-t)-W_1,\ I_2(t_3)=0,\ I_2(t_4)=V,\ I_2(t_5)=0.$$

The solutions of the differential equations in Eq.(1) are represented by

$$0\leq t\leq t_1 = (P-\alpha)(t-t_2)\ t_1\leq t\leq t_2 \qquad (4)$$

The solutions of the differential equations in Eq.(2) are represented by

$$I_1(t)=\frac{P-\alpha}{\beta}[1-e^{\beta(t_2-t)}], t_2\leq t\leq t_p$$
$$=\frac{P-\alpha}{\beta+\{\theta_1-m(\xi)\}}[1-e^{(\beta+\{\theta_1-m(\xi)\})(t_3-t)}]+We^{(\beta+\{\theta_1-m(\xi)\})(t_3-t)}\ t_p\leq t\leq t_3$$
$$=We^{\{\theta_1-m(\xi)\}(t_3-t)}, t_3\leq t\leq t_5$$
$$=\frac{\alpha}{\beta+\{\theta_1-m(\xi)\}}[e^{(\beta+\{\theta_1-m(\xi)\})(T-t)}-1], t_5\leq t\leq T \qquad (5)$$

The solutions of the differential equations in Eq.(3) are represented by

$$I_2(t)=\frac{P-\alpha-\beta W}{\{\theta_2-n(\lambda)\}}[1-e^{\{\theta_2-n(\lambda)\}(t_3-t)}], t_3\leq t\leq t_4 \qquad (6)$$
$$=\frac{\alpha+\beta W}{\{\theta_2-n(\lambda)\}}[e^{\{\theta_2-n(\lambda)\}(t_5-t)}-1], t_4\leq t\leq t_5$$

Using continuity condition on the equations in Eq.(4) at $t=t_1$ we have,

$$t_2=\frac{W_1}{P-\alpha}+t_1 \qquad (7)$$

Using continuity conditions on the equations in Eq.(5) at $t=t_p$ and $t=t_5$ we have respectively,

$$t_3=t_p+\frac{1}{\beta+\{\theta_1-m(\xi)\}}\log\left[\frac{\frac{P-\alpha}{\beta}-\frac{P-\alpha}{\beta+\{\theta_1-m(\xi)\}}-\frac{P-\alpha}{\beta}e^{\beta(t_2-t_p)}}{W-\frac{P-\alpha}{\beta+\{\theta_1-m(\xi)\}}}\right] \qquad (8)$$

And

$$T=t_5+\frac{1}{\beta+\{\theta_1-m(\xi)\}}\log(1+\frac{W(\beta+\{\theta_1-m(\xi)\})}{\alpha}e^{\{\theta_1-m(\xi)\}(t_3-t_5)}) \qquad (9)$$

Using the boundary condition, at $t=t_4, I_2(t_4)=V$ from the equations in Eq.(6) we have,

$$t_4=t_3+\frac{1}{\{\theta_2-n(\lambda)\}}\log(\frac{P-\alpha-\beta W}{P-\alpha-\beta W-V\{\theta_2-n(\lambda)\}}) \qquad (10)$$

And

$$t_5=t_4+\frac{1}{\{\theta_2-n(\lambda)\}}\log(1+\frac{V\{\theta_2-n(\lambda)\}}{\alpha+\beta W}) \qquad (11)$$

Total amount of produced item is given by

$$P_i=\int_{t_1}^{t_4}Pdt=P(t_4-t_1) \qquad (12)$$

Total Production Cost,

$$C_p=k\times P_i=kP(t_4-t_1) \qquad (13)$$

Total Holding Cost is given by

$$C_h = c_{11}\int_{t_2}^{T} I_1(t)dt + c_{12}\int_{t_3}^{t_5} I_2(t)dt$$

$$= c_{11}\frac{P-\alpha}{\beta}[t_p - t_2 + \frac{1}{\beta}(e^{\beta(t_2-t_p)}-1)] + \frac{c_{11}(P-\alpha)}{\beta+\{\theta_1-m(\xi)\}}[(t_3-t_p) + \frac{1}{\beta+\{\theta_1-m(\xi)\}}(1-e^{(\beta+\{\theta_1-m(\xi)\})(t_3-t_p)})]$$

$$+c_{11}W\frac{1}{\beta+\{\theta_1-m(\xi)\}}[1-e^{(\beta+\{\theta_1-m(\xi)\})(t_3-t_p)}]$$

$$-\frac{Wc_{11}}{\{\theta_1-m(\xi)\}}[e^{\{\theta_1-m(\xi)\}(t_3-t_5)}-1]$$

$$+c_{11}\frac{\alpha}{\beta+\{\theta_1-m(\xi)\}}[\frac{1}{\beta+\{\theta_1-m(\xi)\}}\{e^{(\beta+\{\theta_1-m(\xi)\})(T-t_5)}-1\}-(T-t_5)]$$

$$+c_{12}(\frac{P-\alpha-\beta W}{\{\theta_2-n(\lambda)\}})[(t_4-t_3)+\frac{1}{\{\theta_2-n(\lambda)\}}\{e^{\{\theta_2-n(\lambda)\}(t_3-t_4)}-1\}]$$

$$+c_{12}(\frac{\alpha+\beta W}{\{\theta_2-n(\lambda)\}})[-\frac{1}{\{\theta_2-n(\lambda)\}}\{1-e^{\{\theta_2-n(\lambda)\}(t_5-t_4)}\}-(t_5-t_4)] \quad (14)$$

Total Shortages Cost is given by

$$C_s = c_2\int_0^{t_2} S(t)dt$$

$$= c_2[\frac{\alpha t_1^2}{2} - W_1 t_1] + c_2(P-\alpha)[\frac{t_2^2}{2}-\frac{t_1^2}{2}+t_1t_2] \quad (15)$$

