

# BER PERFORMANCE COMPARISON OF MIMO SYSTEMS USING OSTBC WITH ZF AND ML DECODING

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## Abstract

*Multiple Input Multiple Output (MIMO) systems with multiple antenna elements at both transmitter and receiver ends are an efficient solution for wireless communication systems. They provide high data rates by exploiting the spatial domain under the constraints of limited bandwidth and transmit power. Space-Time Block Coding (STBC) is a MIMO transmit strategy which exploits transmit diversity and provides high reliability. Implementation of orthogonal space-time block codes (OSTBCs) for a two transmitter–two receiver system under AWGN (Additive White Gaussian Noise) channel and flat fading channel is performed. Alamouti code is employed for the STBC. The modulation techniques used are BPSK, QPSK and 16-QAM. Decoding is done using the Zero Forcing (ZF) algorithm and Maximum Likelihood (ML) algorithm. The BER Performance of each modulation scheme is compared with the un-coded version of the same. Performance comparison between the two decoding techniques is also done. It is found that ML detection offers a slightly better performance for BPSK and QPSK system than ZF detection.*

## Keywords:

Alamouti Code, Flat Fading, STBC, ZFD, MLD

## 1. INTRODUCTION

Optimal design and successful deployment of high performance wireless networks present a number of technical challenges. These include regulatory limits on usable radio frequency spectrum and a complex time-varying propagation environment affected by fading and multipath.

Fading can be mitigated by diversity, which means that, the information is transmitted not only once but several times, hoping that at least one of the replicas will not undergo severe fading. In order to meet the growing demand for higher data rates at better quality of service (QoS) with fewer dropped connections, boldly innovative techniques that improve both spectral efficiency and link reliability are called for. There exist different diversity techniques but the use of multiple antennas at the transmitter and receiver (MIMO) in a wireless network is a rapidly emerging technology.

Spatial diversity uses multiple antennas to achieve diversity. If the antennas are separate enough, more than half of the wavelength, signals corresponding to different antennas fade independently. A (MIMO) channel is created in which each path from one transmit antenna to one receive antenna can be viewed as one signaling path. Space time coding combines *all* the copies of the received signal in an optimal way to extract as much information from each of them as possible. MIMO systems have two major attractive advantages [1].

- The channel capacity of a multiple-antenna system is considerably higher than that of a single-antenna system

- Fading can be effectively mitigated and hence, link reliability is significantly improved

Multiple antennas play an important role in advanced wireless systems. MIMO is used for high speed packet data mode for third and fourth generation systems. MIMO has also influenced wireless local area networks (WLANs) like IEEE802.11n. Since transmission energy is enhanced by using STBC's, they have been used for transmission in wireless sensor networks, where each node has to operate without battery replacement for a long time and energy consumption is an important constraint. More recently MIMO signal processing has also found its way into power line communications (PLC) [2].

The concept of space-time coding was introduced by Tarokh et al [3]. It mainly deals with space-time trellis codes (STTC). It combines signal processing at the receiver with coding techniques appropriate to multiple transmit antennas and provides significant gain over previous transmit diversity schemes as the delay diversity scheme by Sheshadri and Winters [4]. Alamouti [5] discovered a remarkable scheme for transmission using two transmit antennas. These codes are orthogonal and allow linear processing at the receiver. Decoding methods considered at first were heuristic methods like Zero Forcing Detection (ZFD) but optimal solutions were provided by Maximum Likelihood Detection (MLD).

Space-time codes fully utilize the diversity advantage to improve the error probability behavior. This family of code design performs coding across both time and space (transmit antennas) dimensions. It works with multiple transmit antennas and does not necessarily need multiple receive antennas. The number of transmitted code symbols per time slot are equal to the number of transmit antennas. The design criteria of space time codes apply to the complex domain of the baseband modulated signals rather than to the binary or discrete domain in which the underlying codes are traditionally designed. Current space-time codes include STTC and space-time block codes (STBC).

The paper is organized as follows. Section 2 of the paper deals with the STBC that provides full rate diversity, the Alamouti code. Section 3 deals with the system model used in the simulation. This is followed by the results and analysis in section 4. Finally section 5 provides the conclusion.

## 2. ALAMOUTI CODE

One of the most commonly used STBC is the Alamouti code [5]. Assume a system with  $N = 2$  transmit antennas and one receive antenna, employing Alamouti code. To transmit  $b$  bits/cycle, we use a modulation scheme that maps every  $b$  bits to one symbol from a constellation with  $2^b$  symbols [6].

