

# ADAPTIVE GUARD INTERVAL AND POWER ALLOCATION FOR OFDM-BASED COGNITIVE RADIO

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## Abstract

*The function of the Radio Resource Management module of a CR system is to evaluate the available resources and assign them to meet the Quality of Service (QoS) objectives of the Secondary User (SU), within some constraints on factors which limit the performance of the Primary User (PU). While interference mitigation to the PU spectral band from the SU's transmission, has received a lot of attention in recent literature, the novelty of our work is in considering a more realistic and effective approach of dividing the PU into sub-bands, and ensuring that the interference to each of them is below a specified threshold. The objective of this paper is joint determination of the channel adaptive guard interval, along with sub-carrier power allocation for an orthogonal frequency division multiplexing (OFDM)-based SU, to maximize the SU's throughput, within a power budget and with the aforementioned PU interference constraint. A multiple SU scenario is also considered, which entails assigning sub-carriers to users, besides determining the guard interval and power allocation. Simulation results are provided, which indicate the effectiveness of the proposed algorithms in a CR environment.*

## Keywords:

*Cognitive Radio, OFDM, Interference Mitigation, Guard Interval, Power Allocation*

## 1. INTRODUCTION

A new wireless communication paradigm, called *Cognitive Radio (CR)*, has emerged in recent times to alleviate the imbalance between spectrum allocation and its use [1][2]. CR entails the temporary usage of unused portions of the spectrum (*spectrum holes* or *white spaces*), owned by the licensed users (*Primary Users* or *PUs*) to be accessed by unlicensed users (*Secondary Users* or *SUs*). Built on the platform of *software-defined radio (SDR)*, a CR node has the ability to interact with its environment in real-time and dynamically reconfigure its operating parameters such as frequency range, modulation type or output power, in software, without making any alteration in the hardware [2]. It is anticipated that the *Next-Generation (xG)* communication networks will be based on CR [2].

An essential component of a CR system is the *Radio Resource Management (RRM)* module, the aim of which is to evaluate the available resources and assign them to meet the QoS objectives of the SU, within some constraints on factors (typically interference) which limit the performance of the licensed user or the PU. Furthermore, for optimum spectrum utilization it is necessary to be adaptive to, one or more, time-varying characteristics of the system, such as the wireless channel state, number of users, QoS requirements, etc. RRM seeks to harmonize two contradictory concepts of limited resources and strict QoS requirements, depending on the

instantaneous state of the system, and suitably reconfigure after having detected the new state [3].

OFDM is a widely-deployed multi-carrier modulation technology for various wireless application segments, viz. wireless local area networks (IEEE 802.11a,g), wireless wide area networks (IEEE 802.16), wireless regional area networks (IEEE 802.22) and wireless personal area networks (IEEE 802.15.3a). It also presents a promising solution to enable opportunistic spectrum access in CR networks by dynamically nulling those sub-carriers where the PU claims its spectrum. This variant of OFDM is called *dis-contiguous OFDM (D-OFDM)*. Besides its ability to handle multi-path fading, it offers flexibility of resource allocation on its individual sub-carriers. When OFDM is used in CR transmission, the sub-carriers' power, constellation size, bandwidth, and the length of the guard interval, are parameters which may be reconfigured by the RRM module to improve the performance and achieve the desired system goals [4].

The OFDM transmitter converts digital data into a mapping of subcarrier amplitude and phase. It transforms this spectral representation of the data into the time domain using an Inverse Fast Fourier Transform (IFFT). In order to transmit the OFDM signal, the calculated time domain signal is then mixed up to the required frequency. Before transmission, a guard interval (also known as the cyclic prefix, since it is a cyclic copy of the original symbol) of length greater than the channel delay spread is added to the OFDM symbol. Without sufficient guard interval (GI), two difficulties arise in a slow-fading multi-path environment [5]-[7]. The first is inter-symbol interference (ISI), which is the spreading of the symbol boundaries due to multi-path propagation in radio transmission. Also, the time dispersion of the channel destroys the orthogonality between sub-carriers resulting in inter-carrier interference (ICI). Though typically the GI is selected to be greater than the maximum delay spread of the channel ( $\tau_{max}$ ), it is a well-known fact that insertion of GI decreases the spectral efficiency. Consequently, the system throughput is maximized with a GI that is not necessarily equal to  $\tau_{max}$  [7]-[13].

