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A HYBRID TECHNIQUE FOR PAPR REDUCTION OF OFDM USING DHT PRECODING WITH PIECEWISE LINEAR COMPANDING

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is a fascinating approach for wireless communication applications which require huge amount of data rates. However, OFDM signal suffers from its large Peak-to-Average Power Ratio (PAPR), which results in significant distortion while passing through a nonlinear device, such as a transmitter high power amplifier (HPA). Due to this high PAPR, the complexity of HPA as well as DAC also increases. For the reduction of PAPR in OFDM many techniques are available. Among them companding is an attractive low complexity technique for the OFDM signal's PAPR reduction. Recently, a piecewise linear companding technique is recommended aiming at minimizing companding distortion. In this paper, a collective piecewise linear companding approach with Discrete Hartley Transform (DHT) method is expected to reduce peak-to-average of OFDM to a great extent. Simulation results shows that this new proposed method obtains significant PAPR reduction while maintaining improved performance in the Bit Error Rate (BER) and Power Spectral Density (PSD) compared to piecewise linear companding method.

Keywords:

OFDM, PAPR, Discrete Hartley Transform, Companding Distortion, Piecewise Linear Companding.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing is a modulation format that is being used by many of the latest wireless as well as telecommunications standards. A large number of closely spacing sub-carrier signals those are orthogonal to each other are used to carry data on several parallel data paths or channels. Each of the sub-carrier signals are modulated with a conventional modulation formats like Quadrature Amplitude Modulation (QAM) or Phase-Shift Keying (PSK) at a low symbol rate. Major advantages of OFDM are low complexity in Receiver design ,finds applications which need high data rates, provide immunity to ISI by using cyclic prefix, less sensitive to timing offset than single carrier systems, no need to employing bank of oscillators. It allows simultaneous transmission of subcarriers over a common channel, thus making efficient use of available spectrum, results in high Spectral Efficiency. Due to its adverse benefits, it finds the applications in Digital Audio Broadcasting (DAB), IEEE 802.16 (Wireless Local Area Networks), MBWA (IEEE802.20), DSL and ADSL Modems, DRM and Wireless Metropolitan Area Networks (WMAN) as well as DVB-T.

Despite many advantages, OFDM affect from high fluctuation of the transmitted envelope signal. To characterize the envelope fluctuations of OFDM signal, the term Peak to Average Power Ratio (PAPR) mostly used by relating peak power and mean value power. When non linear power amplifier (HPA) is used, then high PAPR causes serious degradation in performance. Several techniques are proposed to reduce PAPR in OFDM signals [1, 2]. Selective level mapping (SLM) partial transmit sequence (PTS), companding techniques are proposed in [3-9], Discrete Hartley Transform [10-11]. In these techniques, companding schemes gain more attention due to their resilience (flexibility) and simplicity. The concept of companding technique was first introduced in [5], which uses the μ-law companding technique, which aiming at reducing PAPR by increasing the average power of the signal while keeping the peak power remains unchanged. Later on, exponential companding (EC) was developed in [6], which can improve reduction of OFDM's PAPR by altering the dispersion (distribution) of OFDM signals while keeping average power remains constant. In recent times, a new nonlinear companding technique is proposed [7] which changes the Gaussian distributed signal into distribution form by using a linear function format. This nonlinear companding technique reduces the PAPR of OFDM signal at an expense of high computational complexity. Then two piecewise companding (TPWC) technique proposed in [8] which compress large signal amplitudes and expand small ones by using two different piecewise functions. In all above companding techniques, it reduces PAPR by producing companding distortion. Recently, a piecewise linear companding technique was investigated in [9] to decrease the distortion by alters the signals linearly with amplitudes close to the given companded peak amplitude and chopping the signals with amplitudes over a given companded amplitude. The DHT Precoded system [10] shows better PAPR reduction than WHT precoded system and Selected Mapping OFDM system. This DHT precoded OFDM system has the advantage that it doesn't require any additional power requirement, Complex optimization and side information to be sent to Receiver.

This paper starts with the introduction of the topic in section 1. Section 2 presents a Typical OFDM system model with a conception of PAPR problem. Piecewise Linear Companding (PLC) technique and Discrete Hartley Transform (DHT) are introduced in section 3 and section 4. Section 5 provides signal processing steps to implement a PAPR reduction technique by combining Piecewise Linear Companding transform and DHT. Simulation results are conferred in section 6 and finally the Conclusion is given in section 7.

