

CSDE: A CORRELATION-BASED EVOLUTIONARY APPROACH FOR 3D FACE DEPTH ESTIMATION

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Abstract

Estimation of Depth (z-coordinate) of 3D face from 2D (x-, y-) face images based on similarity transform is an optimization problem. In this work Correlation Scale Factor Differential Evolution (csDE) is proposed and used to estimate the optimal depth values which represent the z-coordinate. The different correlations considered to compute the scale factor of Differential Evolution are Spearman's Rho, Kendall's Tau and Pearson Correlation Coefficient. The proposed algorithm is implemented in MATLAB and empirical study is conducted on 2D images taken in lab and 3D Bosphorus database images. Similarity measure is computed between the estimated depth values and Candidate Face model depth values for the 2D images captured in lab. In the case of 3D Bosphorus database similarity measure is computed between the estimated and true depth values provided with the database. The similarity measure obtained using Correlation Scale Factor Differential Evolution for the sample images of 3D Bosphorus database is compared with other similar estimation algorithms.

Keywords:

Structure-from-Motion, Depth Estimation, Orthographic Projection, Differential Evolution, Computer Vision

1. INTRODUCTION

Processing face image is very important in wide range of applications: bio-metrics, computer vision, plastic surgery, gaming industry, face animation and face tracking. The algorithms used in most of the applications require dealing with variation in lighting, pose and expression variation. 2D systems with acceptable performance have been developed in the light of above variations [1]-[4]. Further to improve the performance and requirement of robust processing systems initiated to move from 2D to 3D. 3D data provides geometry invariant to view point and illumination in addition to texture. 3D models are widely used in face recognition/authentication [5]-[8]. An example 3D face model of Bosphorus database is shown in Fig.1.



Fig.1. Textured 3D Model of One Subject of Bosphorus Database

Presently, to acquire the 3D data there are two main stream approaches. The first one uses active sensors based on triangulation or time of flight. These sensors require subject cooperation, post processing and the high cost limit their wide

spread application. The second one is based on developing algorithms to process multiple view 2D images to reconstruct a 3D model [4], [9]-[13].

In surveillance applications 2D cameras are used to ensure security and it is not always possible to get multiple images of subject under consideration to reconstruct a 3D model. An effective 3D model reconstruction algorithm based on single 2D image will be a valuable tool to be used in surveillance application toward law enforcement. In [14], a comprehensive review is presented on 3D reconstruction algorithms based on single image per person. The critical issue to be considered in developing 3D reconstruction algorithm based on single 2D image is the efficient design of the optimization algorithm. The 3D models reconstructed considering the above issue can greatly improve the performance of existing 2D or 3D face image processing systems.

3D-shape reconstruction algorithms developed based on single 2D image are Shape from Shading [15]-[17], 3D Morphable Model [5]-[8], [18], Structure-from-motion [4], [9]-[13], [19], etc. Shape from shading recovers the shape geometry based on reflectance models by exploiting the shading information. To recover a unique solution various constraints including brightness, smoothness, and integrability are explored in sequence and still existence or uniqueness is still a research problem [20].

3D Morphable Model (3DMM) [5] is the foundation of the analysis-by-synthesis framework. In this model 3D point cloud of face is obtained using laser scanner. The 3D data is decomposed into a linearly convex combination of shape and textures. This decomposition is fully controlled by shape and texture parameters. Further statistical signal processing techniques, such as principal component analysis is used to obtain the feature subspace, regarded as generic 3D model.