When OFDM is used for the SU system in a CR scenario, its side-lobes causes interference to the PUs, limiting their performance. The Federal Communications Commission's (FCC) Spectrum Policy Task Force has recommended a metric called the interference temperature, which is intended to quantify and manage the sources of interference in a radio environment [4]. Any transmission in the frequency band of interest (the PU band in the case of CR networks) is considered to be harmful if it increases the noise floor above the interference temperature limit. In an OFDM-based CR system, the amount of interference to the PU band depends on the following parameters: (i) Power

allocated to the SU's sub-carriers, and the effective sub-carrier bandwidth (which depends on the effective symbol length after adding the guard interval); (ii) The spectral distance between the SU's sub-carriers and the PU band; and (iii) The channel conditions between the SU and PU. The issue of interference mitigation in the PU band is receiving increasing attention in recent literature [14]-[25]. In CR, the optimum GI length is the one that maximizes the SU throughput while mitigating the interference to the PU band. In our previous work, we have developed a holistic resource allocation scheme for an OFDM-based CR, which covers the aspects of power allocation, bit loading and sub-carrier bandwidth sizing [23]-[25]. The contribution of this paper is to compute the channel adaptive optimum GI jointly with sub-carrier power allocation, with the objective of maximizing the SU throughput, while mitigating interference to the PU band. We have adopted a realistic and efficient strategy, wherein the PU is divided into sub-bands, and the interference to each of its sub-bands is separately constrained. Both, a single and multiple SU scenario is considered; problems pertaining to the latter are more complex, since they involve assigning sub-carriers to users, besides determination of the optimum GI and power allocation.

To detail the proposed scheme, the paper has been organized as follows: Section II presents related literature. Section III describes the system model and communication scenario for a single SU. Section IV describes the problem formulation and the proposed algorithm for a single SU. Sections V and VI are dedicated to the multiple SU scenario. Section VII presents simulation results and their discussion, while Section VIII concludes the paper.

## 2. RELATED WORK

In the context of adaptive GI length for conventional OFDM, the following references can be cited: Salvatore et al. [7] have studied capacity improvements in IEEE 802.11 via adaptation of the GI to the channel. The selection of the optimum GI is based on searching from a pre-determined set of values which maximize the throughput. Andrea et al. [8] have proposed adaptation of the GI, jointly with power allocation and bit loading. First, a uniform power allocation is executed, followed by GI determination based on the sub-optimum solution of the convex optimization problem. Lastly, the bit allocation is determined with integer granularity to achieve a certain probability of error, indicated by the signal-to-noise ratio (SNR) gap. In another work of literature, Salvatore et al. [9] have executed GI length optimization jointly with power allocation, in which they have compared uniform power allocation, uniform power allocation with peak power constraint and iterative water-filling. Osman and Rahman [10] have proposed adaptation of the CP length for mobile WiMax systems, based on testing and comparing the constellations of the transmitter and receiver; when the multi-path delay spread is greater than the GI length, the constellation diagram is highly distorted. However, the algorithm implemented is unclear.

Though there is considerable amount of work in the context of OFDM-based CR, it mainly focusses on power allocation and bit loading [14]-[25]; the issue of sub-carrier bandwidth sizing and GI determination has been largely neglected. Adaptive sub-

carrier bandwidth sizing has been recently addressed by us [25], while optimizing the GI is the objective of this paper.

## 3. SYSTEM MODEL & COMMUNICATION SCENARIO: SINGLE SU

In the current model, a single SU transceiver is considered, and a PU exists in its radio range (Fig.1). OFDM is the communication technology of the SU, the use of which divides the available bandwidth into frequency-flat sub-carriers. When the PU claims a portion of the spectrum, the SU nulls the corresponding sub-carriers. Let  $N_s$  be the number of active sub-carriers for the SU. The transmission opportunity is detected by the SU in the spectrum sensing phase of its cognitive cycle [1]. The channel power gain of the  $i^{th}$  sub-carrier on the link between the SU transmitter (Tx) and receiver (Rx) is denoted by  $h_i$ . To efficiently control the interference to the PU, the PU spectrum is divided into  $N_p$  sub-bands of equal width, and the gain of the  $j^{th}$  sub-band from the SU Tx to the PU Rx is given by  $g_j$ . The mutual interference model between the PU and SU is assumed [15]. In the present work, we have considered an immobile SU, resulting in no Doppler spread.

Resource allocation strategies in CR require that the channel state information (CSI) be known to the SU Tx. It is assumed that the SU Rx estimates the channel by measuring the received power of the pilot signals sent by the transmitter, and the CSI is fed back to the transmitter [26]-[28]. A robust and low-complexity protocol can be used for the feedback. A block fading propagation channel is assumed where the channel remains constant during the resource allocation and transmission process. The channel sensing and feedback is done once per coherence time. Estimating the channel between the PU Tx and SU Rx, as well as that between the SU Tx and PU Rx, is more challenging, and entails the use of blind estimation techniques [28].