2. FORMULATION OF PAPR PROBLEM

Typically, N independent data symbols are modulated by using baseband modulation schemes like phase-shift keying (PSK) as well as Quadrature Amplitude Modulation (QAM). OFDM signal is nothing but sum of those N independent modulated data symbols. The oversampled time-domain OFDM symbols $X = [x_0, x_1, \ldots, x_{LN-1}]^T$ can be calculated as,

$$x_n = \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} X_k . e^{j2\pi \frac{kn}{NL}}, \quad 0 \le n \le NL-1 \tag{1}$$

where, n = 0, 1,…, LN-1 time is index and L is the upsampling factor.

Usually, $L \ge 4$ is used to characterize the PAPR of the discrete-time signal to PAPR of the continuous-time signal exactly. To ensure the Nyquist criteria of OFDM data signal, which is done by inserting (L-1)N zeros in the centre of N length vector, i.e.

$$X_e = \left[X_0, X_1, ..., X_{\frac{N}{2}-1}, \underbrace{0, ..., 0}_{N(L-1)}, X_{\frac{N}{2}}, ..., X_{N-1} \right]^T . \tag{2}$$

It is clear that $x = IFFT_{LN}\{X_e\}$. For a large N (e.g. $N \ge 64$), the real as well as imaginary parts of OFDM are approximated to be independent and identically distributed Gaussian random variables having zero mean and a variance σ_x^2. Based on this criterion, the OFDM signal's $|x_n|$ amplitude follows a Rayleigh distribution having its probability density function (PDF) as,

$$f_{|x_n|}(x) = \frac{2x}{\sigma_x^2} e^{-\frac{x^2}{\sigma_x^2}}, \ x \ge 0. \tag{3}$$

The cumulative density function (CDF) of $|x_n|$ is therefore

$$\begin{aligned} F_{|x_n|}(x) &= \Pr ob\{|x_n| \le x\} \\ &= \int_0^x \frac{2y}{\sigma_x^2} e^{-\frac{y^2}{\sigma_x^2}} dy = 1 - e^{-\frac{x^2}{\sigma_x^2}}, x \ge 0. \end{aligned} \tag{4}$$

The PAPR of OFDM signal in a given frame is defined as,

$$PAPR_X = \frac{\max_{n \in [0, LN-1]} \left\{ |x_n|^2 \right\}}{E\left\{ |x_n|^2 \right\}}. \tag{5}$$

It is more favorable to consider the OFDM signal's PAPR as a random variable and employ a statistical description given by the CCDF (Complementary Cumulative Distribution Function), which states that the probability of the PAPR of signal (x) exceeds an assigned level $\gamma_0 > 0$, i.e.

$$CCDF_X(\gamma_0) = \Pr ob\{PAPR_X > \gamma_0\} = 1 - (1 - e^{-\gamma_0})^N. \tag{6}$$

The companding transform be adapted to the original signal x_n before it is reformed into analog waveform and then amplified by the HPA.

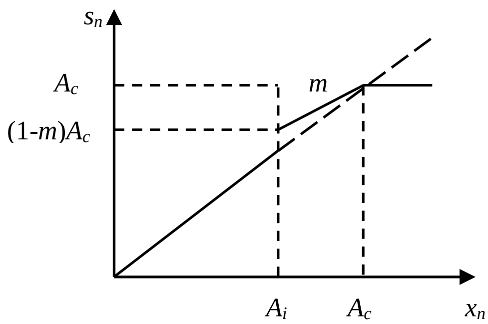

Fig.1. Piecewise linear companding transform

The signal after companding is denoted as $s_n = C(x_n)$, where $s_n = C(\cdot)$ is the companding transform that only changes the amplitude of signal x_n. In the case of additive Gaussian white noise (AWGN) channel, the received signal $r_n = s_n + \upsilon_n$ can be recovered by the inverse companding function $C^{-1}(\cdot)$, namely,

$$x_n^{'} = C^{-1}(s_n + v_n) = x_n + C^{-1}(v_n)$$

where, υ_n is channel noise.