Total preservation cost is given by

$$C_{pr} = (T-t_p)\xi + (t_5-t_3)\lambda \quad (16)$$

Total deteriorated items,

$$D_1 = \{\theta_1-m(\xi)\}\int_{t_p}^{T} I_1(t)dt + \{\theta_2-n(\lambda)\}\int_{t_3}^{t_5} I_2(t)dt$$

$$=\{\theta_1-m(\xi)\}\frac{P-\alpha}{\beta+\{\theta_1-m(\xi)\}}$$

$$[(t_3-t_p)+\frac{1}{\beta+\{\theta_1-m(\xi)\}}\{1-e^{(\beta+\{\theta_1-m(\xi)\})(t_3-t_p)}\}]$$

$$-\frac{W\{\theta_1-m(\xi)\}}{\beta+\{\theta_1-m(\xi)\}}\{1-e^{(\beta+\{\theta_1-m(\xi)\})(t_3-t_p)}\}-W\{e^{\{\theta_1-m(\xi)\}(t_3-t_5)}-1\}$$

$$-\frac{\{\theta_1-m(\xi)\}\alpha}{\beta+\{\theta_1-m(\xi)\}}[\frac{1}{\beta+\{\theta_1-m(\xi)\}}\{1-e^{(\beta+\{\theta_1-m(\xi)\})(T-t_5)}\}+(T-t_5)]$$

$$+(P-\alpha-\beta W)[(t_4-t_3)-\frac{1}{\{\theta_2-n(\lambda)\}}\{1-e^{\{\theta_2-n(\lambda)\}(t_3-t_4)}\}]$$

$$-(\alpha+\beta W)[\frac{1}{\{\theta_2-n(\lambda)\}}\{1-e^{\{\theta_2-n(\lambda)\}(t_5-t_4)}\}+(t_5-t_4)] \quad (17)$$

Total selling price, $S_t = s(P_i - D_1)$ (18)

Total profit, $P_T = S_t - C_p - C_h - C_s - C_{pr}$ (19)

The average Profit, $AP = \frac{P_T}{T}$ (20)

Case II: $t_3 \le t_p \le t_4$

In case-II, during the interval $[t_2,t_3]$ the system is subjected due to the effect of production and demand only in *PW* and inventory items begin to accumulate up to *W* units. During the time interval $[t_3,t_p]$ the stock level at *PW* remains unchanged, during the interval $[t_p,t_5]$ the inventory in *PW* are also lowered at a level below *W* due to deterioration only. Also in the time interval $[t_5,T]$ the remaining stock in *PW* are then fully depleted at $t = T$ due to both demand and deterioration. After $t=t_3$ the produced quantity exceeding W must be stored in *SW* and production continuous up to $t=t_4$ (cf. Fig. 2) and inventory level of *SW* reaches its maximum level *V*. In SW during the time interval $[t_3,t_p]$ the system is subjected due to the effect of production and demand only, during the interval $[t_p,t_4]$ the system is subjected due to the effect of production, demand and deterioration. At the end of production, the inventory in SW would be depleted due to demand and deterioration and it vanishes at $t=t_5$.

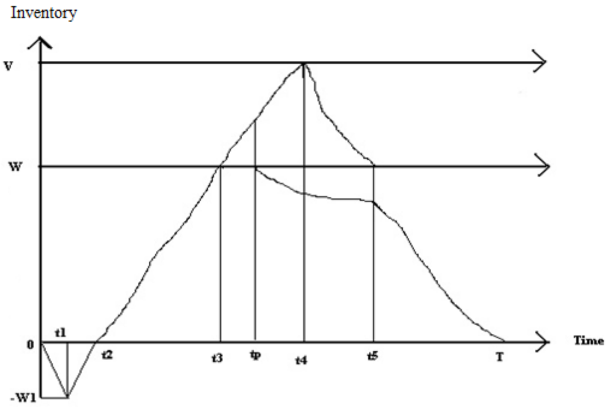

Fig.2. Graphical representation of a two-warehouse production system with stock-dependent demand.

The differential equations describing the inventory level are given as follows:

$$\frac{dS(t)}{dt} = -\alpha, 0 \le t \le t_1 = P-\alpha, t_1 \le t \le t_2 \quad (21)$$

With the boundary conditions, $S(t_1) = -W_1$, $S(t_2) = 0$ and

$$\frac{dI_1(t)}{dt} = P-\alpha-\beta I_1(t), t_2 \le t \le t_3$$
$$= 0, t_3 \le t \le t_p \quad (22)$$
$$= -\{\theta_1-m(\xi)\}I_1(t), t_p \le t \le t_5$$
$$= -\alpha-\beta I_1(t)-\{\theta_1-m(\xi)\}I_1(t), t_5 \le t \le T$$

With the boundary conditions, $I_1(t_2)=0$, $I_1(t_3)=W$, $I_1(T)=0$ and

$$\frac{dI_2(t)}{dt} = P-\alpha-\beta W, t_3 \le t \le t_p$$
$$= P-\alpha-\beta W-\{\theta_2-n(\lambda)\}I_2(t), t_p \le t \le t_4 \quad (23)$$
$$= -\alpha-\beta W-\{\theta_2-n(\lambda)\}I_2(t), t_4 \le t \le t_5$$

With the boundary conditions, $I_2(t_3)=0$, $I_2(t_4)=V$, $I_2(t_5)=0$.

The solutions of the differential equations in Eq.(21) are represented by

$$\begin{aligned} S(t) &= \alpha(t_1 - t) - W_1, 0 \le t \le t_1 \\ &= (P-\alpha)(t-t_2), t_1 \le t \le t_2 \end{aligned} \quad (24)$$

The solutions of the differential equations in Eq.(22) are represented by

$$\begin{aligned} I_1(t) &= \frac{P-\alpha}{\beta}[1-e^{\beta(t_2-t)}], t_2 \le t \le t_3 \\ &= W, t_3 \le t \le t_p \\ &= We^{\{\theta_1 - m(\xi)\}(t_p - t)}, t_p \le t \le t_5 \\ &= \frac{\alpha}{\beta+\{\theta_1 - m(\xi)\}}[e^{(\beta+\{\theta_1-m(\xi)\})(T-t)}-1], t_5 \le t \le T \end{aligned} \quad (25)$$

The solutions of the differential equations in Eq.(23) are represented by

$$\begin{aligned} I_2(t) &= (P-\alpha-\beta W)(t-t_3), t_3 \le t \le t_p \\ &= \frac{(P-\alpha-\beta W)}{\{\theta_2 - n(\lambda)\}}[1-e^{\{\theta_2-n(\lambda)\}(t_4-t)}] + Ve^{\{\theta_2-n(\lambda)\}(t_4-t)}, t_p \le t \le t_4 \\ &= \frac{\alpha+\beta W}{\{\theta_2-n(\lambda)\}}[e^{\{\theta_2-n(\lambda)\}(t_5-t)}-1], t_4 \le t \le t_5 \end{aligned} \quad (26)$$

Using continuity condition on the equations in Eq.(24) at $t = t_1$ we have,

$$t_2 = \frac{W_1}{P-\alpha} + t_1 \quad (27)$$

Using continuity conditions on the equations in Eq.(25) at $t = t_3$ and $t = t_5$ we have respectively,

$$t_3 = t_2 + \frac{1}{\beta}\log[\frac{P-\alpha}{P-\alpha-W\beta}] \quad (28)$$

And

$$T = t_5 + \frac{1}{\beta+\{\theta_1-m(\xi)\}} \log\left[1+\frac{W\left(\beta+\{\theta_1-m(\xi)\}\right)}{\alpha}e^{\{\theta_1-m(\xi)\}(t_p-t_5)}\right] \quad (29)$$