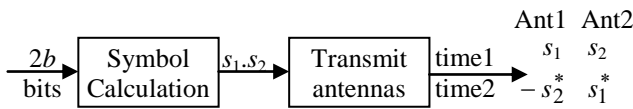


Fig.1. Transmitter Block Diagram for Alamouti Code

For a PSK constellation, first, the transmitter picks two symbols from the constellation each of  $b$  bits. If  $s_1$  and  $s_2$  are the selected symbols for a block of  $2^b$  bits, the transmitter sends  $s_1$  from antenna one and  $s_2$  from antenna two at time one. Then at time two, it transmits  $-s_2^*$  and  $s_1^*$  from antennas one and two, respectively. Therefore, the transmitted codeword is

$$C = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \quad (1)$$

It is clear that the encoding is performed in both time (two transmission intervals) and space domain (across two transmit antennas).  $s_1^*$  is the conjugate of  $s_1$  and orthogonal to  $s_1$ . Similarly,  $-s_2^*$  is the negative of the conjugate of  $s_2$  and is orthogonal to  $s_2$ . The STBC matrix is unitary. Alamouti encoding belongs to the class of orthogonal codes, OSTBC (orthogonal space time block codes) which allows simple decoding.

This STBC achieves symbol transmission rate = 1 at full diversity and has a full rank of 2. The average transmission power of this STBC is  $\frac{s_i}{\sqrt{2}}$  through each of the two antennas.

This is same as the system transmitting  $s_i$  through each antenna.

Here BPSK, QPSK and 16-QAM have been used as the modulation techniques at the transmitter and the receiver. The decoding techniques used are Zero forcing Detection (ZFD) and Maximum Likelihood Detection (MLD).

**2.1 ZERO FORCING DETECTOR**

In this type of detection, all the elements of the channel matrix other than the diagonal elements are forced to zero. Assuming channel matrix  $H$  is invertible, the transmitted symbol at receiver is estimated as,

$$\hat{s} = H^\dagger y \quad (2)$$

where,  $H^\dagger$  denotes the pseudo inverse of matrix  $H$  and is defined as  $H^\dagger = (H^T H)^{-1} H^T$ .

**2.2 MAXIMUM LIKELIHOOD DETECTOR**

One attractive feature of orthogonal STBCs is that MLD can be achieved at the receiver with only linear processing. This is a method that compares the received signal with all possible transmitted vectors and estimates  $s$  according to the Maximum Likelihood principle which can be formalized by the following formula:

$$\hat{s} = \arg \min \|y - Hs_j\|^2 \quad (3)$$

where,  $j = 1, 2, \dots, K$ . This requires an exhaustive search through all  $K$  possible transmitted vectors.

$$K = M^{N_t} \quad (4)$$

where,  $M$  represents the number of constellation points and  $N_t$  is the number of transmit antennas.

The complexity of this algorithm increases as the number of transmit antennas increases. These receivers usually provide the maximum diversity and lower BER.  $\hat{s}$  is the estimated value of data and  $y$  is the received data after passing through the channel.  $H$  is the channel matrix.

**3. SYSTEM MODEL**

The system model used in the simulation is as shown in Fig.2.

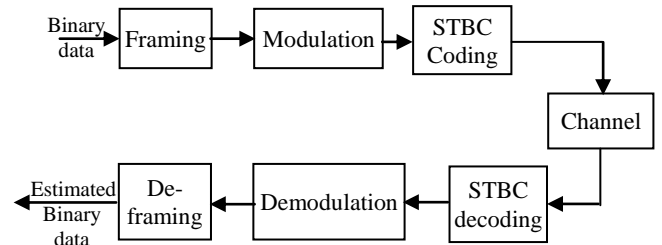


Fig.2. Communication System Model

MIMO systems with two transmit and two receive antennas are considered here. The input binary data has been converted into frames. Each frame is then modulated according to the required scheme and then encoded using the Alamouti principle. The STBC encoded data is sent through transmitters. Various types of channels are simulated namely the AWGN channel and the flat fading Rayleigh channel. The received signals are then STBC decoded after equalization. Demodulation is then performed using ZFD and MLD to calculate the estimated binary input data. Bit error rate (BER) is calculated after de-framing and BER versus Signal to noise ratio (SNR) graphs are plotted.