The maximum achievable throughput of the SU, in bits/sec, is given by [9][29]

$$C = \frac{1}{Tg + \frac{1}{B} \sum_{i=1}^{N_s} \log_2(1 + SINR_i)} \quad (1)$$

and

$$SINR_i = \frac{P_i h_i}{\sigma_i^2 + P_{IN_i}} \quad (2)$$

In which B is the sub-carrier bandwidth,  $T_g$  is the duration of the GI, and  $P_i$  is the power allocated to the  $i^{th}$  SU sub-carrier.  $\sigma_i^2 = \sigma_n^2 + J_i$ , where  $\sigma_n^2$  is the Additive White Gaussian Noise (AWGN) variance and  $J_i$  is the interference from the PU on the  $i^{th}$  SU subcarrier.  $J_i$  depends on the power spectral density (PSD)

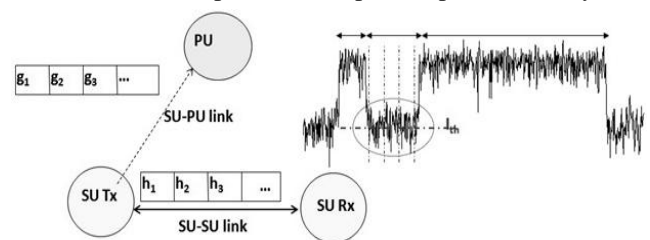


Fig.1. System model for a single Secondary User

of the PU and the channel gain between the PU Tx and SU Rx.  $P_{IN_i}$  is the interference power comprising of ISI and ICI components. Nguyen and Kuchenbecker [30] have mathematically described the interference power for OFDM in case of insufficient GI. The interference power due to ISI,  $P_{ISI}$ , depends on the tail outside the GI of the multi-path channel profile or power delay spectrum  $\rho(\tau)$ , and is given by

$$P_{ISI_i} = P_i \int_{t=0}^{\tau_{\max}-T_g} \int_{\tau=t+T_g}^{\tau_{\max}} \rho(\tau) d\tau dt. \quad (3)$$

The interference power due to ICI,  $P_{ICI}$ , is also well approximated by Eq.(3). The total interference power,  $P_{IN_i}$ , is computed as [30]

$$P_{IN_i} = P_{ISI_i} + P_{ICI_i}. \quad (4)$$

In the assumed CR scenario, the interference from the SU on the  $j^{\text{th}}$  PU sub-band is formulated as [25]

$$I_j = g_j \sum_{i=1}^{N_s} P_i \int_{j^{\text{th}} \text{PUband}} \text{Sinc}^2[(f - f_i)T_s'], \quad (5)$$

where  $T_s$  is the total length of the symbol after adding the guard interval, i.e.  $T_s' = T_s + T_g$ , and  $f_i$  represents the center frequency of the  $i^{\text{th}}$  subcarrier.  $\text{Sinc}(x)$  is the mathematical function commonly defined by  $\text{Sin}(\pi x)/(\pi x)$ .

#### 4. PROBLEM FORMULATION (SINGLE SU)

In the GI optimization problem, our objective is to maximize the SU throughput under a total node power constraint  $P_t$ , in such a way that the interference to the  $j^{\text{th}}$  PU sub-band is less than a threshold  $I_{th}^j$ .  $I_{th}^j = T_{th}^j BW_j$ , where  $T_{th}^j$  is the interference temperature limit for the  $j^{\text{th}}$  PU sub-band and  $BW_j$  is its bandwidth. For simplicity of representation, we assume that the interference threshold is the same for all PU sub-bands and is denoted by  $I_{th}$ .

It is evident from Eq.(1) and Eq.(2), that an increase in the GI causes the signal-to-interference-plus-noise ratio (SINR) to increase, unto a certain point; after which any further increase in GI causes the bandwidth efficiency to decrease, resulting in a fall in the throughput. Also, increasing the GI causes lower interference to the PU band (as indicated by Eq.(5)). The GI that maximizes the throughput can be determined with uniform sub-carrier power, i.e.  $P_i = P_t/N_s$ . However, it is possible that this value of GI does not satisfy the PU interference constraint (this will be true in most cases, unless the power budget is very small). Therefore, the optimization problem entails solving jointly for  $T_g^*$  and  $P_i^*$  to arrive at an optimum OFDM configuration which meets the interference constraint, within the power budget, while maximizing the achievable throughput. The problem is posed as follows

Problem P1:

$$\text{obj} = \max_{T_g, P_i} C \quad (6)$$

subject to

$$I_j \leq I_{th} \forall_j \quad (7)$$

$$\sum_{i=1}^{N_s} P_i \leq P_t \quad (8)$$

$$0 \leq T_g \leq \tau_{\max} \quad (9)$$

$$P_i \geq 0. \quad (10)$$

Since the denominator of the SINR expression (2) depends on  $P_i$ , the problem is clearly not a convex optimization, either with respect to  $T_g$  or  $P_i$  (as explained earlier,  $T_g$  and  $P_i$  are not independent of each other).

The proposed algorithm (*Algorithm 1*) for jointly computing  $T_g^*$  and  $P_i^*$  is depicted in the flow-chart of Fig.2.