Using continuity conditions on the equations in Eq.(26) at $t = t_p$ and boundary condition, at $t = t_4$, $I_2(t_4) = V$, we have respectively,

$$t_4 = t_p + \frac{1}{\{\theta_2-n(\lambda)\}}\log[\frac{(P-\alpha-\beta W)(t_p - t_3 - \frac{1}{\{\theta_2-n(\lambda)\}})}{V - \frac{P-\alpha-\beta W}{\{\theta_2-n(\lambda)\}}}] \quad (30)$$

And

$$t_5 = t_4 + \frac{1}{\{\theta_2-n(\lambda)\}}\log[1+\frac{V\{\theta_2-n(\lambda)\}}{\alpha+\beta W}] \quad (31)$$

Total amount of produced item is given by

$$P_i = \int_{t_1}^{t_4} P dt = P(t_4 - t_1) \quad (32)$$

Total Production Cost,

$$C_p = k \times P_i = kP(t_4 - t_1) \quad (33)$$

Total Holding Cost is given by

$$\begin{aligned} C_h &= c_{11}\int_{t_2}^{T} I_1(t)dt + c_{12}\int_{t_3}^{t_5} I_2(t)dt \\ &= c_{11}\frac{P-\alpha}{\beta}[t_3 - t_2 + \frac{1}{\beta}(e^{\beta(t_2-t_3)}-1)] + c_{11}W(t_p - t_3) \\ &-c_{11}\frac{W}{\{\theta_1-m(\xi)\}}(e^{\{\theta_1-m(\xi)\}(t_p-t_5)}-1) \\ &+\frac{c_{11}\alpha}{\beta+\{\theta_1-m(\xi)\}}[\frac{1}{\beta+\{\theta_1-m(\xi)\}}\{e^{(\beta+\{\theta_1-m(\xi)\})(T-t_5)}-1\} \\ &-(T-t_5)] + c_{12}(P-\alpha-\beta W)[\frac{1}{2}(t_p{}^2 + t_3{}^2) - t_3 t_p] \\ &+c_{12}(\frac{P-\alpha-\beta W}{\{\theta_2-n(\lambda)\}})[(t_4-t_p) + \frac{1}{\{\theta_2-n(\lambda)\}}\{1-e^{\{\theta_2-n(\lambda)\}(t_4-t_p)}\}] \\ &-c_{12}\frac{V}{\{\theta_2-n(\lambda)\}}\{1-e^{\{\theta_2-n(\lambda)\}(t_4-t_p)}\} + c_{12}(\frac{\alpha+\beta W}{\{\theta_2-n(\lambda)\}}) \\ &[\frac{1}{\{\theta_2-n(\lambda)\}}\{e^{\{\theta_2-n(\lambda)\}(t_5-t_4)}-1\} - (t_5-t_4)] \end{aligned} \quad (34)$$

Total Shortages Cost is given by

$$\begin{aligned} C_s &= c_2\int_0^{t_2} S(t)dt \\ &= c_2[\frac{\alpha t_1{}^2}{2} - W_1 t_1] + c_2(P-\alpha)[\frac{t_2^2}{2} - \frac{t_1^2}{2} + t_1 t_2] \end{aligned} \quad (35)$$

Total preservation cost is given by

$$C_{pr} = (T - t_p)\xi + (t_5 - t_p)\lambda \quad (36)$$

Total deteriorated items,

$$\begin{aligned} D_1 &= \{\theta_1 - m(\xi)\}\int_{t_p}^{T} I_1(t)dt + \{\theta_2 - n(\lambda)\}\int_{t_p}^{t_5} I_2(t)dt \\ &= W[1-e^{\{\theta_1-m(\xi)\}(t_p-t_5)}] + \frac{\{\theta_1-m(\xi)\}\alpha}{\beta+\{\theta_1-m(\xi)\}} \\ &\left[\frac{1}{\beta+\{\theta_1-m(\xi)\}}\{e^{(\beta+\{\theta_1-m(\xi)\})(T-t_5)}-1\} - (T-t_5)\right] \\ &+(P-\alpha-\beta W)[(t_4-t_p) + \frac{1}{\{\theta_2-n(\lambda)\}}\{1-e^{\{\theta_2-n(\lambda)\}(t_4-t_p)}\}] \\ &-V\{1-e^{\{\theta_2-n(\lambda)\}(t_4-t_p)}\} \\ &+(\alpha+\beta W)[\frac{1}{\{\theta_2-n(\lambda)\}}\{e^{\{\theta_2-n(\lambda)\}(t_5-t_4)}-1\} - (t_5-t_4)] \end{aligned} \quad (37)$$

Total selling price, $S_t = s(P_i - D_1)$ (38)

Total profit, $P_T = S_t - C_p - C_h - C_s - C_{pr}$ (39)

The average Profit, $AP = \frac{P_T}{T}$ (40)

Case III: $t_4 \le t_p \le t_5$

In case-III, during the interval $[t_2,t_3]$ the system is subjected due to the effect of production and demand only in *PW* and inventory items begin to accumulate up to *W* units. During the time interval $[t_3,t_p]$ the stock level at *PW* remains unchanged, during the interval $[t_p,t_5]$ the inventory in *PW* are also lowered at a level below *W* due to deterioration only. Also in the time interval $[t_5,T]$ the remaining stock in *PW* are then fully depleted at $t = T$ due to both demand and deterioration. After $t=t_3$ the produced quantity exceeding *W* must be stored in *SW* and production continuous up to $t=t_4$ (cf. Fig. 3) and inventory level of *SW* reaches its maximum level *V*. In *SW* during the time interval $[t_3,t_4]$ the system is subjected due to the effect of production and demand only, during the time interval $[t_4,t_p]$ the system is subjected due to the effect of demand only, during the interval $[t_p, t_5]$ the system is subjected due to the effect of demand and deterioration and it vanishes at $t=t_5$.