**3.1 AWGN CHANNEL**

An AWGN channel adds white Gaussian noise to the signal that passes through it as seen in Eq.(5).

$$y = s + n \quad (5)$$

Here the received signal  $y$  is the sum of the transmitted signal,  $s$  and the noise,  $n$ .

**3.2 FLAT FADING CHANNEL**

Flat fading, or frequency non-selective fading, applies by definition to systems where the bandwidth of the transmitted signal is much smaller than the coherence bandwidth of the channel. All the frequency components of the transmitted signal within the same frame undergo the same attenuation and phase shift propagation through the channel.

$$y = Hs + n \quad (6)$$

where,  $y$  and  $s$  are the receive and transmit signals, respectively.  $H$  is the channel matrix and  $n$  is the noise vector.

Assumptions made during simulation are as follows.

- 1) The channel experienced by each transmit antenna is independent from the channel experienced by other transmit antennas.
- 2) The channel experienced between each transmit to the receive antenna is randomly varying in time. However the channel is assumed to be constant over two time slots.
- 3) When the channel is flat fading, convolution operation reduces to a simple multiplication. For the  $i^{th}$  transmit antenna, each transmitted symbol gets multiplied by a randomly varying complex number,  $h_i$ . As the channel under consideration is a flat fading channel, the real and imaginary parts of  $h_i$  are Gaussian distributed having mean  $\mu_{hi} = 0$  and variance  $\sigma_{h_i}^2 = \frac{1}{2}$ .
- 4) On the receive antenna, the noise  $n$  has the Gaussian probability density function given by Eq.(7)
- 5) 
$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}$$
 with  $\mu = 0$  and  $\sigma^2 = \frac{N_0}{2}$  (7)
- 6) The channel estimation is not done at the receiver. The channel coefficients  $h_i$  are assumed to be known at the receiver.

**4. RESULTS ANALYSIS**

Monte Carlo simulations are carried out using MATLAB and the BER vs. SNR graphs are plotted for both the channels.

**4.1 ZERO FORCING DETECTOR**

**4.1.1 AWGN Channel:**

For STBC coded BPSK, the  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 8.35 dB and for  $10^{-5}$  it is 9.69 dB. The corresponding  $E_b/N_0$  values of the un-coded BPSK are 11.4 dB and 12.6 dB. The coding gain is 3.05 dB and 2.91 dB for BER of  $10^{-4}$  and  $10^{-5}$  respectively.

For STBC coded QPSK, the  $E_b/N_0$  values at BER of  $10^{-4}$  is 8.67 dB and for  $10^{-5}$  it is 10.04 dB. The values of SNR for the same BER for the un-coded QPSK are 11.80 dB and 12.84 dB respectively. The coding gain for the above BER values is 3.13 dB and 2.8 dB and 2.8 dB.

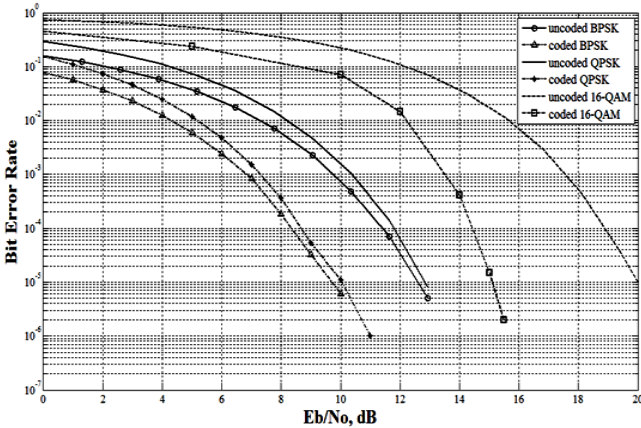


Fig.3. Comparison between Un-coded and STBC Coded Modulation Schemes for AWGN Channel with ZF Decoding

For STBC coded 16-QAM, the  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 14.43 dB and for  $10^{-5}$  it is 15.10 dB. Un-coded 16-QAM has values of 18.9 dB and 20 dB for the BER of  $10^{-4}$  and  $10^{-5}$ . The coding gain for the above BER values is approximately 4.47 dB and 4.9 dB. It can be seen that as the BER values decrease, the BER performance of the coded STBC increases.

