DOI: 10.21917/ijsc.2010.0016

A NEW APPROACH ON SHORTEST PATH IN FUZZY ENVIRONMENT

A. Nagoorgani[1] and A. Mumtaj Begam[2]

[1]*P.G & Research Department of Mathematics, Jamal Mohamed College, Tamil Nadu, India*
E-mail: ganijmc@yahoo.co.in

[2]*Department of Computer Science & Information Technology, Dr. Umayal Ramanathan College for Women, Tamil Nadu, India*
E-mail: mumtajbegam73@yahoo.com

Abstract

This paper introduces a new type of fuzzy shortest path network problem using triangular fuzzy number. To find the smallest edge by the fuzzy distance using graded mean integration representation of generalized fuzzy number for every node. Thus the optimum shortest path for the given problem is obtained.

Keywords:
Fuzzy Number, Graded Mean Integration Representation, Fuzzy Distance, Order Relation

1. INTRODUCTION

The shortest path problem was one of the first network problems studied in terms of operations research. In some applications, the numbers associated with the edges of networks may represent characteristics other than lengths, and we may want the optimum paths, where optimum can be defined by different criteria. The shortest-path problem is the most common problem in the whole class of optimum path problems. Consider the edge weight of the network as uncertain; which means that it is either imprecise or unknown.

In 1965, Zadeh [1] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line. Okada and Gen (1994) first tried to resolve the issue of incomparability of intervals using a ranking strategy of their own but their method seems to work in an *ad hoc* manner and so the result is not always unique and self-explanatory when used in an algorithm for solving the shortest path problem. In 1997, Heilpern [5] proposed three definitions of the distance between two fuzzy numbers. These include that mean distance method is generated by expect values of fuzzy numbers, distance method is combined by a Minkowski distance and the h-levels of the closed intervals of fuzzy numbers, and geometrical distance method is based on the geometrical operation of trapezoidal fuzzy numbers. All of them use real number to calculate the distance. Here, the fuzzy distance between two trapezoidal fuzzy numbers is measured by using graded mean integration representation [2, 3, 4] and the fuzzy number.

2. PREMILINARIES

2.1TRIANGULAR FUZZY NUMBER

The fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is a triangular number, denoted by (a_1, a_2, a_3), its membership function $\mu_{\tilde{a}}$ is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & , if\ 0 \le x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & , if\ a_1 \le x \le a_2 \\ 1 & , if\ x = a_2 \\ \dfrac{x-a_3}{a_2-a_3} & , if\ a_2 \le x \le a_3 \\ 0 & , if\ x \ge a_3 \end{cases}$$

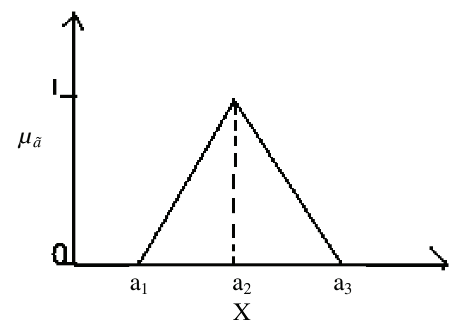


Fig.1. Membership function of a fuzzy number $\tilde{a}$

2.2 POSITIVE FUZZY NUMBER

A fuzzy number $\tilde{a}$ is called a positive fuzzy number if its membership function is such that $\mu_{\tilde{a}}(x) = 0\ \forall\ x < 0$.

2.3 ADDITION OF TWO FUZZY NUMBERS

Let $\tilde{a}$ and $\tilde{b}$ two triangular fuzzy numbers. An addition of fuzzy numbers is $\tilde{c} = \tilde{a} \oplus \tilde{b}$ defined by the membership function.

$\mu_{\tilde{a}}(t) = Sup\ min\{\mu_{\tilde{a}}(u), \mu_{\tilde{b}}(v)\}$
$t = u + v$

Addition of $\tilde{a}$ and $\tilde{b}$ is represented as $(a_1, a_2, a_3) \oplus (b1, b_2, b_3)$. Therefore, the function principle is

$$\tilde{c} = \tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)$$
$$= (a_1+b_1, a_2+b_2, a_3+b_3).$$

2.4 ORDER RELATION

Consider an order relation among fuzzy numbers. A variety of methods for the ordering and ranking of fuzzy numbers has been proposed in the literature. These methods have been reviewed and tested by Bortolan and Degani [6].

Rule:

Let $\tilde{a}$ and $\tilde{b}$ two triangular fuzzy numbers such that $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b}$=(b1, b_2, b_3).Then $\tilde{a} \leq \tilde{b}$ iff the following inequalities are

1. $a_1 \leq b_1$ 2. $a_2 \leq b_2$ 3. $a_3 \leq b_3$

Suppose A = (a_1, a_2, a_3) is a triangular fuzzy number. The graded mean integration representation of A becomes

$$P(A)=\frac{a_1+4a_2+a_3}{6}$$

2.5 THE FUZZY DISTANCE

Let A= (a_1, a_2, a_3), B = (b_1, b_2, b_3) be two triangular fuzzy numbers, and their Graded mean integration representation are P(A), P(B) respectively. Assume

Si = (a_i -P(A)+ b_i –P(B))/2, i =1,2,3;

Ci = | P(A)-P(B) | + Si, i =1,2,3;

then the fuzzy distance of A,B is C = (c_1, c_2, c_3) .

3. ALGORITHM

Initialization

Step-1:

Let S = (0,0,0)

Compute Fuzzy distance

Step-2:

To calculate the graded mean value for each adjacent vertex from the current vertex.

Graded mean value by

$$P(A)=\frac{a+4b+c}{6}$$

where A = (a, b, c) is the edge weight of the node (i, j).