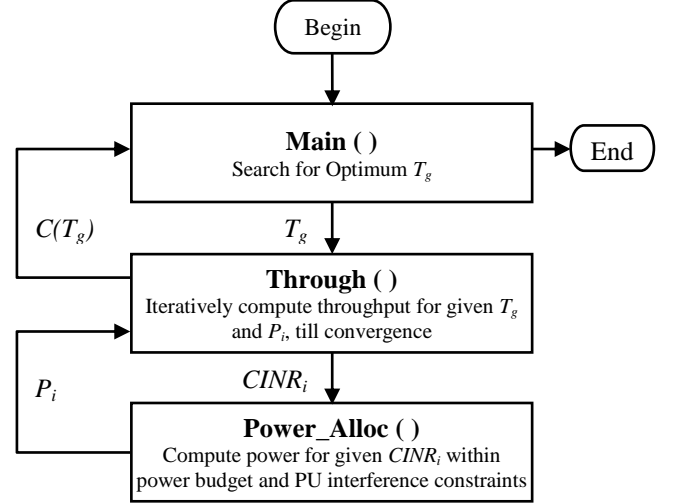


Fig.2. Flow Chart of Algorithm-1

It is motivated by the aforementioned discussion on variation in throughput with  $T_g$ , and the corresponding impact on PU interference. The algorithm consists of three modules:

- (i) The *Main()* module executes the search on  $T_g$  to look for the value which maximizes the throughput. Initially, a crude search is conducted with a larger step size of  $T_g$  to identify the optima. Then a fine search, with a small step size, is conducted in the vicinity of this crude optima to locate the global optima. The selection of the step size should consider the trade-off between computational complexity and performance of the algorithm.
- (ii) The *Through()* module computes the throughput for a given  $T_g$  and  $P_i$ . It starts with a uniform power allocation to generate the channel-gain-to-interference-plus-noise ratio (CINR), and subsequently in every iteration it uses the power allocation generated by *Power Alloc*. The module terminates if,

$$|\text{throughput}(\text{iteration}-1) - \text{throughput}(\text{iteration})| \leq \epsilon.$$

- (iii) The *Power Alloc()* module computes the sub-carrier power allocation within the power budget and PU interference constraints, given  $CINR_i$ .

The details of each of the modules of *Algorithm-1* are given below.

##### Module 1:

<i>Main()</i>
1) Initialize $T_g = 0$ . %Crude search

```

Initialize the step size for the crude search as 'step'.
Initialize  $C_{prev}(T_g) = C_{new}(T_g) = 0$ .
2) While  $\{(C_{prev}(T_g) \leq C_{new}(T_g)) \text{ AND } (T_g \leq \tau_{max})\}$  Do
{
Assign  $C_{prev}(T_g) = C_{new}(T_g)$ .
 $C_{new}(T_g) = \text{function Through}(T_g)$ .
 $T_g = T_g + \text{step}$ .
}
3)  $T_g = T_g - \text{step}$ . % Crude search ends
4) Initialize the step size for the fine search as 's'. %Fine search
5) Calculate the throughput for  $T_g, T_g + s, T_g - s$  and
represent them as  $C(T_g), C(T_g + s), C(T_g - s)$ , respectively.
6) While  $\{(C(T_g) < C(T_g + s)) \text{ OR } (C(T_g) < C(T_g - s))\}$  Do
{
 $\text{step} = \text{step}/2$ ;
If  $\{(C(T_g) < C(T_g + s))\}$ 
 $T_g = T_g + \text{step}$ .
end If
If  $\{C(T_g) < C(T_g - s)\}$ 
 $T_g = T_g - \text{step}$ .
end If
Calculate the throughput for  $T_g, T_g + s, T_g - s$  and
represent them as  $C(T_g), C(T_g + s), C(T_g - s)$ , respectively.
}
 $T_{gopt} = T_g$  % Fine search ends

```

**Module 2:**

```

function Through( $T_g$ )
1) Initially assume uniform power allocation across the
subcarriers, i.e.  $P_i = P/N_s$ .
Compute the throughput  $C_{prev}(P_i)$  using (1).
Initialize  $C_{new}(P_i) = C_{prev}(P_i)$ 
2) While  $\{C_{prev}(P_i) > C_{new}(P_i)\}$  Do
{
Assign  $C_{prev}(P_i) = C_{new}(P_i)$ .
Compute  $CINR_i = \frac{h_i}{\sigma_i^2 + P_{IN_i}}$ .
Compute the power allocation  $P_i$  using function
Power_Alloc( $CINR_i$ ).
Compute the throughput  $C_{new}(P_i)$  using (1).
}
return  $C_{new}(P_i)$ .