Structure-from-motion (SFM) [4], [9]-[13] is a popular approach based on inverting the projection process. A special case of this projection is when the surface to scanning distance is small and is modelled as orthographic projection. In the case of availability of multiple view of 2D images SFM is used to recover the depth of shape under consideration. Koo and Lam [11] employed genetic algorithm and estimated the pose followed by depth of important face features utilizing multiple view 2D face images. However, the use of genetic algorithm needs specification of population size, mutation scheme, selection and crossover and fine tuning all these parameters is an intensive task. Sun et al. [12] recovered the sparse 3D model from single 2D non-frontal image using non-linear least squares (NLS) optimization. NLS optimization requires good initial start and obtained estimate may not represent the desired solution. Differential Evolution (DE) optimization an evolutionary optimization algorithm is widely used to solve the complex engineering optimization problems in varied fields. Classical DE optimization requires specification of

two parameters crossover and scale factor to solve the optimization problem at hand. In [21] the authors estimated the pose and depth information of face using classical DE. The range of the scale factor in the classical DE is from [0 1] and requires trial and error to select the suitable scale factor.

In this work, we propose Correlation Scale factor DE (csDE) an improvement to classical DE and is used to estimate the pose and depth values of 3D face features. Candide face model is shown in Fig.2 and is a widely used face mask in face processing applications. Candide model [22] depth values are used to measure the correlation of population members during the progress of the proposed optimization algorithm. In summary the contributions are as follows. Correlation Scale Factor (csDE) optimization is proposed; the parameter F (Scale Factor) is adapted based on correlation and only requires specification of Cr (crossover rate); the proposed algorithm is used to estimate the depth values of the facial features from 2D images; the experimental results are demonstrated on 2D images taken in lab and on Bosphorus Database.

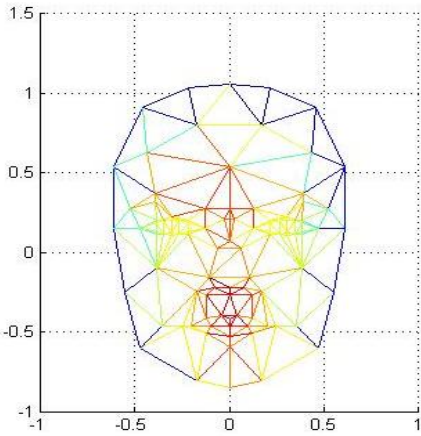


Fig.2. Candide-3 Face Model

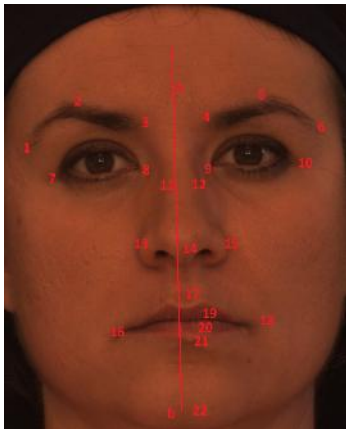


Fig.3. 22 Features Marked on Sample 2D Image of Bosphorus Database

The remainder of the paper is organized as follows: in section 2, the problem of depth estimation from similarity transform is presented. In section 3, Correlation Scale Factor Differential Evolution algorithm is described. Experimental results are presented in section 4, and conclusion is presented in section 5.

2. PROBLEM FORMULATION

2.1 SIMILARITY TRANSFORM

The Point \mathbf{P} in $\mathbf{R}^{3 \times 3}$ space is projected on to $\mathbf{R}^{2 \times 2}$ plane point \mathbf{p} by the similarity transform [23] given the image dependent scale (s) and rotation matrix $\mathbf{R}_{2 \times 3}$. Under orthographic projection 3D face model \mathbf{C} is projected to the corresponding 2D face \mathbf{p} using similarity transform and is given below:

$$\mathbf{p} = s * \mathbf{R}_{2 \times 3} * \mathbf{C} + \mathbf{T} \quad (1)$$

where s , $\mathbf{R}_{2 \times 3}$ and \mathbf{T} denote the scale of the image, rotation matrix and translation matrix between the frontal view 3D face model \mathbf{C} and the non-frontal-view face image \mathbf{p} , respectively. The successive rotations around the x-, y-, and z-axes, by angles ϕ , Ψ , θ , respectively, and can be written as the product of these three rotations as follows:

$$R = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \psi & 0 & -\sin \psi \\ 0 & 1 & 0 \\ \sin \psi & 0 & \cos \psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad (2)$$

$\mathbf{R}_{2 \times 3}$ contains the first two rows of the 3×3 rotation matrix R .

