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AN ACCURATE MODELING OF DELAY AND SLEW METRICS FOR ON-CHIP VLSI RC INTERCONNECTS FOR RAMP INPUTS USING BURR'S DISTRIBUTION FUNCTION

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Abstract
This work presents an accurate and efficient model to compute the delay and slew metric of on-chip interconnect of high speed CMOS circuits foe ramp input. Our metric assumption is based on the Burr's Distribution function. The Burr's distribution is used to characterize the normalized homogeneous portion of the step response. We used the PERI (Probability distribution function Extension for Ramp Inputs) technique that extends delay metrics and slew metric for step inputs to the more general and realistic non-step inputs. The accuracy of our models is justified with the results compared with that of SPICE simulations.

Keywords:
Delay and Slew Calculation, Burr's Distribution Function, Probability Distribution Function, Cumulative Distribution Function, Interconnect, Moment Matching

1. INTRODUCTION

As the scale of process technology is steadily shrinking towards the ultra deep sub micrometer regime and the size of the design increases to a large extent, length of interconnect is getting longer [1]. So, efficient and accurate computation of delay and slew metric is crucial for enhancing the switching speed of present day devices. The timing verification is very complicated issue in IC design process because of the statistical variations in the device and interconnects delays. Different proposals have been made to meet the timing constraints. The Elmore delay metric [2] is a popular method for the fast calculation of the interconnect trees. This is because it is simple, closed-form, and easy to evaluate. However with development of the technology, interconnect delay is becoming comparable in value to cell delay or even dominates it, so in this case much more accurate interconnect delay metrics are desired [3]. Many approaches primarily concentrated to find the interconnect delay rather than gate delay so that one can increase the speed of the circuit by simply decreasing interconnect length. Accurate and exact calculation of propagation delay and slew in VLSI interconnects are critical to the design of high speed systems. Current techniques are based on either simulation or analytical models. Slew rate indicates the rate of change of the signal. An increase in slew enhances the delay through the line. Hence an accurate estimation of the slew metric is thus essential for efficient design of high speed CMOS integrated circuits. As the design parameters like gate oxide thickness, channel length reach their threshold, computation of slew metric and interconnect delay become crucial for both performance and physical design optimization for high speed CMOS integrated circuits[3]. AWE [4] can approach towards SPICE-like accuracy by computing and matching higher order moments of the impulse response, but AWE is not a simple closed-form formula and in involves computational complexity, in particular it involves finding a solution of a non-linear equation. So the desired delay and slew metric should be not only highly accurate but also simple and closed-form.

In this paper, a closed form delay and slew metrics are presented based on the Burr's probability distribution function. It is shown that the moment matching to the Burr's distribution parameters produces explicit expression for delay and slew metrics. PERI technique [5] is used for extending the delay metric derived for a step input into a delay metric for a ramp input for RC trees and it is valid over all input slew conditions. Note that, the delay metric reduces to the Elmore delay of the circuit under the limiting case of an infinitely slow ramp, a fact first proved in [4] to establish the Elmore delay as an upper bound.

The rest of the paper is organized as follows: Section 2 discusses the basic theory, expressions of circuit moments in terms of impulse response and expressions of mean and variance in terms of circuit moments. Section 3 describes the properties of Burr's distribution function and discusses the method to calculate the proposed delay and slew matrices. Section 4 shows the experimental results and the comparison with other established matrices. Finally Section 5 concludes the paper.

2. BASIC THEORY

2.1 MOMENTS OF A LINEAR CIRCUIT RESPONSE

Let h (t) is the circuit impulse response in the time domain and let H (s) be the corresponding transfer function. By definition, H(s) is the Laplace transform of h (t)

$$H(s) = \int_0^\infty h(t) e^{-st} dt \quad (1)$$

Applying a Taylor series expansion of e^{-st} about s = 0 yields:

$$H(s) = \int_0^\infty h(t)\left\{1 - st + \frac{1}{2!}s^2t^2 - \frac{1}{3!}s^3t^3 + ...\right\}dt$$

$$= \sum_{i=0}^{\infty} \frac{(-1)^i}{(i)!} s^i \int_0^\infty t^i h(t) dt \quad (2)$$

The i^{th} circuit-response moment [7], m_i is defined as:

$$\hat{m}_i = \frac{(-1)^i}{(i)!}\int_0^\infty t^i h(t) dt \quad (3)$$

From (2) and (3), the transfer function H (s) can be expressed as:

$$H(s) = \hat{m}_0 + \hat{m}_1 s + \hat{m}_2 s^2 + \hat{m}_3 s^3 + ... \quad (4)$$