Table.1. BER Performance Comparison of different Modulation Schemes under AWGN channel for ZF Decoding

Modulation/Coding	$E_b/N_0$ requirement in dB for BER of		Coding Gain in dB for BER of	
	$10^{-4}$	$10^{-5}$	$10^{-4}$	$10^{-5}$
BPSK – Un-coded	11.40	12.60	3.05	2.91
BPSK – STBC coded	8.35	9.69		
QPSK – Un-coded	11.80	12.84	3.13	2.8
QPSK – STBC coded	8.67	10.04		
16-QAM – Un-coded	18.90	20.00	4.47	4.9
16-QAM – STBC coded	14.43	15.10		

**4.1.2 Flat Fading Channel:**

For STBC coded BPSK, the  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 13.2 dB and for  $10^{-5}$  it is 16 dB. The corresponding  $E_b/N_0$  values of the un-coded BPSK are 36.3 dB and 47.73 dB. The coding gain is 23.1 dB and 31.73 dB for BER of  $10^{-4}$  and  $10^{-5}$  respectively.

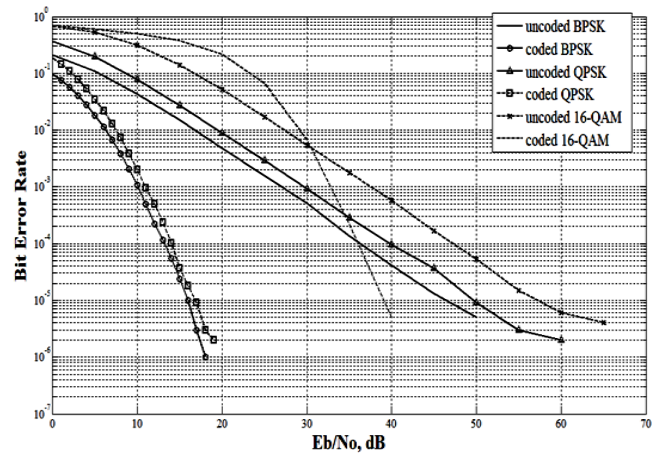


Fig.4. Comparison between Un-coded and STBC Coded Modulation Schemes for Flat Fading Channel with ZF Decoding

Table.2. BER Performance Comparison of different modulation schemes under flat fading channel for ZF decoding

Modulation/Coding	$E_b/N_0$ requirement in dB for BER of		Coding Gain in dB for BER of	
	$10^{-4}$	$10^{-5}$	$10^{-4}$	$10^{-5}$
BPSK – Un-coded	36.3	47.73	23.1	31.73
BPSK – STBC coded	13.2	16.00		
QPSK – Un-coded	39.8	49.63	25.8	32.76

QPSK – STBC coded	14.0	16.87		
16-QAM – Un-coded	47.2	57.20	11.2	18.13
16-QAM – STBC coded	36.0	39.07		

For STBC coded QPSK, the  $E_b/N_0$  values at BER of  $10^{-4}$  is 14 dB and for  $10^{-5}$  it is 16.87 dB. The values of BER for the un-coded QPSK are 39.8 dB and 49.63 dB respectively. The coding gain for the different BER values is 25.8 dB and 32.76 dB.

For STBC coded 16-QAM, the  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 36 dB and for  $10^{-5}$  it is 39.07 dB. Un-coded 16-QAM has values of 47.25 dB and 57.2 dB for the BER of  $10^{-4}$  and  $10^{-5}$ . The coding gain for both the above BER values is approximately 11.2 dB and 18.13 dB.

## 4.2 MAXIMUM LIKELIHOOD DETECTOR

### 4.2.1 AWGN Channel:

For STBC coded BPSK,  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 8.33 dB and for  $10^{-5}$  it is 9.55 dB. The coding gain is 3.07 dB and 3.05 dB for BER of  $10^{-4}$  and  $10^{-5}$  respectively.

For STBC coded QPSK, the  $E_b/N_0$  values at BER of  $10^{-4}$  is 8.77 dB and for  $10^{-5}$  it is 9.68 dB. The coding gain for the above BER values is 3.03 dB and 3.16 dB.