2.2 FACE IMAGES DEPTH ESTIMATION

Let n be the number of feature points in a face image. The matrix $C = [X, Y, Z]^T$ is a $3 \times n$ matrix, which represents the adapted face model coordinates in $\mathbf{R}^{3 \times 3}$ space. X and Y are measured from the frontal face image being adapted. To reconstruct the sparse 3D model the number of facial features required are $n=22$. The sample image from Bosphorus database with 22 features marked is shown in Fig.3. The shape feature from frontal image represented as $(x_i, y_i)_{i=1,2,\dots,22}$ along with depth values of the corresponding features represented as $(z_i)_{i=1,2,\dots,22}$ are used and subjected to rotation followed by translation to get the corresponding features in non-frontal image $(p_i, q_i)_{i=1,2,\dots,22}$.

$$\begin{bmatrix} p_i \\ q_i \end{bmatrix} = k \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}, \quad i = 1, 2, \dots, n \quad (3)$$

where k is the scale factor and (t_x, t_y) represents the translations along the x and y axes. Denoting $M = (x, y, z)_{3 \times 22}^T$ and $N = (p, q)_{2 \times 22}^T$, the Eq.(2) can be written as:

$$N = kR_{2 \times 3}M + t \quad (4)$$

The translation term t can be eliminated if features in non-frontal face image and frontal face image are centered at origin, then the Eq.(3) can be written as:

$$\begin{aligned} N &\leftarrow N - \bar{N} \\ M &\leftarrow M - \bar{M} \\ N &= kR_{2 \times 3}M \end{aligned} \quad (5)$$

where $\bar{N} = \frac{1}{n} \sum_{i=1}^n \begin{bmatrix} p_i \\ q_i \end{bmatrix}$ and $\bar{M} = \frac{1}{n} \sum_{i=1}^n \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$. Let $A = kR_{2 \times 3}$ then

Eq.(5) can be rewritten as

$$N = AM \tag{6}$$

The Eq.(6) is valid if the rotation about x-, y-, z-axis and actual depth values are substituted. If the estimated rotation matrix and depths of the features of the face image under consideration fit the non-frontal image N , then the following equation will be a minimum.

$$d = \|N - AM\|_2^2 \tag{7}$$

Assume $f(\mathbf{x}) = N - AM$,

where $\mathbf{x} = [M_{z_1}, M_{z_2}, M_{z_3}, \dots, M_{z_n}, \phi, \psi, \theta]$ as the parameter vector including the depth values of the feature points and rotation parameters then Eq.(8) can be expressed as:

$$d = \min_{\mathbf{x}} \|f(\mathbf{x})\|_2^2 \tag{8}$$

$$f(\mathbf{x}) = N - AM.$$

The parameters Φ, Ψ, θ, k , and the depth values, M_z can be obtained by minimizing the distance, d , in Eq.(8). The estimation of parameters of a non-linear equation is not an easy task and the gradient based algorithms may easily stuck in local minima. Therefore to search for optimal solution we employ the proposed csDE optimization.

2.3 CORRELATION SCALE FACTOR DIFFERENTIAL EVOLUTION

The csDE optimization starts with the initial population $\mathbf{x}_{i,0} = (x_{1,i,0}, x_{2,i,0}, \dots, x_{D,i,0}) | i = 1, 2, \dots, NP$ randomly generated according to uniform distribution $X_j^{low} \leq x_{j,i,0} \leq X_j^{up}$, for $j = 1, 2, \dots, D$, where NP is the population size, D is the dimension of the problem, and X_j^{low} and X_j^{up} are the upper and lower limits of the j^{th} component of the vector. After initialization, csDE enters a loop of evolutionary operations: mutation, crossover and selection similar to classical Differential Evolution [24] and is shown in Fig.4.