2.2 CENTRAL MOMENTS

Central moments are distribution theory concepts.
Let us consider the moment definition given again:

$$\hat{m}_q = \frac{(-1)^q}{(q)!}\int_0^\infty t^q h(t)dt \tag{5}$$

The mean of the impulse response is given by,

$$\mu = \frac{\int_0^\infty th(t)dt}{\int_0^\infty h(t)dt} = \frac{-m_1}{m_0} \tag{6}$$

It is straight forward to show that the first few central moments can be expressed in terms of circuit moments as follows [6]:

$$\mu_0 = m_0, \mu_1 = 0, \mu_2 = 2m_2 - \frac{m_1^2}{m_0}, \mu_3 = -6m_3 + 6\frac{m_1 m_2}{m_0} - 2\frac{m_1^3}{m_0^2} \tag{7}$$

The central moments have the following geometrical interpretations [6]:

μ_0 is the area under the curve. It is generally unity, or else a simple scaling factor is applied.

μ_2 is the variance of the distribution. It measures the spread or the dispersion of the curve from the center. A larger variance results a larger spread of the curve.

μ_3 is a measure of the skewness of the distribution; for a unimodal function its sign determines if the mode (global maximum) is to the left or to the right of the expected value (mean). Its magnitude is a measure of the distance between the mode and the mean.

2.3 SECOND AND THIRD CENTRAL MOMENTS IN RC TREES

The second and third central moments [6] are always positive for RC tree impulse responses. From (8), it is obvious that the second order central moment is positive.

$$\mu_2 = \int_0^\infty (t-\mu)^2 h(t)dt \tag{8}$$

The impulse response, h (t), at any node in an RC tree is always positive. Hence, the second central moment, μ_2 is always positive.

2.4 MOMENTS OF PROBABILITY FUNCTION

A probability function is a real valued set function where the domain is a subset of the sample space, S, and the range is a real number in the interval [0, 1]. Generally, a function Pr {*} should satisfy the three Kolmogorov axioms [8], or equivalent conditions, in order to be considered as the probabilities function:

i. Pr {S} = l;
ii. Pr {A} ≥0 for all A $\subset$ S;
iii. Pr{ A $\cup$ B } = Pr{ A } + Pr{ B } if A $\cap$ B = ϕ , A $\subset$ S , B $\subset$ S .

The distribution function of a continuous random variable T denoted as F_T (t) provides the value of Pr {T≤ t} for any real number $-\infty \le t \le \infty$. The associated probability density function (PDF) denoted as f_T (t) is the derivative of the distribution function with respect to t, thus,

$$\left.\begin{aligned} f_T(t) &= \frac{dF_T(t)}{dt} \\ F_T(t) &= \int_{-\infty}^{t} f_T(t)dt \end{aligned}\right\} \tag{9}$$

The median, $t_{(0.5)}$, is defined by

$$P_r\{T \le t_{(0.5)}\} = F_T(t_{(0.5)}) = \int_{-\infty}^{t_{0.5}} f_T(t)dt = 0.5 \tag{10}$$

Whereas, the expected value or mean, E [T] of a continuous random variable T with distribution f_T (t) is

$$E[T] = \int_{-\infty}^{\infty} tf_T(t)dt \tag{11}$$

The mean is also the first moment of the distribution (or PDF). In general, the i^{th} moment m_i of the distribution is,

$$m_i = E[T^i] = \int_{-\infty}^{\infty} t^i f_T(t)dt \tag{12}$$

2.5 RELATION BETWEEN PROBABILITY DENSITY FUNCTIONS AND CIRCUIT RESPONSES

Any function f(t) can be treated as a probability density function [6] if it is defined in the range [a, b] and satisfies

$$\begin{cases} f(t) \ge 0 & \forall\ t \\ \int_a^b f(t)dt = 1 \end{cases} \tag{13}$$

If f (t) is equal to zero outside of the range [a, b], we can replace the integration limits in Eq. (13) with -∞ and ∞. Elmore was the first to apply moments for delay approximation of a limited class of circuit responses by observing that the impulse response of a circuit can be treated as a probability density function. Elmore used this observation to justify the approximation of the 50% point of a monotonic step response (the median point of the impulse response) by the first moment (mean of the impulse response). It was shown that the impulse response corresponding to an RC tree is unimodal with positive skew [6]. From this it follows that the mode is less than the median which is less than the mean and vice versa [7-8]: (Skew > 0) if and only if (mode <median < mean). This proved that the Elmore delay is an upper bound for the 50% step response delay, and was shown to hold for finite input signal rise time. An important observation [6] is that because of the variation in impulse response shapes along an interconnect path, the relative accuracy of the Elmore delay bound can be quite poor. Especially for interconnects associated with deep submicron technologies, more than one moment is needed to capture the waveform shape-characteristics.