```

**Module 3:**

```

function Power_Alloc( $CINR_i$ )**
1) Initialize all  $\lambda_j$  and  $\mu$ .
%  $\lambda_j, \mu$  represent the Lagrangian multipliers for the convex
optimization problem of power allocation with PU interference
constraint and power budget, respectively. For details, we would
like to refer the readers to [23]
2) Compute  $P_i$  by substituting the above  $\lambda_j$  and  $\mu$  in

$$P_i^* = \max\left(\frac{1}{\lambda_j g_j Q_{j,i} + \mu} - \frac{1}{CINR_i}, 0\right)$$


```

$$\text{where } Q_{j,i} = \int_{j^{\text{th}} \text{ PUb and}} \text{Sinc}^2[(f - f_i)T_s] \text{23}.$$

Compute the total power allocated as  $P_s = \sum P_i$   
Calculate the interference caused to each PU sub-band,  $I_j$ ,  
as given by (5).

```

3) For each PU sub-band calculate the difference between
the interference generated and the threshold, as  $\text{diff}_j = I_j - I_{th}$ .
Calculate the difference between the total power allocated
and the power budget, as  $\text{diff}_p = P_s - P_r$ .
4) For each PU sub-carrier; If( $\text{diff}_j > 0$ )
 $\lambda_j = \lambda_j + a_j * \text{diff}_j$ 
end If
If( $\text{diff}_p > 0$ )
 $\mu = \mu + b * \text{diff}_p$ 
end If
5) If( $\{\text{diff}_j > 0\}$  OR ( $\text{diff}_p > 0$ ))
Goto Step2.
Else
End Algorithm
end If
return  $P_i^*$ .

```

\*\*In this module, the Lagrange multipliers  $\lambda_j$  and  $\mu$  are updated in proportion to  $\text{diff}_j$  and  $\text{diff}_p$  respectively.  $a_j$  and  $b$  are the step sizes, given by  $a_j = \text{diff}_j / \max(\text{diff}_j)$  and  $b = 1/N_s$ . The process is iteratively repeated until the power budget and PU interference constraints are satisfied.

The worst-case computational complexity of *Algorithm 1* is given by  $O(|X| + |Y|)$  in which  $X$  and  $Y$  represent the complexities of the crude search and fine search respectively.

The crude search in *Main()* is conducted over  $\tau_{max}/\text{step}$  points, and each of the points involves  $M$  iterations of *Through()*. (When executing the algorithm, we have observed that *Through()* has converged in approximately 4 iterations.) The *Power Alloc* has a complexity of  $\max(C_1 - P_r, C_2 - I_{th})N_s N_p$ , where  $C_1$  and  $C_2$  represent the initial total power ( $\sum P_i$ ) and initial maximum interference among all the PU sub-bands ( $\max I_j$ ), respectively. Thus,

$$X = \max(C_1 - P_r, C_2 - I_{th})N_p N_s M \frac{\tau_{max}}{\text{step}}. \quad (11)$$

The fine search involves a complexity given by  $Y = \max(C_1 - P_r, C_2 - I_{th}) \times N_p \times \log \text{step} \times \left( N_s + \left( \frac{\text{step}}{2} \right) \log \text{step} \right)$  (12)

## 5. SYSTEM MODEL & COMMUNICATION SCENARIO: MULTIPLE SUS

In this scenario, we assume that there are  $K$  SU transceivers, and the PU is in the radio range of all of them (Fig.3). The assumptions on the propagation channel are the same as in the single user case (Section III). The multi-user scenario is more complex than the single user situation, since it also involves assigning sub-carriers to users. All the CSI estimated at the receivers is now required to be sent to a centralized controller, which is responsible for coordinating the resource allocation in the multi-user CR network. A centralized mode involves considerable signaling overheads, especially in fast fading environments. In a slow fading environment as is assumed in

this work, the centralized architecture will compensate for the overheads with near-optimum solutions.

*Note:* Since the multiple SUs are in close proximity, we assume that their channel delay profiles are similar; consequently it is reasonable to assume the same  $T_g$  for all of them.

The throughput of the  $k^{\text{th}}$  user on the  $i^{\text{th}}$  sub-carrier is defined as [23]

$$c_{k,i} = \log_2(1 + \text{SINR}_{k,i}) \quad (13)$$

and

$$\text{SINR}_{k,i} = \frac{p_{k,i} h_{k,i}}{\sigma_{k,i}^2 + P_{\text{IN}_{k,i}}}, \quad (14)$$

where  $p_{k,i}$  is the power allocated to the  $i^{\text{th}}$  sub-carrier assigned to the  $k^{\text{th}}$  user;  $h_{k,i}$ ,  $\sigma^2$ , and  $P_{\text{IN}_{k,i}}$  denote the channel power gain, AWGN variance and interference power, respectively, of  $k^{\text{th}}$  user on  $i^{\text{th}}$  sub-carrier.