3. PIECEWISE LINEAR COMPANDING

The incoming original data signal x_n is to be companded with a given predefined peak amplitude A_c. The Piecewise Linear Companding method (PLC) which is shown in fig.1, used to clip the signals have amplitudes more than A_c and linearly transforms the signals having amplitudes close to A_c. Then, the companding transform of the Piecewise Linear Companding technique is

$$s(n) = C\{r(n)\} = \begin{cases} r(n) & |x(n)| \le A_i \\ mx(n) + (1-m)A_c & A_i < |x(n)| \le A_c \\ \operatorname{sgn}(x(n))A_c & |x(n)| > A_c \end{cases} \tag{7}$$

where, sgn(x) is the sign function.

Next, the inverse companding function at the Receiver end is,

$$y(n) = C^{-1}(r(n)) = \begin{cases} r(n) & |r(n)| \le A_i \\ \dfrac{(r(n) - (1-m)A_c)}{m} & (1-m)A_c < |r(n)| \le A_c \\ \operatorname{sgn}(r(n))A_c & |r(n)| > A_c \end{cases} \tag{8}$$

where A_c, A_i and m are the parameters specified by the Piecewise Linear Companding technique [9].

The theoretical value of PAPR value of OFDM signal using the Piecewise Linear Companding technique is getting by using the determined value of A_c. As the average signal power to be maintained as constant, then with a known theoretical PAPR preset ($PAPR_{preset}$) value, A_c can be determined by using,

$$A_c = \sigma_x 10^{PAPR_{preset}/20}.$$

By using this A_c value the resting parameters A_i and m can be attained by solving

$$\begin{aligned} &\int_{A_i}^{A_c} \left(mx + (1-m)A_c\right)^2 f_{|x_n|}(x)dx + \int_{A_c}^{\infty} A_c^2 f(x)dx \\ &= \int_{A_i}^{\infty} x^2 f_{|x_n|}(x)dx \end{aligned} \tag{9}$$

4. DISCRETE HARTLEY TRANSFORM

The Discrete Hartley Transform (DHT) is a linear transform, in this N real numbers $[x_0, x_1, \ldots x_{N-1}]$ is transformed into N real numbers $[H_0, H_1, \ldots. H_{N-1}]$. The precoding matrix P can be written as,

$$P=\begin{bmatrix} P_{00} & \cdots & P_{0(N-1)} \\ \vdots & \ddots & \vdots \\ P_{(N-1)0} & \cdots & P_{(N-1)(N-1)} \end{bmatrix} \quad (10)$$

and p_{mn} is given by,

$$p_{mn}=cas\left(\frac{2\pi nk}{N}\right)$$

where, $cas(\theta)=\cos(\theta)+\sin(\theta)$, k = 0, 1, 2,… (N-1)

The complex baseband OFDM signal with N subcarriers can be written as,

$$x(t)=\frac{1}{\sqrt{NL}}\sum_{k=0}^{NL-1}P.X_k e^{j2\pi k\Delta ft},\ 0\le t\le N. \quad (11)$$

Here P is precoding matrix of order (N×N) given in Eq.(10), m and n are the integers from 0 to (N-1). The modulated OFDM vector signal with N subcarriers can be signified as, $x_N = ifft(X_N)$.

The PAPR of OFDM signal in Eq.(7) can be written as,

$$PAPR=\frac{\max\left(\left|x(t)^2\right|\right)}{E\left[\left|x(t)^2\right|\right]}. \quad (12)$$

According to Eq.(11),the N-point DHT can be expressed as,

$$H_k=\sum_{n=0}^{N-1}x_n\left[\cos\left(\frac{2\pi nk}{N}\right)+\sin\left(\frac{2\pi nk}{N}\right)\right], k=0,1,...,(N-1) \quad (13)$$

The DHT is also invertible transform which allow us to recover the x_n from H_k and inverse can be obtained by simply multiplying DHT of H_k by (1/N).

5. PROPOSED TECHNIQUE

In this section, the authors propose a hybrid companding transform (DHT Precoded OFDM with PLC) to reduce the PAPR of OFDM signal by combining Piecewise Linear Companding transform (PLC) and DHT transform.