3. BURR DISTRIBUTION MODEL

Elmore delay model believes the similarity between non-negative impulse responses and probability density functions (PDF). In theory, Elmore's assumption can be easily extended beyond simply estimating the median by the mean, if one considers higher order moments to characterize the distribution function. Once characterized, the delay can be approximated via table-lookup of the median value for the representative distribution family. In this work, a novel delay and slew metrics are proposed using Burr probability distribution. The Burr's distribution is a two parameter continuous distribution [9]. Since both are unimodel and have non-negative skewness [10], one can match the impulse response of the generalized RC network to the characterize parameters of the Burr distribution. The PDF of Burr's distribution [9] is shown in Fig. 1. The probability density function of the burr's distribution $f_{c,k}$ (x), is a function of one variable *x* and two parameters *c* and *k* (positive real numbers)[9].

$$f(x) = \frac{kcx^{c-1}}{(1+x^c)^{k+1}} \tag{14}$$

Mean for the burr distribution is given as,

$$Mean \quad (E(x)) = kB\left(k - \frac{1}{c}, 1 + \frac{1}{c}\right) \tag{15}$$

Where B (x, y) is the incomplete beta function.

$$Median \quad = \left[2^{\frac{1}{k}} - 1\right]^{\frac{1}{c}} \tag{16}$$

Variance (σ^2)=E(x^2)- [E(x)]^2 (17)

Where E(x^2) is given in terms of beta function as,

$$E\left(x^2\right) = kB\left[k - \frac{2}{c}, 1 + \frac{2}{c}\right] \tag{18}$$

The relation between beta and gamma function is given as,

$$B(x, y) = \frac{\Gamma x \Gamma y}{\Gamma(x+y)} \tag{19}$$

By using the gamma function approximation [11], we get,

$$\Gamma(az+b) = \sqrt{2\pi} e^{-az} (az)^{az+b-\frac{1}{2}} \tag{20}$$

By using equation (19), we get the Mean as,

$$E(x) = k\left[\frac{\Gamma\left(k - \frac{1}{c}\right)\Gamma\left(1 + \frac{1}{c}\right)}{\Gamma(k+1)}\right] \tag{21}$$

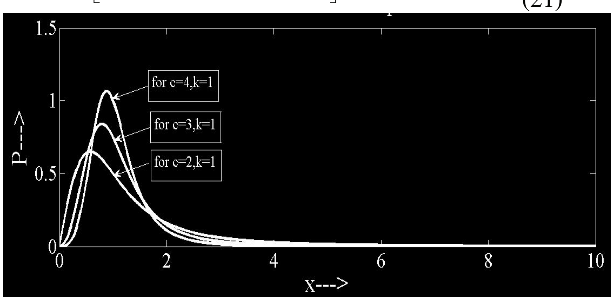

Fig.1. Examples of several members from the burr's distribution family. Each family member corresponds to specific values of the distribution parameters c and k

By using (20), we get,

$$\Gamma\left(k - \frac{1}{c}\right) = \sqrt{2\pi} e^{-k} (k)^{k - \frac{1}{c} - \frac{1}{2}} \tag{22}$$

$$\Gamma\left(1 + \frac{1}{c}\right) = \sqrt{2\pi} e^{-1} \tag{23}$$

And $\Gamma(k+1) = \sqrt{2\pi} e^{-k} (k)^{k+\frac{1}{2}}$ (24)

By substituting equations (22), (23) and (24) in (21) we get

$$Mean \quad = E(x) = 0.9196 \quad (k)^{\frac{1}{c}} \tag{25}$$

Similarly we get E(x^2) as,

$$E(x^2) = k\frac{\Gamma\left(k - \frac{2}{c}\right)\Gamma\left(1 + \frac{2}{c}\right)}{\Gamma(k+1)} \tag{26}$$