The  $N_s$  active SU sub-carriers will be assigned to the contending users, and their sum throughput is given by

$$C_m = \frac{1}{T_g + \frac{1}{B}} \sum_{k=1}^K \sum_{i=1}^{N_s} c_{k,i}(p_{k,i}), \quad (15)$$

We define  $\zeta_{k,i} = p_{k,i} * \rho_{k,i}$ , where

$$\rho_{k,i} = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ sub carrier is allocated to } k^{\text{th}} \text{ user;} \\ 0 & \text{if the } i^{\text{th}} \text{ sub carrier is not allocated to } k^{\text{th}} \text{ user} \end{cases} \quad (16)$$

The interference to the  $j^{\text{th}}$  PU band is given by

$$I_j = \sum_{k=1}^K \sum_{i=1}^{N_s} g_{k,i} \zeta_{k,i} Q_{j,i} \leq I_{th} \quad \forall j, \quad (17)$$

where  $g_{k,j}$  is the channel power gain between  $k^{\text{th}}$  SU and  $j^{\text{th}}$  primary band.

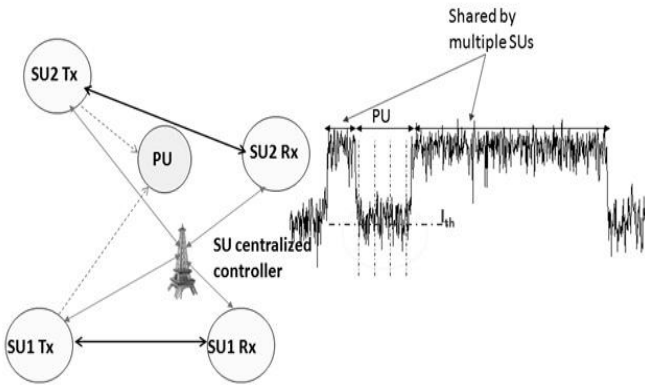


Fig.3. System model for multiple Secondary Users

The assignment of sub-carriers to users and power allocation, within a total power budget and PU interference constraints, for sum throughput maximization, is a convex optimization problem. We would like to refer the readers to our earlier work [25], in which we have applied KKT (Karush Kuhn Tucker) [32] conditions to solve the problem. The solution is of the form,

$$\zeta_{k,i}^* = \max \left( \left[ \left( \frac{1}{\sum_{j=1}^{N_p} \lambda_j g_{k,j} Q_{j,i} + \mu} - \frac{1}{\text{CINR}_{k,i}} \right) \rho_{k,i}^* \right], 0 \right) \quad (18)$$

$$\text{and } \rho_{k,i}^* = \begin{cases} 1 & H_{k,i} > H_{k,i} \quad \forall k; \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

in which

$$H_{k,i}(\lambda_j, \mu) = \log_2 \left( \frac{\text{CINR}_{k,i}}{\sum_{j=1}^{N_p} \lambda_j g_{k,j} Q_{j,i} + \mu} \right) - \left( 1 - \frac{\sum_{j=1}^{N_p} \lambda_j g_{k,j} Q_{j,i} + \mu}{\text{CINR}_{k,i}} \right), \quad (20)$$

$\lambda_j$  and  $\mu$  are the Lagrangian multipliers corresponding to the PU interference constraint and power budget, respectively.  $k$  denotes the optimum assignment of the  $i^{\text{th}}$  sub-carrier to the  $k^{\text{th}}$  user.

## 6. PROBLEM FORMULATION (MULTIPLE SUS)

The joint power allocation and GI determination problem for the multi-user scenario is posed as follows,

**Problem P2:**

$$\text{obj} = \max_{T_g, \zeta_{k,i}} C_m \quad (21)$$

subject to

$$I_j \leq I_{th} \quad \forall j \quad (22)$$

$$\sum_{k=1}^K \sum_{i=1}^{N_s} \zeta_{k,i} \leq P_t \quad (23)$$

$$0 \leq T_g \leq \tau_{max} \quad (24)$$

$$\sum_{k=1}^K \rho_{k,i} = 1 \quad \forall i \quad (25)$$

$$\zeta_{k,i} \geq 0 \quad (26)$$

The proposed algorithm (*Algorithm 2*) for joint sub-carrier power allocation and GI determination for multiple SUs, also consists of three modules, *Main()*, *Through()* and *Power Alloc Multi()*. The *Main()* and *Through()* modules are similar to the corresponding single user modules, except that the parameter passed by *Through()* to *Power Alloc Multi()* is now  $\text{CINR}_{k,i}$ . The *Power Alloc Multi()* module uses  $H_{k,i}$  as the metric to assign sub-carriers to users, and concurrently computes the power allocation  $\zeta_{k,i}$ , which is returned to *Through()*.

The details of *Power Alloc Multi()* are given below.