The incoming input data stream is firstly transformed by Discrete Hartley Transform, and then the transformed data stream is given as input to IFFT signal processing unit. The OFDM system with proposed technique is shown at bottom of this page. The hybrid companding transform technique processing steps are given below:

Step 1: Firstly DHT transform is applied to the sequence X, i.e. $Y = HX$

Step 2: Apply IFFT to DHT transformed signal, y = IFFT(Y), where, $y = [y(1)\ y(1)\ \ldots\ y(N)]^T$

$$y_n=\frac{1}{\sqrt{NL}}\sum_{k=0}^{N-1}Y_k.e^{j2\pi\frac{kn}{NL}},\quad 0\le n\le NL-1$$

Step 3: Apply piecewise linear companding transform to y, i.e. $s(n) = C\{y(n)\}$

$$s(n)=C(y(n))=\begin{cases} y(n) & |y(n)|\le A_i \\ my(n)+(1-m)A_c & A_i<|y(n)|\le A_c \\ \operatorname{sgn}(y(n))A_c & |y(n)|>A_c \end{cases}$$

Step 4: Apply piecewise linear decompanding transform to the received signal $r(n)$, i.e. $\hat{y}(n) = C^{-1}\{y(n)\}$

$$\hat{y}(n)=C^{-1}(r(n))=\begin{cases} r(n) & |r(n)|\le A_i \\ \dfrac{(r(n)-(1-m)A_c)}{m} & (1-m)A_c<|r(n)|\le A_c \\ \operatorname{sgn}(r(n))A_c & |r(n)|>A_c \end{cases}$$

Step 5: Apply FFT transform to the signal $\hat{y}(n)$ i.e. $\hat{Y}$ = FFT($\hat{y}$), where $\hat{y} = [\hat{y}(1)\ \hat{y}(2)\ \ldots\ \hat{y}(N)]^T$

Step 6: Apply inverse DHT transform to the signal $\hat{Y}$ i.e. $\hat{X}=H^T\hat{Y}$. Then the signal $\hat{X}$ is demapped to bit Stream.

6. SIMULATION RESULTS

Computer simulation results are conferred in this section to appraise the performance of the proposed technique i.e. hybrid companding transform (DHT Precoded OFDM with PLC) with reference to the PAPR reduction, BER as well as PSD performance. The number of subcarrier to be N = 256 and oversampling factor L is 4 as per IEEE 802.16 (WiMAX) standards.

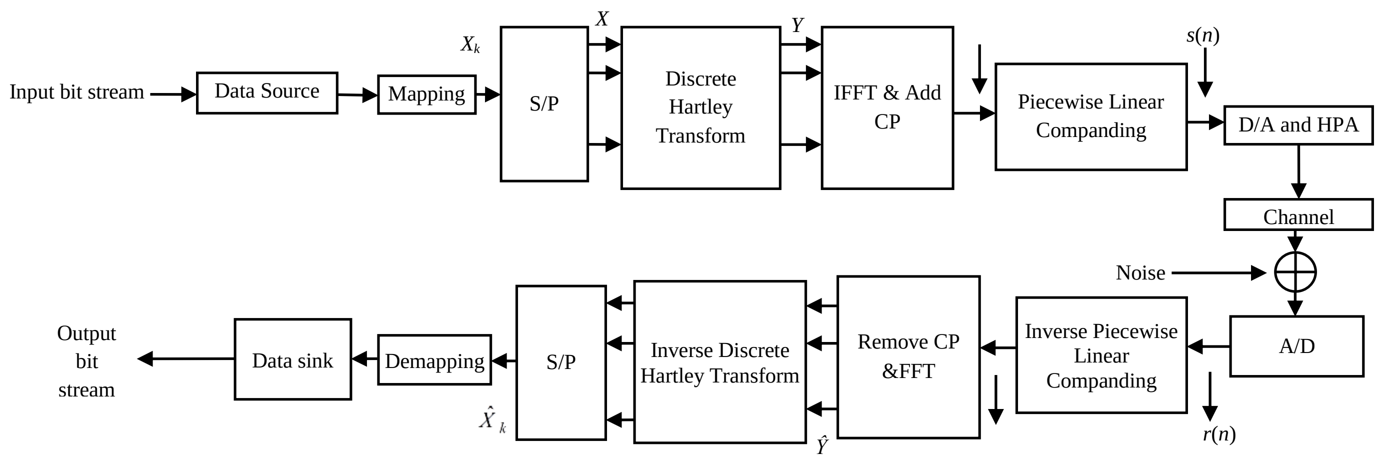

Fig.2. Block diagram of OFDM system with DHT and Piecewise Companding

The baseband modulation techniques 4-QAM and 16-QAM are considered here. Solid State Power Amplifier (SSPA) is considered as a model for input-output characteristics of the nonlinear region by passing companded signals through HPA. This SSPA model is formulated by,

$$\left|z(t)\right| = \frac{\left|y(t)\right|}{\left(1+\left(\frac{\left|y(t)\right|}{A_{sat}}\right)^{2p}\right)^{\frac{1}{2p}}}. \quad (14)$$

where, A_{sat} is the saturation level, and knee factor (p) = 2 is more commonly used.