By using (20) we get,

$$\Gamma\left(k - \frac{2}{c}\right) = \sqrt{2\pi} e^{-k} (k)^{k - \frac{2}{c} - \frac{1}{2}} \tag{27}$$

$$\Gamma\left(1 + \frac{2}{c}\right) = \sqrt{2\pi} e^{-1} \tag{28}$$

Substituting (27), (28) and (24) in (26) we get,

$$E(x^2) = 0.9196 \quad (k)^{2 - \frac{2}{c}} \tag{29}$$

Substituting (25) and (29) in (17) we get,

$$Variance \quad (\sigma^2) = 0.9196 \; k^{\frac{2}{c}}\left[k^{-\left(\frac{4}{c}+1\right)} - 0.9196\right] \tag{30}$$

3.1 CALCULATION FOR THE PARAMETERS *c* & *k*

Since Burr's distribution has two parameters *c* and *k* for characterization. So, by matching the two moments completely represents this model. Hence, the mean and variance of burr's distribution can be presented in terms of moments as given bellow.

$$\left.\begin{aligned} &E(x) = -m_1 \\ &and \\ &Variance \quad = V(x) = 2m_2 - m_1^2 \end{aligned}\right\} \tag{31}$$

By solving (25), (30) and (31) we get

$$c = \frac{\ln 2.5 m_2 m_1^2}{\ln\left(\frac{0.9196}{-m_1}\right)} \tag{32}$$

$$k = \left[\frac{-m_1}{0.9196}\right]^{\left(\frac{\ln 2.5 m_2 m_1^2}{\ln\left(\frac{0.9196}{-m_1}\right)}\right)} \tag{33}$$

3.2 CALCULATION OF MEDIAN OR 50% DELAY METRIC FOR RAMP INPUT

The Median of the burr's distribution [9] is defined as

$$Median \quad = \left[2^{\frac{1}{k}} - 1\right]^{\frac{1}{c}} \tag{34}$$

Substituting the values of c and k from (32), (33) in (34), we have,

$$median = \left[2^{\left(\frac{0.9196}{-m_1}\right)^{\frac{-2}{3}\ln\left(\frac{2.4709}{m_2 m_1^2}\right)}} - 1\right]^{\left(\frac{2}{3}\ln\left(\frac{2.4709}{m_2 m_1^2}\right)\right)} \tag{35}$$

So, we can write the closed form delay expression as,

$$Delay(50\%) = \left[2^{\left(\frac{0.9196}{-m_1}\right)^{\frac{-2}{3}\ln\left(\frac{2.4709}{m_2 m_1^2}\right)}} - 1\right]^{\left(\frac{2}{3}\ln\left(\frac{2.4709}{m_2 m_1^2}\right)\right)} \tag{36}$$

The expression presented above is the 50% delay metric for step input for generalized RC network. From (36) we can see that the median i.e. 50% delay metric is the simple function of the first two circuit moments. This is our proposed closed form model using burr's distribution.

Let us assume that the input waveform is a ramp with slope *T*, as shown in Fig. 2(a) [12]. The PDF of this waveform is a uniform distribution with mean $\mu(I) = T/2$ and standard deviation $\sigma(I) = \frac{T^2}{12}$. Thus, the delay of the output ramp is as shown in Fig.2(b).

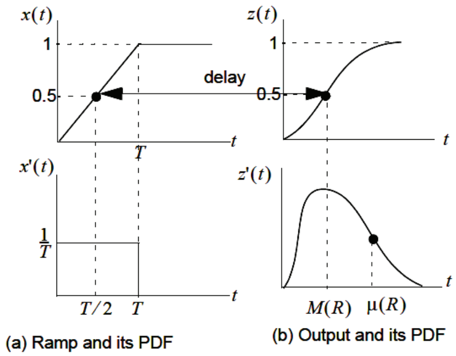

Fig.2. Ramp input and its corresponding response of an RC network

If μ(s) = - m_1 is the Elmore delay and $M(S)$ is the step delay metric as given by equation (36). The delay estimation for the ramp response [5] is given by,

$$D(R) = (1-\alpha)\,\mu(s) + \alpha\, M(S) \tag{37}$$

Where α is a constant and is given by,

$$\alpha = \left(\frac{2m_2 - m_1^2}{2m_2 - m_1^2 + \frac{T^2}{12}}\right)^{\frac{5}{2}} \tag{38}$$