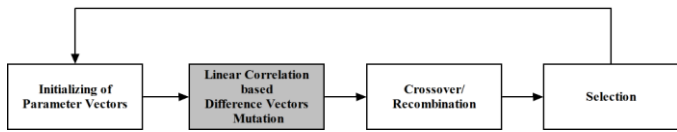


Fig.4. Stages of Operation of Differential Evolution Optimization

2.3.1 Mutation:

At each generation g , this operation creates mutation vectors $\mathbf{v}_{i,g}$ based on the current parent population $\mathbf{x}_{i,g} = (x_{1,i,g}, x_{2,i,g}, \dots, x_{D,i,g}) | i = 1, 2, \dots, NP$.

$$\mathbf{v}_{i,g} = \mathbf{x}_{r_0,g} + F_i (\mathbf{x}_{r_1,g} - \mathbf{x}_{r_2,g})$$

where the indices r_0, r_1 and r_2 are distinct integers uniformly chosen from the set $\{1, 2, \dots, NP\}$, $\{i\}$, $\mathbf{x}_{r_1,g} - \mathbf{x}_{r_2,g}$ is a difference

vector to mutate the parent, $\mathbf{x}_{r_0,g}$. F_i is the mutation factor and is defined as follows:

$$F_i = 1 - c(\mathbf{x}_{r_0,g}, M_{zc})$$

where $c(\mathbf{x}_{r_0,g}, M_{zc})$ is the linear correlation [25] between the solution vector $\mathbf{x}_{r_0,g}$ representing the depth values and M_{zc} the candidate depth values. The value of c is in the range $[0, 1]$; if the value of c is 1, then F_i value is 0 and the parent vector is not mutated as it may be a potential optimal solution. If $c < 1$, i.e. parent vector is not similar to the candidate depth values, then the parent vector is added to linear correlation scaled difference vector to obtain mutant vector $\mathbf{v}_{i,g}$, hence the name Correlation Scale Factor Differential Evolution. The different non-parametric correlation coefficients are Pearson correlation coefficient, Kendall's Tau, Spearman's rho and are described below:

2.3.2 Pearson Correlation Coefficient:

Let $M_{zc} = (M_{zc_1}, M_{zc_2}, \dots, M_{zc_n})$ represent depth values of the feature points from the CANDIDE model having the same position as the feature points displayed in Fig.3 in frontal view. Therefore, M_{zc} should be closely correlated to the corresponding values of the 3D face model. Thus, Pearson's linear correlation coefficient, $(\mathbf{x}_{r_0,g}, M_{zc})$, of M_{zc} and the depth values, $\mathbf{x}_{r_0,g}$, is defined as:

$$c(M_{zc}, \mathbf{x}_{r_0,g}) = \frac{1}{n} \sum_{i=1}^n \left(\frac{M_{zc_i} - \bar{M}_{zc}}{S_{M_{zc}}} \right) \left(\frac{x_{r_0,i,g} - \bar{x}_{r_0,g}}{S_{x_{r_0,g}}} \right) \tag{9}$$

where \bar{M}_{zc} and $S_{M_{zc}}$ are the mean and standard deviation of M_{zc} , respectively, and $\bar{x}_{r_0,g}$ and $S_{x_{r_0,g}}$ represent the mean and standard deviation of $\mathbf{x}_{r_0,g}$, respectively. A higher absolute value of $c(\mathbf{x}_{r_0,g}, M_{zc})$ means that $\mathbf{x}_{r_0,g}$ is closer to M_{zc} .