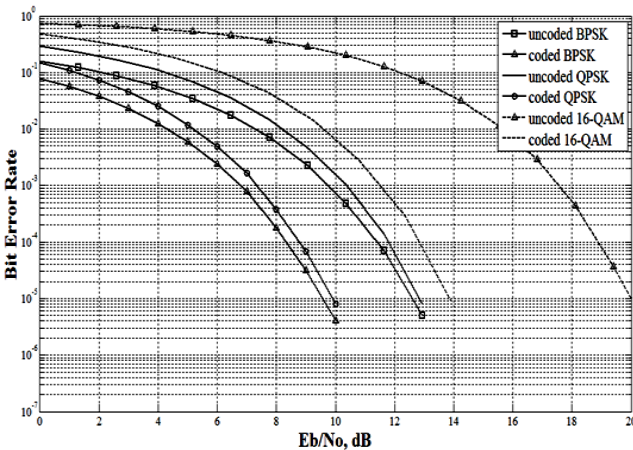


Fig.5. Comparison between Un-coded and STBC Coded Modulation Schemes for AWGN Channel with ML Decoding

For STBC coded 16-QAM, the  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 12.83 dB and for  $10^{-5}$  it is 13.86 dB. The coding gain for both the BER values is approximately 6.07 dB and 6.14 dB. Un-coded values of BPSK, QPSK and 16-QAM are the same as for ZFD.

Table.3. BER Performance Comparison of different Modulation Schemes under AWGN Channel for ML Decoding

Modulation/Coding	$E_b/N_0$ requirement in dB for BER of		Coding Gain in dB for BER of	
	$10^{-4}$	$10^{-5}$	$10^{-4}$	$10^{-5}$
BPSK – Un-coded	11.40	12.60	3.07	3.05
BPSK – STBC coded	8.33	9.55		
QPSK – Un-coded	11.80	12.84	3.03	3.16

QPSK – STBC coded	8.77	9.68		
16-QAM – Un-coded	18.90	20.00	6.07	6.14
16-QAM – STBC coded	12.83	13.86		

### 4.2.2 Flat Fading Channel:

For STBC coded BPSK, the  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 13.06 dB and for  $10^{-5}$  it is 15.50 dB. The coding gain is 23.24 dB and 32.23 dB for BER of  $10^{-4}$  and  $10^{-5}$  respectively.

For STBC coded QPSK, the  $E_b/N_0$  values at BER of  $10^{-4}$  is 14 dB and for  $10^{-5}$  it is 16.36 dB. The coding gain for the above BER values is 25.8 dB and 33.27 dB.

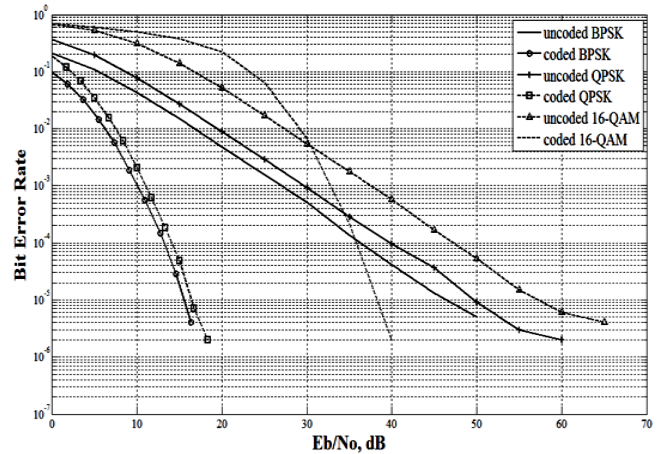


Fig.6. Comparison between Un-coded and STBC Coded Modulation Schemes for Flat Fading Channel with ML Decoding

For STBC coded 16-QAM, the  $E_b/N_0$  value needed to attain a BER of  $10^{-4}$  is 35.77 dB and for  $10^{-5}$  it is 38.26 dB. The coding gain for both the BER values is approximately 11.48 dB and 18.94 dB.

Table.4. BER Performance Comparison of different modulation schemes under flat fading channel for ML decoding

Modulation/Coding	$E_b/N_0$ requirement in dB for BER of		Coding Gain in dB for BER of	
	$10^{-4}$	$10^{-5}$	$10^{-4}$	$10^{-5}$
BPSK – Un-coded	36.30	47.73	23.24	32.23
BPSK – STBC coded	13.06	15.50		
QPSK – Un-coded	39.80	49.63	25.8	33.27
QPSK – STBC coded	14.00	16.36		
16-QAM – Un-coded	47.25	57.20	11.48	18.94
16-QAM – STBC coded	35.77	38.26		

In a flat fading channel, BER performance improvement of MLD over ZFD is as follows. For BPSK system, with MLD the BER performance improvement w.r.t coding gain is 1.55% as compared to the same system with ZFD. For QPSK system with MLD the BER improvement w.r.t coding gain, as compared to ZFD is similar to the BPSK system and for 16-QAM system

with MLD the BER performance improvement is 4.29% over that of ZFD.