<i>function Power_Alloc_Multi(CINR<sub>k,i</sub>)</i>
1) Initialize all $\lambda_j$ and $\mu$ .
2) Initialize all $\lambda_{jold}$ and $\mu_{old}$ to zero.
3) Assign each sub-carrier $i$ to that user $k$ that will maximize the function $H_{k,i}$ .
4) Compute $\zeta_{k,i}^*$ by substituting the above $\lambda_j$ and $\mu$ in (18).

```

Compute the total power allocated as  $P_s = \sum_k \sum_i \zeta_{k,i}$ 
Calculate the interference caused to each PU sub-band,  $I_j$ 
using (17).
5) For each PU sub-band calculate the difference between
the interference generated and the threshold, as  $diff_j = I_j - I_{th}$ .
Calculate the difference between the total power allocated
and the power budget, as  $diff_p = P_s - P_r$ .
6)  $\lambda_{j,old} = \lambda_j \forall j$  and  $\mu_{old} = \mu$ 
If  $\{(\max(diff_j) < 0) \text{ AND } (diff_p < 0)\}$ 
 $\lambda_j = (\lambda_{j,old} + \lambda_j)/2 \quad \forall j$ 
 $\mu = (\mu_{old} + \mu)/2$ 
Goto Step3.
end If
7) For each PU sub-carrier, If  $(diff_j > 0)$ 
 $\lambda_j = \lambda_j + a_j * diff_j$ 
end If
If  $(diff_p > 0)$ 
 $\mu = \mu + b * diff_p$ 
end If
8) If  $\{(\max(diff_j) > 0) \text{ OR } (diff_p > 0)\}$ 
Goto Step3.
Else
End Algorithm
end If
return  $\zeta_{k,i}^*$ 
    
```

For the worst-case computational complexity of *Algorithm 2*, we consider the fact that the *Power Alloc Multi()* module involves assigning sub-carriers to the  $K$  SUs. Therefore, the complexity of this algorithm is given by (referring to the corresponding single-user algorithm complexity in Eq.(11) and Eq.(12)),

$$O(|KX| + |KY|) \tag{27}$$

### 7. SIMULATION RESULTS & DISCUSSION

For the single user case, we assume that the total system bandwidth for the PU and SU is 6 MHz wide, of which the SU occupies a contiguous band of 5 MHz, while the PU occupies 1 MHz bandwidth. The SU transceiver uses 128 sub-carrier OFDM for communication. An exponential power delay profile is considered for the channel, i.e.  $\rho(\tau) = \frac{1}{D_e} e^{-\tau/D_e}$ , where  $D_e$  is the rms delay spread and is assumed to be 1  $\mu$  sec. [31]. The maximum delay spread is  $\tau_{max} = 3 \mu$  secs. The Rayleigh multi-path fading is defined in the time domain by  $\sum_{l=0}^{L-1} h_l \delta(i - lT)$  where  $h_l$  is the complex amplitude of path  $l$  and  $L$  is the number of channel taps.  $h_l = a_l + j b_l$ , where  $a_l$  and  $b_l$  are distributed as  $N(0, \rho(\frac{l\tau_{max}}{L})^2 / 2)$ , and the frequency domain channel is given by its Fourier Transform. AWGN variance is assumed to be  $\sigma^2 = 1e-9$ . The power budget of the SU is  $P_r = 1$  W.

The PU band is divided into 8 sub-bands, and we attempt to mitigate the interference to each of them. We set the interference temperature,  $T_{th} = 1e-5$  W/Hz for each PU sub-band. Without

loss of generality, it is assumed that the interference induced by the PU to the SU is negligible.

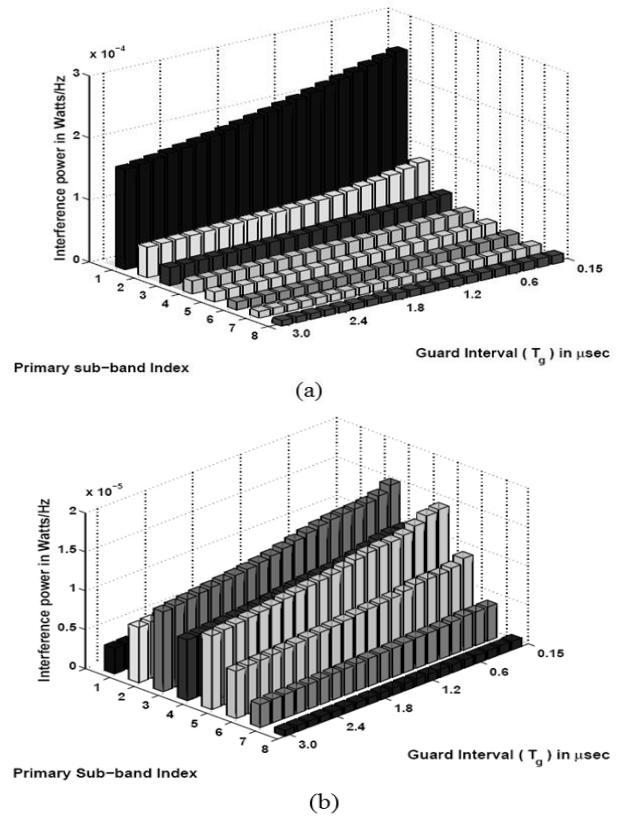


Fig.4. Interference to the PU band: (a) Without signal gains (b) With channel gains