6.1. PERFORMANCE IN PAPR REDUCTION

The Fig.3 and Fig.4 plots the Simulated Complementary Cumulative Distribution Function (CCDF) of PAPR of the proposed technique (DHT Precoded OFDM with PLC) and previous companding techniques with 4-QAM &16-QAM as the modulation techniques. The authors observed from Fig.3, Fig.4 the proposed technique can draw good PAPR reduction. Given that CCDF = 10^{-3}, from Fig.3 the proposed technique with $PAPR_{preset}$ = 4dB is 0.9dB, $PAPR_{preset}$ = 4.5dB is 1.4dB and $PAPR_{preset}$ = 5dB is 2dB superior over piecewise linear companding technique with their respective $PAPR_{preset}$ value as well as OFDM precoded with DHT also.

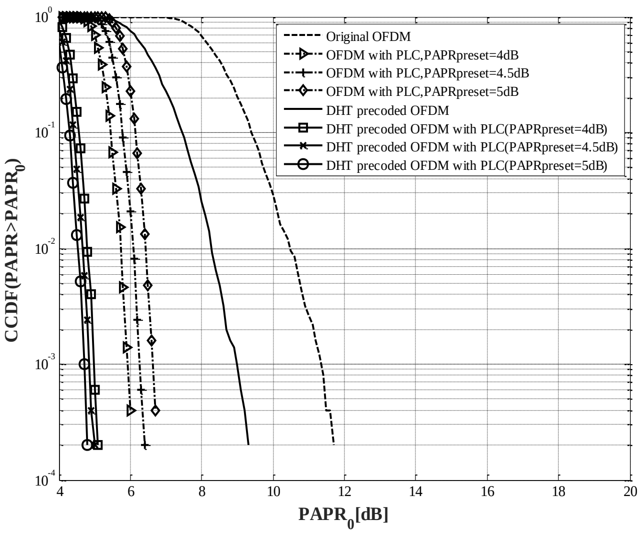

Fig.3. CCDFs of original OFDM signal and different PAPR reduction techniques with 4-QAM modulation

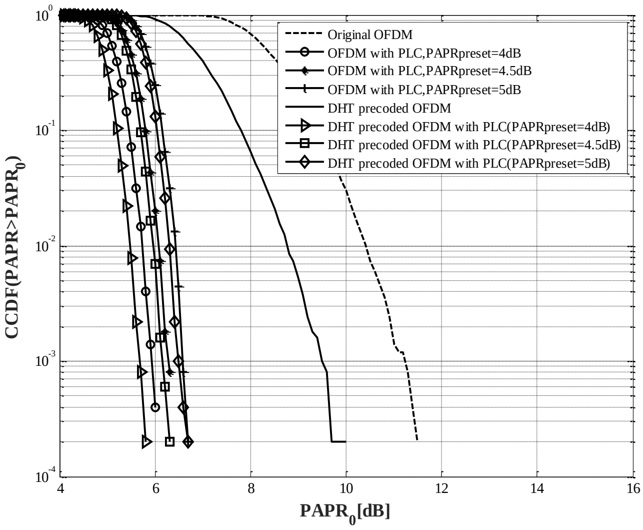

Fig.4. CCDFs of original OFDM signal and different PAPR reduction techniques with 16-QAM modulation

6.2. BER PERFORMANCE

The impact of the proposed hybrid companding transform technique on the BER performance is discussed in this section. BER versus Signal to Noise Ratio (SNR) curves with different companding transforms under an AWGN channel using 4-QAM and 16-QAM are shown in Fig.5 and Fig.6 respectively. From Fig.5, it is observed that BER performance is improved with 4-QAM modulation, with the proposed technique.

At BER level of 10^{-3}, the proposed technique with $PAPR_{preset}$ = 4dB exceeds the Piecewise Linear Companding (PLC) technique by 0.5dB.