## 5. CONCLUSION

BPSK and QPSK performances are similar. 16-QAM performance in terms of coding gain is 20.17% compared to that of BPSK and QPSK for AWGN channel. MLD offers a better performance than ZFD for BPSK and QPSK and 16-QAM in an AWGN channel. The improvement in the BER performance of MLD is also greater in a Rayleigh fading channel.

Sphere Decoding can be carried out for the above system and performance comparison of the three decoding techniques can be conducted.

## REFERENCES

- [1] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-Time Block Coding for Wireless Communications: Performance Results", *IEEE Journals on Selected Areas in Communications*, Vol. 17, No. 3, pp. 451-460, 1999.
- [2] L. T. Berger, Andreas Schwager, Daniel Schneider and Pascal Pagani, "MIMO Power Line Communications: Narrow and Broadband Standards, EMC, and Advanced Processing", *Devices, Circuits, and Systems*, 2014.
- [3] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction", *IEEE Transactions on Information Theory*, Vol. 44, No. 2, pp. 744-765, 1998.
- [4] N. Sheshadri and J. H. Winters, "Two signaling schemes for improving the error performance of frequency division duplex (FDD) transmission systems using transmitter antenna diversity", *International Journal of Wireless Information Networks*, Vol. 1, No. 1, pp. 49-60, 1994
- [5] S. Alamouti, "A Simple Transmitter Diversity Technique for Wireless Communications", *IEEE Journal on Selected Areas of Communications*, Vol. 16, No. 8, pp. 1451-1458, 1998.
- [6] Hamid Jafarkhani, "Space Time Coding Theory and Practice", Cambridge University Press, 2005.
- [7] Vahid Tarokh, Hamid Jafarkhani and A. R. Calderbank, "Space-Time Block Coding for Wireless Communications: Performance Results", *IEEE Journal on Selected Areas in Communications*, Vol. 17, No. 3, pp. 451-460, 1999.
- [8] A. Wittenben, "Base station modulation diversity for digital SIMULCAST", *IEEE Vehicular Technology Conference*, pp. 848-853, 1993.
- [9] G. J. Foschini and M. J. Gans, "On Limits of Wireless Communications in Fading Environments when Using Multiple Antennas", *Wireless Personal Communications*, Vol. 6, pp. 311-335, 1998.
- [10] J. H. Winters, J. Salz and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems", *IEEE Transactions on Communications*, Vol. 42, No. 234, pp. 1740-1751, 1994.
- [11] Richard Roy and Björn Ottersten, "Spatial division multiple access wireless communication systems", Patents Publication no. US 5515378, 1996.
- [12] Arogyaswami Paulraj and Thomas Kailath, "Increasing capacity in wireless broadcast systems using distributed transmission/directional reception (DTDR)", Patents Publication no. US 5345599, 1994.
- [13] G. D. Golden, G. J. Foschini, R. A. Valenzuela and Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture", *Electronic Letters*, Vol. 35, pp. 14-16, 1999.
- [14] H. El Gamal and M. O. Damen, "Universal space-time coding", *IEEE Transactions on Information Theory*, Vol. 49, No. 5, pp. 1097-1119, 2003.
- [15] Meixia Tao and R. S. Cheng, "Generalized layered space-time codes for high data rate wireless communications", *IEEE Transactions on Wireless Communications*, Vol. 3, No. 4, pp. 1067-1075, 2004.
- [16] M. Mohammad, S. Al-Ghadhban, B. Woerner and W. Tranter, "Comparing Decoding algorithms for multi-layer Space Time Block codes", *Proceedings of the IEEE South East Conference*, pp. 147-152, 2004.
- [17] Theodore S. Rappaport, "Wireless Communication-Principles and Practice", Prentice-Hall, 2005.
- [18] Shu Lin and Daniel J. Costello, "Error Control Coding: Fundamentals and Applications", Prentice-Hall, 1983.