Step-3:

To evaluate the fuzzy distance between two edges using the following equations

$S_i=[a_i\text{-}P(A)+b_i\text{-}P(B)/2]$ $\quad i=1,2,3$

and

$c_i=|P(A)\text{-}P(B)|+S_i$ $\quad i=1,2,3$

and then the fuzzy distance of A, B is C = (c1, c2, c3) for each adjacent node.

Step-4:

Compare fuzzy distance among all adjacent nodes using order relation.

1. $a_i \leq a_j$ 2. $b_i \leq b_j$ 3. $c_i \leq c_j$.

where i = 1,2,3 and j = 1,2,3.

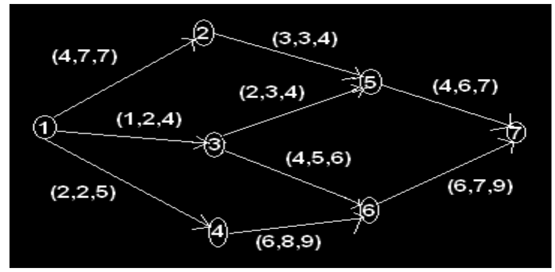
Step-5:

Select the smallest fuzzy distance edge among them and continue until destination node is reached.

Step-6:

Finally, the Fuzzy shortest path is obtained for the given network.

3.1 NUMERICAL EXAMPLE



In this example, the given graph contains 7 nodes and 9 edges. Start with node 1, the distance s = (0,0,0). From node 1, the adjacent nodes are 2, 3 & 4.

$$P(1,1) = (0,0,0) = \frac{0}{6} = 0 = 0$$

$$P(1,2) = (4,7,7) = \frac{4+28+7}{6} = \frac{39}{6} = 6.5$$

$$S1 = \frac{(0-0+4-6.5)}{2} = -1.25$$

$$S2 = \frac{(0-0+7-6.5)}{2} = +0.25$$

$$S3 = \frac{(0-0+7-6.5)}{2} = +0.25$$

$$C1 = |0-6.5| + (-1.25) = 5.25$$

$$C2 = |0-16.5| + (0.25) = 6.75$$

$$C3 = |0-6.5| + (0.25) = 6.75$$

C = (5.25,6.75,6.75)

$$P(1,1) = (0,0,0) = \frac{0}{6} = 0 = 0$$

$$P(1,3) = (1,2,4) = \frac{1+8+4}{6} = \frac{13}{6} = 2.17$$

$$S1 = \frac{(0-0+2.17-1)}{2} = 0.585$$

$$S2 = \frac{(0-0+2.17-2)}{2} = 0.085$$

$$S3 = \frac{(0-0+2.17-4)}{2} = -0.915$$

$$C1 = |0-2.17| + 0.585 = 2.755$$

DOI: 10.21917/ijsc.2010.0016

C2 = $\left|0-2.17\right|+0.085=2.255$

C3 = $\left|0-2.17\right|+0.915=1.255$

C = (2.755,2.255,1.255)

P(1,1) = (0,0,0) = $\frac{0}{6}=0=0$

P(1,4) = (2,5,5) = $\frac{2+8+5}{6}=\frac{15}{6}=2.5$

S1 = $\frac{(0-0+2.5-2)}{2}=0.25$

S2 = $\frac{(0-0+2.5-2)}{2}=0.25$

S3 = $\frac{(0-0+2.5-5)}{2}=-1.25$

C1 = $\left|0-2.5\right|+0.25=2.75$

C2 = $\left|0-2.5\right|+0.25=2.75$

C3 = $\left|0-2.5\right|+(-1.25)=1.25$

C = (2.75,2.75,1.25)

In order to compare (1,2), (1,3) & (1,4) fuzzy distance among the adjacent edges. We get (1,4) is the smallest among them according to order relation. From node 4, the only one adjacent node is 6. Then choose (4,6) edge and then from node 6, the only one adjacent node is 7 which is the destination node. Now the process is terminated and the fuzzy shortest path 1 – 4 – 6 – 7 is obtained.

It can be easily seen that the procedure is very simple and needs light computational load. This example shows how the algorithm works.

4. CONCLUSIONS

This paper defines a solution for a shortest path problem with fuzzy triangular number. The main issue dealt with was developing a fuzzy distance for triangular fuzzy number and with minimum number of process steps. This algorithm can be implemented using fuzzy numbers graded mean integration chosen by the decision maker, the algorithm can return a single path as solution. This algorithm is executed for various network and verified. It provides the better output for different types of network. Such as the network may have more number of vertices or else more number of edges.

REFERENCES

[1] L. A. Zadeh, 1965, “Fuzzy set”, *Information and Control,* Vol.8, pp. 338-353.

[2] S. H. Chen and C. H. Hsieh, 1998, “Graded mean integration representation of generalized fuzzy number”, *Proc. of Conference of Taiwan Fuzzy System Association*, Taiwan.

[3] Shan Huo Chen and Chih Hsun Hsieh, 1999, “Graded Mean Integration Representation of Generalized Fuzzy Number”, *Journal of the Chinese Fuzzy System Association*, Vol. 5, No. 2, pp. 1-7.

[4] Shan Huo Chen and Chih Hsun Hsieh, 2000, “Representation, Ranking, Distance, and Similarity of L-R type fuzzy number and Application”, *Australia Journal of Intelligent Information Processing Systems*, Vol. 6, No. 4, pp.217-229.

[5] S. Heilpern, 1997, “Representation and application of fuzzy numbers”, *Fuzzy sets and Systems*, Vol. 91, No.2, pp.259-268.

[6] Bortolan G. and Degani. R, 1985, “A review of some methods for ranking fuzzy subsets”, Fuzzy sets and Systems, Vol.15, pp.1-19.