Where 0<T<∞ is the slope of the ramp input as shown in Fig.2(a). From (36), (37) and (38), we get

$$D(R) = \left(1 - \left(\frac{2m_2 - m_1^2}{2m_2 - m_1^2 + \frac{T^2}{12}}\right)^{\frac{5}{2}}\right)(-m_1) + \left(\frac{2m_2 - m_1^2}{2m_2 - m_1^2 + \frac{T^2}{12}}\right)^{\frac{5}{2}} \left[2^{\left(\frac{0.9196}{-m_1}\right)^{\frac{-2}{3}\ln\left(\frac{2.4709}{m_2 m_1^2}\right)}} - 1 \right]^{\frac{2}{3}\ln\left(\frac{2.4709}{m_2 m_1^2}\right)} \tag{39}$$

The above derived equation (39) is the delay metric equation for the Burr's Distribution function for ramp input. This is our proposed closed form delay model for model for ramp input for on-chip VLSI interconnect using burr probability distribution.

3.2 PROPOSED SLEW MODEL

Burr's cumulative distribution function [9], as a function of t, is given by,

$$F(t) = 1 - \frac{1}{(1 + t^c)^k} \qquad t \geq 0 \tag{40}$$

If $F(t)$ satisfies the following conditions: they are $0 \leq F(t) \leq 1$ *and* $\lim_{t\to\infty} F(t) = 1$, $\lim_{t\to 0} F(t) = 0$

After applying Binomial approximations [7],

$$t \approx \left[\frac{1}{k} F(t)\right]^{\frac{1}{c}} \tag{41}$$

Now, let T_{LO} and T_{HI} be 10% and 90% delay points, respectively. Matching to these points to the CDF yields from equation (41),

$$T_{LO} = \left[\frac{0.1}{k}\right]^{\frac{1}{c}} \tag{42}$$

And

$$T_{HI} = \left[\frac{0.9}{k}\right]^{\frac{1}{c}} \tag{43}$$

The Burr's slew metric is calculated by using equations (42) and (43) as,

$$BSM = T_{HI} - T_{LO} = \left[\frac{0.9}{k}\right]^{\frac{1}{c}} - \left[\frac{0.1}{k}\right]^{\frac{1}{c}} \tag{44}$$

By substituting the values of the c and k from (32) and (33) in (44), we get,

$$BSM = \frac{0.9196}{-m_1}\left[(0.9)^{\frac{\ln\left(\frac{0.9196}{-m_1}\right)}{\ln 2.5 m_2 m_1^2}} - (0.1)^{\frac{\ln\left(\frac{0.9196}{-m_1}\right)}{\ln 2.5 m_2 m_1^2}}\right] \tag{45}$$

This is the proposed closed form model for the slew metric for step input for on-chip VLSI interconnects based on the burr's distribution. From the above equation it can be seen that Burr Slew Metric [BSM] is the mere function of the first two circuit moments.

The output slew is the root-mean square of the step slew and input slew [5]. For ramp slew,

$$Slew(R) = \sqrt{Slew^2(S) + Slew^2(I)} \tag{46}$$

Further, (46) exhibits the right limiting behavior:

As $Slew(I) \to \infty$ we get $Slew(R) \to \infty$

and as $Slew(I) \to 0$, we have $Slew(R) \to Slew(S)$.

Where Slew(S) is the step slew metric which is given by equation (45) and Slew (I) is the input slew which is given as,

$$Slew^2(I) = T^2/12 \tag{47}$$

From equations (45), (46) and (47), we get,

$$Slew(R) = \sqrt{\left(\frac{0.916}{-m_1}\left[(0.9)\left(\frac{3}{2\ln\left(\frac{2.4709}{m_2 m_1^2}\right)}\right) - (0.1)\left(\frac{3}{2\ln\left(\frac{2.4709}{m_2 m_1^2}\right)}\right)\right]\right)^2 + \frac{T^2}{12}} \tag{48}$$

The above derived equation (48) is the slew metric equation for the Burr's Distribution function for ramp input. This is our proposed closed form model using burr's distribution.

4. EXPERIMENTAL RESULTS

We have implemented the proposed delay and slew estimation method using burr's distribution and applied it to widely use actual interconnect RC networks as shown in Fig. 3. For each RC network source we put a driver, where the driver is a voltage source followed by a resistor.