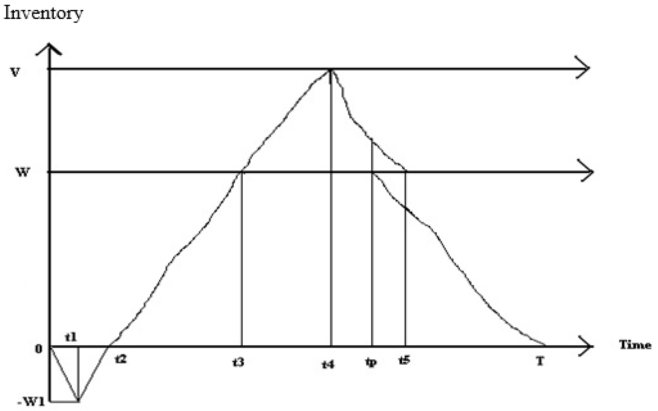

Fig.3. Graphical representation of a two-warehouse production system with stock-dependent demand.

The differential equations describing the inventory level are given as follows:

$$\frac{dS(t)}{dt} = -\alpha, 0 \le t \le t_1 = P-\alpha, t_1 \le t \le t_2 \tag{41}$$

With the boundary conditions, $S(t_1) = -W_1$, $S(t_2) = 0$ and

$$\begin{aligned} \frac{dI_1(t)}{dt} &= P-\alpha-\beta I_1(t), t_2 \le t \le t_3 \\ &= 0, t_3 \le t \le t_p \\ &= -\{\theta_1 - m(\xi)\} I_1(t), t_p \le t \le t_5 \\ &== -\alpha - \beta I_1(t) - \{\theta_1 - m(\xi)\} I_1(t), t_5 \le t \le T \end{aligned} \tag{42}$$

With the boundary conditions, $I_1(t_2) = 0, I_1(t_3) = W,\ I_1(T) = 0$ and

$$\begin{aligned} \frac{dI_2(t)}{dt} &= P-\alpha-\beta W, t_3 \le t \le t_4 \\ &= -\alpha - \beta W, t_4 \le t \le t_p \\ &= -\alpha - \beta W - \{\theta_2 - n(\lambda)\} I_2(t), t_p \le t \le t_5 \end{aligned} \tag{43}$$

With the boundary conditions $I_2(t_3) = 0, I_2(t_4) = V, I_2(t_5) = 0$.

The solutions of the differential equations in Eq.(41) are represented by

$$\begin{aligned} S(t) &= \alpha(t_1 - t) - W_1, 0 \le t \le t_1 \\ &= (P-\alpha)(t-t_2), t_1 \le t \le t_2 \end{aligned} \tag{44}$$

The solutions of the differential equations in Eq.(42) are represented by

$$\begin{aligned} I_1(t) &= \frac{P-\alpha}{\beta}[1 - e^{\beta(t_2 - t)}], t_2 \le t \le t_3 \\ &= W, t_3 \le t \le t_p \\ &= We^{\{\theta_1 - m(\xi)\}(t_p - t)}, t_p \le t \le t_5 \\ &= \frac{\alpha}{\beta + \{\theta_1 - m(\xi)\}}[e^{(\beta + \{\theta_1 - m(\xi)\})(T-t)} - 1], t_5 \le t \le T \end{aligned} \tag{45}$$

The solutions of the differential equations in Eq.(43) are represented by

$$\begin{aligned} I_2(t) &= (P-\alpha-\beta W)(t-t_3), t_3 \le t \le t_4 \\ &= (\alpha + \beta W)(t_4 - t) + V, t_4 \le t \le t_p \\ &= \frac{\alpha + \beta W}{\{\theta_2 - n(\lambda)\}}[e^{\{\theta_2 - n(\lambda)\}(t_5 - t)} - 1], t_p \le t \le t_5 \end{aligned} \tag{46}$$

Using continuity condition on the equations in Eq.(44) at $t = t_1$ we have,

$$t_2 = \frac{W_1}{P-\alpha} + t_1 \tag{47}$$

Using continuity conditions on the equations in Eq.(45) at $t = t_3$ and $t = t_5$ we have respectively,

$$t_3 = t_2 + \frac{1}{\beta}\log[\frac{P-\alpha}{P-\alpha-\beta W}] \tag{48}$$

And

$$T = t_5 + \frac{1}{\beta + \{\theta_1 - m(\xi)\}}\log(1 + \frac{W(\beta + \{\theta_1 - m(\xi)\})}{\alpha}e^{\{\theta_1 - m(\xi)\}(t_p - t_5)}) \tag{49}$$

Using continuity conditions on the equations in (46) at $t = t_p$ and boundary condition, at $t = t_4$, $I_2(t_4) = V$, we have respectively,

$$t_4 = t_3 + \frac{V}{P-\alpha-\beta W} \tag{50}$$

And

$$t_5 = t_p + \frac{1}{\{\theta_2 - n(\lambda)\}}\log(1 + \{\theta_2 - n(\lambda)\}(t_4 - t_p) + \frac{V\{\theta_2 - n(\lambda)\}}{\alpha + \beta W}) \tag{51}$$

Total amount of produced item is given by

$$P_i = \int_{t_1}^{t_4} P dt = P(t_4 - t_1) \tag{52}$$

Total Production Cost, $C_p = k \times P_i = kP(t_4 - t_1)$ (53)

Total Holding Cost is given by

$$C_h = c_{11}\int_{t_2}^{T} I_1(t)dt + c_{12}\int_{t_3}^{t_5} I_2(t)dt \tag{54}$$

$$= c_{11}\frac{P-\alpha}{\beta}[t_3 - t_2 + \frac{1}{\beta}(e^{\beta(t_2-t_3)} - 1)] + c_{11}W(t_p - t_3)$$
$$-c_{11}W\frac{1}{\{\theta_1 - m(\xi)\}}[e^{\{\theta_1 - m(\xi)\}(t_p - t_5)} - 1] +$$
$$\frac{c_{11}\alpha}{\beta + \{\theta_1 - m(\xi)\}}[\frac{1}{\beta + \{\theta_1 - m(\xi)\}}\{e^{(\beta + \{\theta_1 - m(\xi)\})(T - t_5)} - 1\} - (T - t_5)]$$
$$+c_{12}(P - \alpha - \beta W)[\frac{1}{2}(t_4{}^2 + t_3{}^2) - t_3t_4] + c_{12}(\alpha + \beta W)[t_4t_p - t_4{}^2 - \frac{1}{2}(t_p{}^2 - t_4{}^2)]$$
$$+c_{12}V(t_p - t_4) + c_{12}\frac{\alpha + \beta W}{\{\theta_2 - n(\lambda)\}}[\frac{1}{\{\theta_2 - n(\lambda)\}}\{e^{\{\theta_2 - n(\lambda)\}(t_5 - t_p)} - 1\} - (t_5 - t_p)]$$

Total Shortages Cost is given by

(55)