It is also observed that in 16-QAM (Fig.6), the BER performance of proposed technique has performance floor at high SNR because of the output of the proposed companding function is not continuous. At a BER level of 10^{-2}, the proposed technique with $PAPR_{preset}$ = 4dB is superior over the piecewise linear technique by 0.1dB of their respective $PAPR_{preset}$ value.

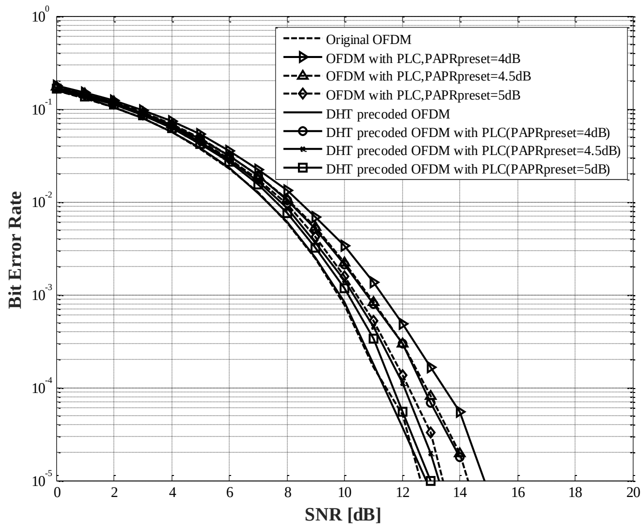

Fig.5. BER Performance of original OFDM signal and different techniques over AWGN channel with 4-QAM modulation

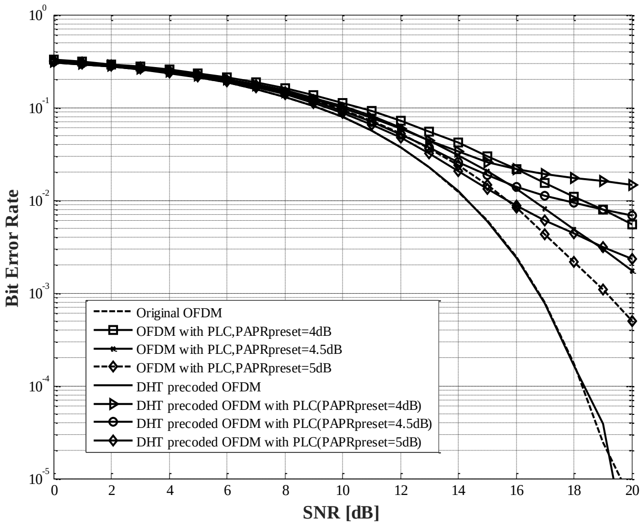

Fig.6. BER Performance of original OFDM signal and different techniques over AWGN channel with 16-QAM modulation

The Fig.7 and Fig.8 illustrates the BER performance using 4-QAM, 16QAM modulation with SSPA passing through an AWGN channel. It can be seen that the BER performance of the proposed technique with SSPA model is also sufficient.

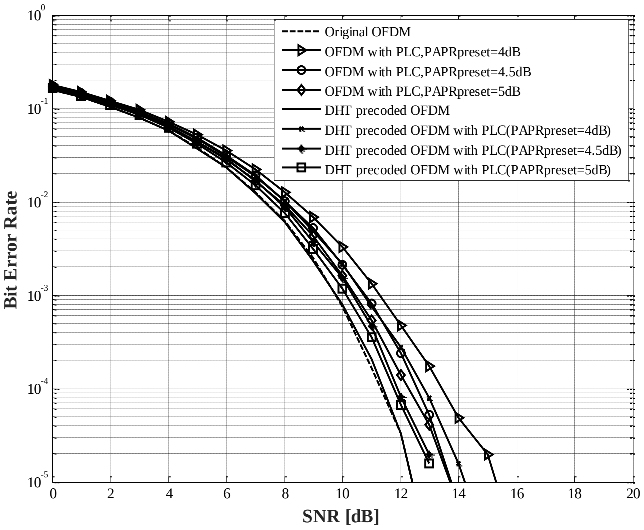

Fig.7. BER Performance of original OFDM signal and different techniques with SSPA over AWGN channel with 4-QAM modulation

6.3. PSD PERFORMANCE

The PSD performances for the original OFDM signal and the other companded OFDM signals are given in Fig.9 and Fig.10. The PSDs are computed by means of nonparametric method to get PSD comparison among different companding techniques.