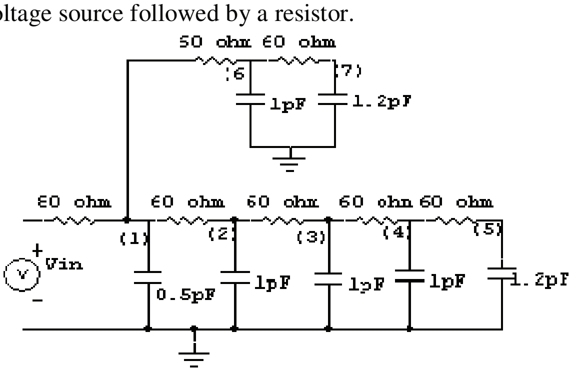

Fig.3. An RC Tree Example

In order to verify the efficiency of our model, we have extracted 208 routed nets containing 2024 sinks from an industrial ASIC design in 0.18 μm technology. We choose the nets so that the maximum sink delay is at least 10 ps and the delay ratio between closet and furthest sinks in the net is less than 0.2. It ensures that each net has at least one near end sink. We classify the 2244 sinks as it was taken in PERI [5] into the following three categories:

- 1187 far-end sinks have delay greater or equal to 75% of the maximum delay to the furthest sink in the net.
- 670 mid-end sinks which have delay between 25% and 75% of the maximum delay and,
- 367 near-end sinks which have delay less than or equal to 25% of the maximum delay**.**

In order to find the delay and slew at node 5 for ramp input, a saturate ramp of time period of T=100 ps is used. For both calculation of delay and slew, the relative error is less than 2 %. The calculated average, minimum, maximum values are compared along with standard deviation for the delay calculated by using PERI and with those found using the proposed model. The comparative results are summarized in Table 1.

Table.1 Comparison between Proposed Delay Model and PERI

Step Delay	Delay Metric Using Two Moments							
For Ramp	PERI Method				Our Proposed Model			
Sinks	Avg	SD	max	min	Avg	SD	max	Min
Near	1.25	0.33	2.25	0.50	1.26	0.39	2.13	0.45
Mid	1.14	0.09	1.47	0.97	1.17	0.10	1.48	0.85
Far	1.00	0.01	1.03	0.98	1.10	0.05	1.05	0.99
Total	1.08	0.17	2.25	0.50	1.07	0.15	2.23	0.46

The obtained average, minimum, maximum values are compared along with standard deviation for slew calculated by using PERI method and with those found using our proposed model. The comparative results are summarized in Table 2.

In the final step of the experiments, we have estimated the effect of input slew on the delay and slew estimation for the same seven node RC network as shown in the Fig. 3. We have chosen 10/90 input slew values of 250 and 500 picoseconds, respectively and delays and slews were estimated for each node. The comparative result of our proposed model with PERI [5] and RICE [13] are shown in the Table 3 and Table 4 for delay and slew, respectively.

Table.2 Comparison between Proposed Slew Model and PERI

Step Slew	Slew Metric Using Two Moments							
For Ramp	PERI Method				Our Proposed Model			
Sinks	Avg	SD	max	Min	Avg	SD	max	Min
Near	1.01	0.21	1.89	0.65	1.02	0.21	1.80	0.61
Mid	0.89	0.07	1.21	0.75	0.95	0.07	1.27	0.78
Far	1.13	0.06	1.25	0.98	1.17	0.08	1.25	0.95
Total	1.04	0.15	1.89	0.66	1.04	0.16	1.81	0.64

Table.3 Delay Comparison for Each Node between Proposed Delay Model, PERI and RICE

Input Slew	250(ps)			500(ps)		
Node	PERI	RICE	Proposed Model	PERI	RICE	Proposed Model
1	207	210	209	235	272	273
2	383	383	387	407	409	407
3	484	482	481	505	498	501
4	707	705	709	724	716	717
5	851	849	852	867	859	866
6	461	461	464	484	487	489
7	925	923	937	941	933	934

Table 4 Slew Comparison for Each Node between Proposed Slew Model, PERI and RICE

Input Slew	250(ps)			500(ps)		
Node	PERI	RICE	Proposed Model	PERI	RICE	Proposed Model
1	1614	1659	1657	1671	1758	1723
2	1725	1733	1728	1779	1816	1710
3	1996	2003	2005	2042	2079	2073
4	2173	2164	2163	2215	2219	2218
5	2230	2223	2227	2271	2261	2263
6	1743	1748	1749	1796	1826	1811
7	2237	2233	2237	2279	2276	2274

5. CONCLUSION

In this paper, we have proposed an efficient and accurate interconnect delay and slew metric for high speed VLSI designs for ramp input. We have used burr distribution to find the desired matrices. It is found that the proposed matrices are a simple function of first two moments. Our model has Elmore delay as upper bound but with significantly less error and does not require any complex look up table. The novelty of our approach is justified by the calculated delay from the experiments performed on the industrial nets.

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