Total preservation cost is given by

$$C_{pr} = (T - t_p)\xi + (t_5 - t_p)\lambda \quad (56)$$

Total deteriorated items,

$$D_1 = \{\theta_1 - m(\xi)\}\int_{t_p}^{T} I_1(t)dt + \{\theta_2 - n(\lambda)\}\int_{t_p}^{t_5} I_2(t)dt \quad (57)$$

$$= -W[e^{\theta_1(t_p - t_5)} - 1]$$
$$+\frac{\{\theta_1 - m(\xi)\}\alpha}{\beta + \{\theta_1 - m(\xi)\}}[\frac{1}{\beta + \{\theta_1 - m(\xi)\}}\{e^{(\beta + \{\theta_1 - m(\xi)\})(T - t_5)} - 1\} - (T - t_5)]$$
$$+(\alpha + \beta W)[-\frac{1}{\{\theta_2 - n(\lambda)\}}\{1 - e^{\{\theta_2 - n(\lambda)\}(t_5 - t_p)} - (t_5 - t_p)]$$

Total selling price, $S_t = s(P_i - D_1)$ (58)

Total profit, $P_T = S_t - C_p - C_h - C_s - C_{pr}$ (59)

The average Profit, $AP = \frac{P_T}{T}$ (60)

Case IV: $t_5 \le t_p \le T$

In case-IV, during the interval $[t_2,t_3]$ the system is subjected due to the effect of production and demand only in *PW* and inventory items begin to accumulate up to *W* units. During the time interval $[t_3,t_5]$ the stock level at *PW* remains unchanged, during the interval $[t_5,t_p]$ the inventory in *PW* are also lowered at a level below *W* due to demand only. Also in the time interval $[t_p,T]$ the remaining stock in *PW* are then fully depleted at $t = T$ due to both demand and deterioration. After $t=t_3$ the produced quantity exceeding *W* must be stored in *SW* and production continuous up to $t=t_4$ (cf. Fig. 4) and inventory level of SW reaches its maximum level *V*. In *SW* during the time interval $[t_3,t_4]$ the system is subjected due to the effect of production and demand only, during the time interval $[t_4,t_5]$ the system is subjected due to the effect of demand only and it vanishes at $t=t_5$.

The differential equations describing the inventory level are given as follows:

$$\frac{dS(t)}{dt} = -\alpha, 0 \le t \le t_1 = P - \alpha, t_1 \le t \le t_2 \quad (61)$$

With the boundary conditions, $S(t_1) = -W_1$, $S(t_2) = 0$.

$$\frac{dI_1(t)}{dt} = P - \alpha - \beta I_1(t), t_2 \le t \le t_3$$
$$= 0, t_3 \le t \le t_5$$
$$= -\alpha - \beta I_1(t), t_5 \le t \le t_p$$
$$= -\alpha - \beta I_1(t) - \{\theta_1 - m(\xi)\}I_1(t), t_p \le t \le T \quad (62)$$

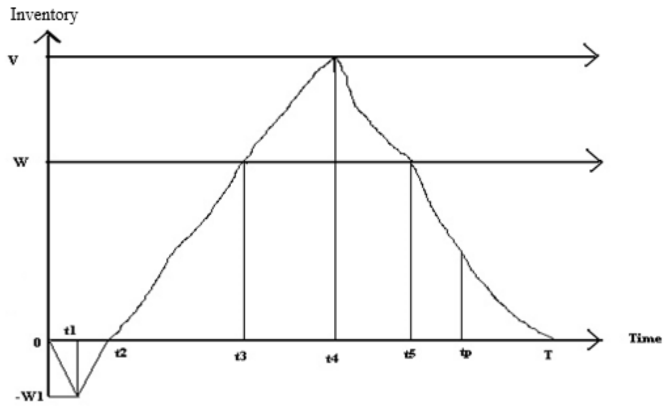

Fig.4. Graphical representation of a two-warehouse production system with stock-dependent demand

With the boundary conditions, $I_1(t_2) = 0$, $I_1(t_3) = W$, $I_1(T) = 0$ and

$$\frac{dI_2(t)}{dt} = P - \alpha - \beta W, t_3 \le t \le t_4$$
$$= -\alpha - \beta W, t_4 \le t \le t_5 \quad (63)$$

With the boundary conditions $I_2(t_3) = 0$, $I_2(t_4) = V$, $I_2(t_5) = 0$.

The solutions of the differential equations in Eq.(61) are represented by

$$S(t) = \alpha(t_1 - t) - W_1, 0 \le t \le t_1$$
$$= (P - \alpha)(t - t_2), t_1 \le t \le t_2 \quad (64)$$

The solutions of the differential equations in Eq.(62) are represented by

$$I_1(t) = \frac{P-\alpha}{\beta}[1 - e^{\beta(t_2 - t)}], t_2 \le t \le t_3$$
$$= W, t_3 \le t \le t_5$$
$$= -\frac{\alpha}{\beta}[1 - e^{\beta(t_5 - t)}] + We^{\beta(t_5 - t)}, t_5 \le t \le t_p \quad (65)$$
$$= \frac{\alpha}{\beta + \{\theta_1 - m(\xi)\}}[e^{(\beta + \{\theta_1 - m(\xi)\})(T - t)} - 1], t_p \le t \le T$$

The solutions of the differential equations in Eq.(63) are represented by

$$I_2(t) = (P - \alpha - \beta W)(t - t_3), t_3 \le t \le t_4$$
$$= (\alpha + \beta W)(t_5 - t), t_4 \le t \le t_5 \quad (66)$$

Using continuity condition on the equations in Eq.(64) at $t = t_1$ we have,

$$t_2 = \frac{W_1}{P - \alpha} + t_1 \quad (67)$$

Using continuity conditions on the equations in Eq.(65) at $t = t_3$ and $t = t_5$ we have respectively,

$$t_3 = t_2 + \frac{1}{\beta}\log[\frac{P-\alpha}{P-\alpha-\beta W}] \quad (68)$$

And

$$T = t_p + \frac{1}{\beta+\{\theta_1 - m(\xi)\}}\log\left[\begin{matrix}1+\frac{W(\beta+\{\theta_1-m(\xi)\})}{\alpha}e^{\beta(t_5-t_p)} \\ -\frac{\beta+\{\theta_1-m(\xi)\}}{\beta}(1-e^{\beta(t_5-t_p)})\end{matrix}\right] \quad (69)$$