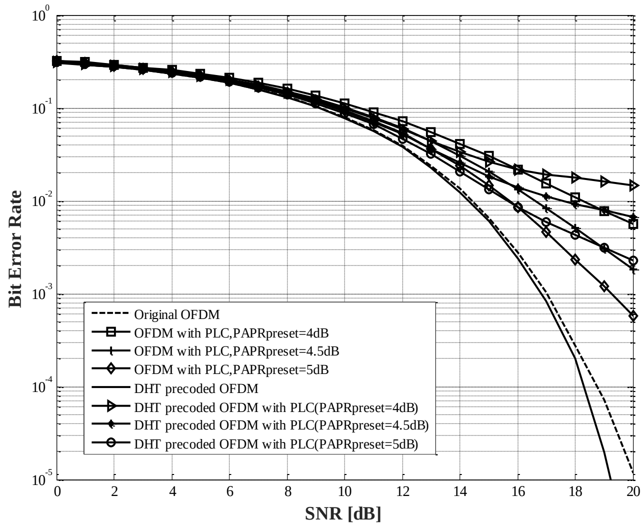

Fig.8. BER Performance of original OFDM signal and different techniques with SSPA over AWGN channel with 16-QAM modulation

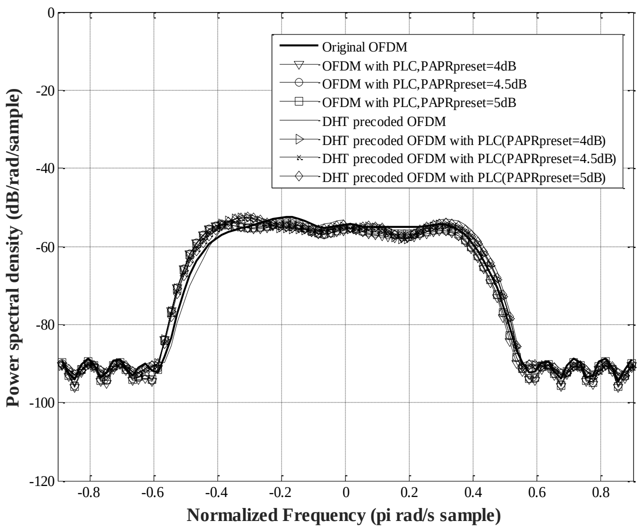

Fig.9. PSD's of original OFDM signal and different PAPR reduction techniques with 4-QAM modulation

7. CONCLUSION

In this paper, a hybrid companding transform (DHT Precoded OFDM with PLC) is proposed to reduce PAPR in OFDM signals and the Simulation results are compared with the existing Piecewise linear companding (PLC) technique. The PAPR is reduced by 2dB when considering the preset value as 5dB. The BER performance under an AWGN channel using 4-QAM is improved by 0.5dB and maintaining the same performance in 16QAM. The PAPR reduction as well as the improvement in BER performance is achieved without sacrificing the PSD performance.

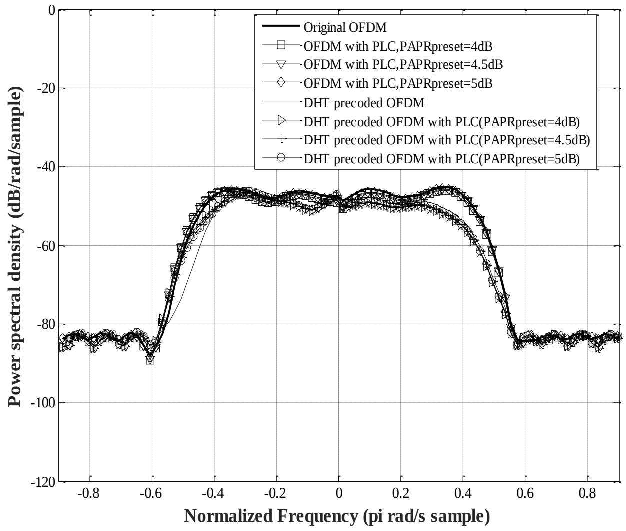

Fig.10. PSD's of original OFDM signal and different PAPR reduction techniques with 16-QAM modulation

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