2.3.3 Kendall's T Coefficient:

To assess the ordinal association between the estimated depth value and candidate depth values Kendall rank coefficient K is computed. The first step in computing K is to calculate the values of the $n(n-1)/2$ paired sign statistics $Q((X_i, Y_i), (X_j, Y_j))$, X represents member of population and Y represents candidate depth value for $1 \leq i < j \leq n$, where

$$Q((a, b), (c, d)) = \begin{cases} 1, & (d-b)(c-a) > 0 \\ -1, & (d-b)(c-a) < 0 \end{cases} \tag{10}$$

That is, for each pair of subscripts (i, j) with $i < j$, score 1 if $(Y_j - Y_i)(X_j - X_i)$ is positive and score -1 if $(Y_j - Y_i)(X_j - X_i)$ is negative. The Kendall statistic K is then

$$K = \frac{1}{n-1} \sum_{i=1}^{n-1} \sum_{j=i+1}^n Q((a, b), (c, d)) \tag{11}$$

This is corresponding to adding up the 1's and -1's from the paired sign statistics.

2.3.4 Spearman rho:

To compute the Spearman rank correlation coefficient r_s , we first order the $n \times r_{0,g}$ population member from least to greatest and let R_i denote the rank of $x_{r_0,g,i}$, $i = 1, \dots, n$, in this ordering. Similarly,

we separately order the n candidate depth values M_{zc} from least to greatest and let S_i denote the rank of M_{zci} , $i=1, \dots, n$, in this ordering. The spearman rank correlation coefficient is defined as the Pearson product moment sample correlation of the R_i and S_i . When no ties within a sample are present, this is equivalent to two computationally efficient formulae:

$$r_s = \frac{12 \sum_{i=1}^n \left(R_i - \frac{n+1}{2} \right) \left(S_i - \frac{n+1}{2} \right)}{n(n^2 - 1)} \quad (12)$$

where $D_i = S_i - R_i$, $i = 1, \dots, n$.

2.3.5 Crossover

After mutation, a ‘‘binomial’’ crossover operation forms the final trial vector $\mathbf{u}_{i,g} = (\mathbf{u}_{1,i,g}, \mathbf{u}_{2,i,g}, \dots, \mathbf{u}_{D,i,g})$:

$$u_{j,i,g} = \begin{cases} v_{j,i,g}, & \text{if } \text{rand}_j(0,1) \leq CR_i \text{ or } j = j_{\text{rand}} \\ x_{j,i,g}, & \text{otherwise} \end{cases} \quad (13)$$

where $\text{rand}_j(a,b)$ is a uniform random number on the interval (a,b) and newly generated for each j , $j_{\text{rand}} = \text{randint}_i(1,D)$ is an integer randomly chosen from 1 to D and newly generated for each i , and the crossover probability, $CR_i \in [0,1]$, roughly corresponds to the average fraction of vector components that are inherited from the mutation vector.

2.3.6 Selection

The selection operation selects the better one from the parent vector $\mathbf{x}_{i,g}$ and the trial vector $\mathbf{u}_{i,g}$ according to their fitness values $f(\cdot)$. For example, if we have a minimization problem, the selected vector is given by

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{u}_{i,g}, & \text{if } f(\mathbf{u}_{i,g}) \leq f(\mathbf{x}_{i,g}) \\ \mathbf{x}_{i,g}, & \text{otherwise} \end{cases} \quad (14)$$

and used as a parent vector in the next generation.

3. EXPERIMENTAL RESULTS

The estimation algorithm performance was evaluated on images taken in lab using Canon A3300 IS, and Bosphorus Database [24]. The images taken in lab are used to measure the pose estimation accuracy. The Bosphorus database contains 3D face images that include large sets of expressions, systematic pose variations, and different types of occlusions. One prominent advantage of this database is that both 2D and 3D facial-feature points are available. Therefore, an adequate experimental comparison using this database can be performed to evaluate any algorithm without considering the marking errors. Moreover, the estimated depth values can be compared to the true depth values from the database. The optimization algorithm is implemented in MATLAB running on a laptop with 4GB RAM. The csDE requires initial population of $N \times D$ and the value of crossover (Cr). The value of Cr is chosen 0.2.