Using boundary condition, at $t = t_4$, $I_2(t_4) = V$, we have respectively,

$$t_4 = t_3 + \frac{V}{P-\alpha-\beta W} \quad (70)$$

And

$$t_5 = t_4 + \frac{V}{\alpha+\beta W} \quad (71)$$

Total amount of produced item is given by

$$. \ P_i = \int_{t_1}^{t_4} P dt = P(t_4 - t_1) \quad (72)$$

Total Production Cost, $C_p = k \times P_i = kP(t_4 - t_1)$ (73)

Total Holding Cost is given by

$$C_h = c_{11}\int_{t_2}^{T} I_1(t)dt + c_{12}\int_{t_3}^{t_5} I_2(t)dt$$

$$= c_{11}\frac{P-\alpha}{\beta}(t_3-t_2) + c_{11}\frac{P-\alpha}{\beta^2}\{e^{\beta(t_2-t_3)}-1\} + c_{11}W(t_5-t_3)$$

$$-c_{11}\frac{\alpha}{\beta}(t_p-t_5) - c_{11}\frac{\alpha}{\beta^2}\{e^{\beta(t_5-t_p)}-1\} - c_{11}\frac{W}{\beta}\{e^{\beta(t_5-t_p)}-1\}$$

$$-c_{11}\frac{\alpha}{(\beta+\{\theta_1-m(\xi)\})^2}\{1-e^{(\beta+\{\theta_1-m(\xi)\})(T-t_p)}\} - c_{11}\frac{\alpha}{\beta+\{\theta_1-m(\xi)\}}(T-t_p)$$

$$+c_{12}[(P-\alpha-\beta W)(\frac{t_4^{\ 2}}{2}+\frac{t_3^{\ 2}}{2}-t_3t_4) + (\alpha+\beta W)(\frac{t_5^{\ 2}}{2}+\frac{t_4^{\ 2}}{2}-t_5t_4)] \quad (74)$$

Total Shortages Cost is given by $C_s = c_2\int_0^{t_2} S(t)dt$

$$= c_2[\frac{\alpha t_1^{\ 2}}{2} - W_1t_1] + c_2(P-\alpha)[\frac{t_2^2}{2} - \frac{t_1^2}{2} + t_1t_2] \quad (75)$$

Total preservation cost is given by

$$C_{pr} = (T - t_p)\xi \quad (76)$$

Total deteriorated items,

$$D_1 = \{\theta_1 - m(\xi)\}\int_{t_p}^{T} I_1(t)dt = -\frac{\alpha\{\theta_1-m(\xi)\}}{(\beta+\{\theta_1-m(\xi)\})^2}$$
$$[1-e^{(\beta+\{\theta_1-m(\xi)\})(T-t_p)}] - \frac{\alpha\{\theta_1-m(\xi)\}}{\beta+\{\theta_1-m(\xi)\}}(T-t_p) \quad (77)$$

Total selling price, $S_t = s(P_i - D_1)$ (78)

Total profit, $P_T = S_t - C_p - C_h - C_s - C_{pr}$ (79)

The average Profit, $AP = \frac{P_T}{T}$ (80)

5. SOLUTION PROCEDURE - MODIFIED GENETIC ALGORITHM

Here we propose a nobility in crossover operator of the GA using probabilistic selection (Boltzmann Probability), IVF comparison crossover and sigmoid random mutation among a set of potential solutions to obtain a new set of solutions. The proposed algorithm is continued until terminating conditions are encountered. The proposed modified GA and its procedures are presented further.

5.1 PROBABILISTIC SELECTION

In the present study, we proposed a predefined value, for example probability of selection parameter (p_s). Each solution randomly generates a number $r \in [0,1]$. If $r < p_s$, the corresponding chromosome is stored to form the matting pool. To maximize the profit, selecting a chromosome in the neighborhood of the maximum solution of the entire solution space, so it propagates a higher convergency rate. From the initial population to the best fitted chromosome for EPQ model is chosen as the most maximum fitness (because EPQ model is a maximizing model) value. To form the matting pool, we use the Boltzmann-Probability of each chromosome from the initial population.

5.2 IVF COMPARISON CROSSOVER

In IVF cases, in additional to the original parents (father and mother), a surrogate mother actively participates in the process of production a child. A new approach with three parents (first two are original parents and the third one say dummy parent) has been used to produce offspring. In the proposed crossover strategy, three parents (i.e., chromosomes) are randomly selected from the matting pool. Further, comparing the profits from each chromosome. On the basis of the above idea, we construct the crossover algorithm as follows:

Step 1: Start the algorithm.

Step 2: Initialize the three parents (P_{r_1} , P_{r_2} and P_{r_3}) depending on the probability of crossover p_c.

Step 3: Generate a random number in between zero and the node (e.g., a_i).

Step 4: Update parents by placing a_i in the position of each parent.

Step 5: In the first child place a_i in the first position.

Step 6: Find the minimum cost between a_i and each next node of given parents.

Step 7: Place s_1 (for example) of the first child at the second place, and update each parent with s_1 in the second place.

Step 8: Repeat steps 6 and 7 up to the end of the nodes.

Step 9: End the algorithm.

Sigmoid random mutation : In the present study, in the place of constant p_m, we dynamically update the mutation probability by making it decreasing with generations. In our current implementation mechanism, sigmoid function returns a value in [0, 1] depending on generation number and adjustment parameter

λ. We understand the diminishing requirement of perturbation as the quality of solution increases with generation. Following the expression of sigmoid function, it returns a value between 0 and 1, used as the probability of mutation. Increasing generation compels the value of p_m to decrease with increasing return . In the initial few iterations, high value of p_m maintain the exploration in the solution space and gradually it stabilizes for convergence. The mutation process is as follows:

(a) *Generation dependent* p_m: To acquire the probability of mutation (p_m) by $p_m = \lambda(1+e^{-g}), \lambda \in [0,1]$, where g is the current generation number.

(b) *Selection for Mutation*: To select the chromosome for mutation, produce a random number $r \in [0,1]$. When $r < p_m$, the corresponding chromosome is selected for mutation. Here, p_m decreases smoothly as the generation increases. In a single point random mutation, two solutions are randomly chosen from each chromosome and interchanged to create the new offspring set.

5.3 MODIFIED GA PROCEDURE

Procedure name: Modified GA.

Input: Maxgen (S_0), population size (pop_size), probability of selection (p_s), probability of crossover (p_c), probability of mutation (p_m).

Output: Optimum and near-optimum solutions.

Step 1. Start.

Step 2. Set the initial generation $t \leftarrow 0$.

Step 3. (Initialization) Randomly generate initial population $p(t)$ where $f(x_i)$, i=1,2,..,pop_size are the chromosomes, and a_k number of nodes in each chromosome represent a solution of the EPQ.

Step 4. Evaluate the fitness of each solution of the initial population $p(t)$.

Step 5. Check the condition while ($t \le S_0$) do up to step 14.

Step 6. Update the generation $t \leftarrow t+1$.

Step 7. Selection Procedure.

Step 8. Determine the Boltzmann Probability (p_B).

Step 9. Create the mating pool based on p_s and p_B .

Step 10. Invoke the crossover procedure based on p_c.

Step 11. Invoke mutation based on p_m.

Step 12. Store new offspring into the offspring set.

Step 13. Compare the fitness and store the local and near-optimum solutions.

Step 14. Repeat Steps 5 to 14.

Step 15. (Optimum Solution) Store global optimum and near-optimum results.

Step 16. Stop.

The above model is solved by using Modified GA approach, discussed in [5]. Our MGA consists of parameters, population size N=50, probability of crossover (p_c)= 0.2, probability of mutation (p_m) = 0.2, and maximum generation = 50. A real number presentation is used here. In this representation, each chromosome X is a string of n numbers of MGA, which denote the decision variable. For each chromosome X, every gene, which represents the independent variables, is randomly generated between their boundaries until it is feasible. In this MGA, arithmetic crossover and random mutation are applied to generate new off springs.

6. NUMERICAL ANALYSIS:

The optimal average profit of the above said two warehouses production model with constant production rate for non-instantaneous deterioration goods having stock dependent demand and preservation technology has been treated with numerical data. An example is presented to illustrate the effect of the model developed here with the numerical data.

Here s = 4.5, k = 2.0, c_{11} = 0.15, c_{12} = 0.10, c_2 = 0.15, θ_1 =0.15, θ_2 =0.17, W_1=50, W=350, V=150, b_1 = 0.05, b_2 = 0.06 in appropriate units and $m(\xi) = \theta_1(1-e^{-b_1\xi})$, $n(\lambda) = \theta_2(1-e^{-b_2\lambda})$.

Now according to the proposed computation procedure(GA) the results listed in the following tables.

From the numerical illustrations in Table.1, it is observed that, for fixed α=25 and β=0.09 average profit increases when P increases and these observations are realistic.

6.1 SENSITIVITY ANALYSIS

To discuss the importance of preservation technology, here we take the sensitivity analysis for different values of α, β and P with and without preservation technology (Table.2).

Table.1. Optimal solutions for illustrated example of cases I, II, III and IV

P	α	β	Case-I					Case-II				
			t_1	t_p	ξ	λ	Av. Pr.	t_1	t_p	ξ	λ	Av. Pr.
87.5	25	0.09	1.140	8.885	1.196	7.692	49.497	1.007	12.875	11.136	2.904	60.472
90.0	25	0.09	1.070	9.208	1.196	7.468	50.956	1.147	13.749	11.094	1.532	60.834
92.5	25	0.09	1.105	9.721	1.280	6.166	51.472	1.063	14.490	11.136	1.042	61.776
P	α	β	Case-III					Case-IV				
			t_1	t_p	ξ	λ	Av. Pr.	t_1	t_p	ξ	λ	Av. Pr.
87.5	25	0.09	1.434	15.972	1.084	1.042	65.642	1.000	19.943	1.084	1.042	73.470
90.0	25	0.09	1.161	16.352	1.084	1.042	66.599	1.014	19.715	1.000	1.028	74.184
92.5	25	0.09	1.014	16.979	1.084	1.042	67.272	1.014	19.715	1.000	1.028	74.994

Table.2. Sensitivity analysis of the demand parameter α when β = 0.09 (a) With Preservation Technology

P	α	**Case-I**					**Case-II**				
		t_1	t_p	ξ	λ	**Av. Pr.**	t_1	t_p	ξ	λ	**Av. Pr.**
87.5	20.0	2.596	9.624	1.000	7.790	34.628	1.105	11.735	9.876	3.548	46.110
	22.5	1.525	8.885	1.000	7.790	42.812	1.063	12.305	10.660	1.042	53.475
	25.0	1.140	8.885	1.196	7.692	49.497	1.007	12.875	11.136	2.904	60.472
90.0	20.0	1.525	8.885	1.000	7.790	37.175	1.063	12.305	9.680	1.042	47.029
	22.5	1.420	9.170	1.140	7.286	43.057	1.007	12.875	10.604	2.904	54.031
	25.0	1.070	9.208	1.196	7.468	50.956	1.147	13.749	11.094	1.532	60.834
92.5	20.0	1.420	9.170	1.140	7.286	37.379	1.007	12.875	9.456	2.428	47.564
	22.5	1.070	9.208	1.196	7.146	45.140	1.147	13.749	10.450	1.532	54.437
	25.0	1.105	9.721	1.280	6.166	51.472	1.063	14.490	11.136	1.042	61.776
P	α	**Case-III**					**Case-IV**				
		t_1	t_p	ξ	λ	**Av. Pr.**	t_1	t_p	ξ	λ	**Av. Pr.**
87.5	20.0	1.350	14.927	1.084	1.042	52.858	1.014	19.962	1.000	1.042	61.997
	22.5	1.105	15.098	1.084	1.042	59.930	1.000	19.658	1.000	1.042	67.530
	25.0	1.434	15.972	1.084	1.042	65.642	1.000	19.943	1.084	1.042	73.470
90.0	20.0	1.238	15.402	1.084	1.042	53.457	1.014	19.715	1.000	1.028	62.369
	22.5	1.014	15.573	1.084	1.042	60.283	1.014	19.962	1.084	1.042	68.601
	25.0	1.161	16.352	1.084	1.042	66.599	1.014	19.715	1.000	1.028	74.184
92.5	20.0	1.434	16.200	1.084	1.042	53.857	1.014	19.829	1.084	1.042	62.940
	22.5	1.014	16.352	1.084	1.042	60.791	1.014	19.886	1.084	1.042	69.137
	25.0	1.014	16.979	1.084	1.042	67.272	1.014	19.715	1.000	1.028	74.994