3.1 RESULTS BASED ON LAB IMAGES

The sample face images acquired in lab with variation in y-direction (pan) from -45 to $+45$ degrees and are shown in Fig.6. Initial experiments were carried out on a subject displaying head-pose variations along the y direction are shown in Fig.5 to serve

as an illustration of the described method. The pose parameters, Φ , Ψ , and θ , were initially set to zero and the scale parameter, k , was set to 1 for the first iteration. The estimated poses are represented using Delaunay triangulation in Fig.6.



Fig.5. Sample Images with Variations in y-Direction

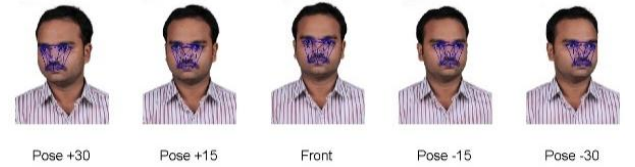


Fig.6. Pose Estimation Results

3D Bosphorus database has multiple views and captured emotions of subjects aiming at improving the recognition, reconstruction and classification of recognized emotions. In this work, the task is to estimate the 3D Face shape features. For this purpose, only the first 30 subjects with clear multiples views of face features are considered for simulation. Further, we have used only five non-frontal-view face images, PR_D, PR_SD, PR_SU, PR_U and YR_R10 as in these images all the 22 features are visible and are shown in Fig.7. When one of these non-frontal view images is used for training we can obtain one set of depth values for all the 22 features. To measure the depth estimation accuracy we have computed three different correlation coefficients of the true depth values and the estimated depth values for the considered five non frontal view images using csDE optimization scheme and DE are given in Table.1. From Table.1, the computed Mean values of csDE are higher and Standard Deviation Values are minimum in comparison to DE_S [1], SM [11] and NLS [12].

3.2 RESULTS BASED ON 3D BOSPHORUS DATABASE

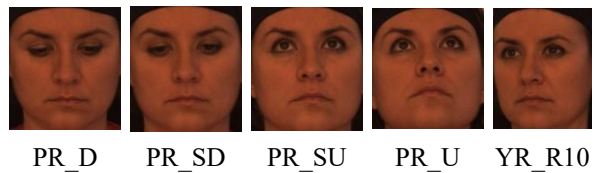


Fig.7. Face Images of bs000 from the Bosphorus Database Displaying Different Poses

Table.1. Correlation Coefficients of the True Depth Values and the Estimated Depth Values Obtained for Different Training Face Images with LcorDE and DE

	Correlation	PR_D	PR_SD	PR_SU	PR_U	YR_R10	Mean	Std
csDE	Pearson	0.9975	0.9983	0.9979	0.9966	0.9992	0.9979	0.0009
	Kendall Tau	0.9976	0.9978	0.9962	0.9958	0.9979	0.9970	0.0009
	Spearman's rho	0.9991	0.9984	0.9982	0.9975	0.9991	0.9984	0.0006

DE_SI [21]	F = 0.6; Cr = 0.2	0.8939	0.8315	0.5409	0.5258	0.8113	0.7207	0.1737
SM [11]	-	0.1962	0.2787	0.5568	0.7283	0.5128	0.4545	0.2408
NLS [12]	-	0.8916	0.8655	0.8454	0.8573	0.9016	0.8723	0.0236

4. CONCLUSION

In this paper, we have proposed a novel optimization scheme Correlation Scale Factor Differential Evolution optimization. The novelty of the proposed csDE optimization is that the adaptive scale factor for differential evolution requires specification of only crossover rate. The proposed algorithm is devised to estimate the sparse depth values representing geometry of 3D face from 2D images. The depth estimation results are reported based on three different correlation coefficients and compared with SM, DE and NLS. The csDE with spearman rho has higher mean value and lower standard deviation. The results presented here demonstrate the feasibility and efficiency of the proposed optimization algorithm to estimate the depth values of facial features from 2D images.

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