(b) Without Preservation Technology

P	α	**Case-I**			**Case-II**		
		t_1	t_p	**Ave. Profit**	t_1	t_p	**Ave. Profit**
87.5	20.0	2.008	9.037	11.785	1.049	8.106	12.346
	22.5	1.021	8.410	20.066	1.014	8.410	20.840
	25.0	1.420	9.151	30.362	1.140	8.866	30.852
90.0	20.0	2.092	9.455	15.185	1.161	8.524	15.907
	22.5	1.420	9.151	24.965	1.140	8.866	25.370
	25.0	1.070	9.208	36.640	1.014	9.151	36.773
92.5	20.0	1.420	9.151	19.756	1.140	8.866	20.082
	22.5	1.070	9.208	30.790	1.014	9.151	30.907
	25.0	1.217	9.816	43.424	1.014	9.626	43.788
P	α	**Case-III**			**Case-IV**		
		t_1	t_p	**Ave. Profit**	t_1	t_p	**Ave. Profit**
87.5	20.0	1.014	14.556	54.431	1.000	19.962	63.157
	22.5	1.056	15.060	60.983	1.000	19.962	69.047
	25.0	1.028	15.573	67.748	1.007	19.981	74.656
90.0	20.0	1.161	15.326	54.524	1.014	19.962	63.753
	22.5	1.014	15.649	61.409	1.000	19.962	69.848
	25.0	1.126	16.390	67.830	1.000	19.981	75.707
92.5	20.0	1.077	15.877	55.037	1.014	19.981	64.289
	22.5	1.014	16.390	61.830	1.014	19.962	70.421

	25.0	1.014	17.093	68.536	1.000	19.962	76.464

Table.4. Sensitivity analysis of the demand parameter β when P=90 (a) With Preservation Technology

α	β	**Case-I**					**Case-II**				
		t_1	t_p	ξ	λ	**Av. Pr.**	t_1	t_p	ξ	λ	**Av. Pr.**
20.0	0.085	1.081	8.323	1.000	8.238	34.299	1.112	12.058	9.876	1.042	44.045
	0.090	1.525	8.885	1.000	7.790	37.175	1.063	12.305	9.680	1.042	47.029
	0.095	1.161	8.695	1.196	7.146	39.560	1.161	12.723	9.876	2.022	49.761
22.5	0.085	2.295	9.873	1.196	7.888	39.168	1.161	12.685	10.660	2.022	50.660
	0.090	1.420	9.170	1.140	7.286	43.057	1.007	12.875	10.604	2.904	54.031
	0.095	1.448	9.341	1.196	6.852	46.473	1.014	13.274	10.660	2.358	57.286
25.0	0.085	1.941	9.921	1.196	7.468	45.738	1.014	13.179	11.374	4.738	57.693
	0.090	1.070	9.208	1.196	7.468	50.956	1.147	13.749	11.094	1.532	60.834
	0.095	2.514	10.826	1.196	4.612	54.914	1.000	14.034	11.192	3.940	64.404
α	β	**Case-III**					**Case-IV**				
		t_1	t_p	ξ	λ	**Av. Pr.**	t_1	t_p	ξ	λ	**Av. Pr.**
20.0	0.085	1.434	15.364	1.084	1.042	50.148	1.014	19.962	1.084	1.042	59.950
	0.090	1.238	15.402	1.084	1.042	53.457	1.014	19.715	1.000	1.028	62.369
	0.095	1.014	15.402	1.084	1.042	56.804	1.014	19.962	1.084	1.000	65.162
22.5	0.085	1.049	15.459	1.084	1.042	57.595	1.014	19.867	1.084	1.000	65.939
	0.090	1.004	15.573	1.084	1.042	60.283	1.014	19.962	1.084	1.042	68.601
	0.095	1.266	16.200	1.084	1.042	62.794	1.014	19.943	1.084	1.042	71.124
25.0	0.085	1.238	16.200	1.084	1.042	63.685	1.014	19.943	1.084	1.042	71.908
	0.090	1.161	16.352	1.084	1.042	66.599	1.014	19.715	1.000	1.028	74.184
	0.095	1.308	16.884	1.084	1.042	69.244	1.014	19.829	1.084	1.042	76.726

(b) Without Preservation Technology

α	β	**Case-I**			**Case-II**		
		t_1	t_p	**Ave. Profit**	t_1	t_p	**Ave. Profit**
20.0	0.085	1.525	8.752	9.740	1.196	8.448	9.835
	0.090	2.092	9.455	15.185	1.161	8.524	15.907
	0.095	1.189	8.714	21.417	1.189	8.733	21.900
22.5	0.085	1.245	8.828	18.523	1.280	8.866	18.602
	0.090	1.420	9.151	24.965	1.140	8.866	25.370
	0.095	1.147	9.037	32.149	1.049	8.942	32.373
25.0	0.085	1.238	9.208	29.000	1.105	9.094	29.112
	0.090	1.070	9.208	36.640	1.014	9.151	36.773
	0.095	1.070	9.398	44.443	1.049	9.417	44.495
α	β	**Case-III**			**Case-IV**		
		t_1	t_p	**Ave. Profit**	t_1	t_p	**Ave. Profit**
20.0	0.085	1.035	14.946	51.821	1.000	19.962	61.188
	0.090	1.161	15.326	54.524	1.014	19.962	63.753
	0.095	1.028	15.421	57.697	1.007	19.962	66.374
22.5	0.085	1.014	15.402	58.523	1.007	19.962	67.250
	0.090	1.014	15.649	61.409	1.000	19.962	69.848
	0.095	1.000	15.934	64.425	1.014	19.962	72.341

25.0	0.085	1.014	15.972	65.205	1.014	19.962	73.117
	0.090	1.126	16.390	67.830	1.000	19.981	75.707
	0.095	1.014	16.599	71.073	1.000	19.981	78.181

Sensitivity analyses are performed for different values of α, β and P. It is observed that if β is fixed for different values of α as P increases, average profit increases. And for the fixed value of P for different values of β as α increases, average profit increases. All these observations agree with the reality (Table.2(a), Table.2(b), Table.3(a), Table.3(b)).

6.2 DISCUSSIONS

In sensitivity analyses we observe that preservation technology plays a vital role in production based economic quantity model with deterioration. In Table.2(a) and Table.2(b) when β is fixed for different values of α as P increases and in Table.3(a) and Table.3(b) for the fixed value of P for different values of β as α increases, it is observed that in Case-I and Case-II preservation technology is beneficial as time span of deterioration is longer than the Case-III and Case-IV. But in Case-III and Case-IV preservation technology is not effective as the time span of deterioration is small and the setup cost for preservation technology is high.

7. CONCLUSION AND FUTURE SCOPE

In this paper, a realistic production-inventory model for non-instantaneous deteriorating items with two warehouses has been considered under stock-dependent demand and fully backlogged shortages using preservation technology over an infinite time horizon. The model is solved numerically by Modified Genetic Algorithm (MGA) and then compared. Sensitivity analyses are also performed for different parameters to study the effect of the decision variables. Finally, for future research, one can incorporate more realistic assumptions in the proposed model considering stochastic nature of demand, and production rate with more warehouses. The similar problems can be formulated with multi-items with budget and space constraints.

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