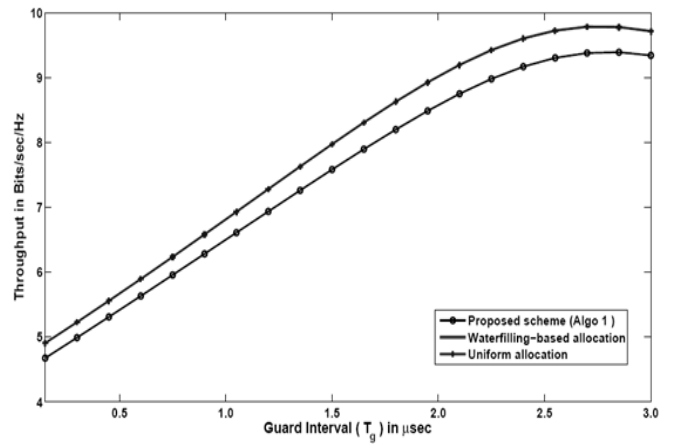


Fig.5. Throughput vs. GI

We have observed the effect of varying  $T_g$  on the PU interference with water-filling based power allocation (Fig.4). As  $T_g$  increases, the interference to the PU sub-bands decreases (as is indicated in Eq.(3)). It is also obvious that the interference decreases with increasing spectral distance. The results in Fig.4(a) and 4(b) are plotted without and with the PU channel gains, respectively. Similar results are expected with uniform power allocation.

In Fig.5, we analyze the SU throughput, while increasing  $T_g$  unto  $\tau_{max}$ . Although not plotted, it is expected that the SINR will increase with an increase in  $T_g$ , however, the same cannot be

said about the throughput. It is observed (Fig.5) that unto a certain point, an increase in  $T_g$  results in a corresponding increase in throughput; after which, any further increase in  $T_g$  results in the symbol duration becoming relatively smaller, and the throughput reduces. The effect has been observed with uniform power allocation, water-filling and the proposed algorithm (the plots for uniform power allocation and water-filling have nearly overlapped). The proposed algorithm (Algorithm 1) yields the lowest throughput, since it also mitigates interference to the PU band, while the others do not. These results have been averaged over 100 independent realizations of the channel.

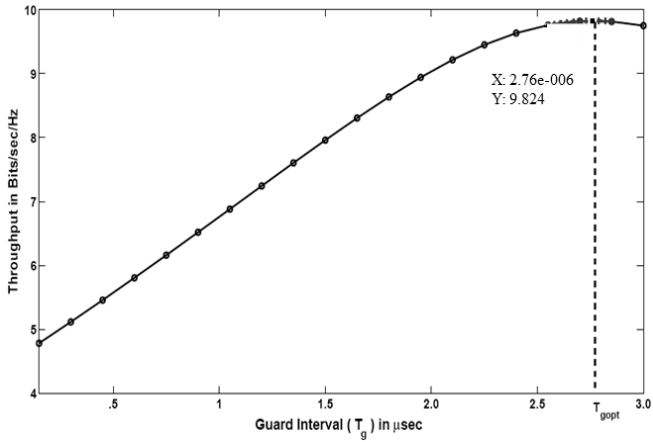


Fig.6. Execution of Algorithm 1

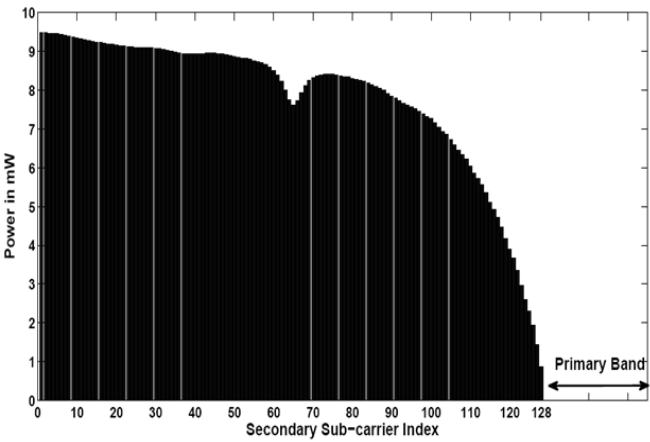


Fig.7. SU Power profile

The results of executing *Algorithm 1* are indicated in Fig.6. Initially a crude search was conducted by varying  $T_g$  in a step size of  $\tau_{max}/20$ , as indicated by the markers. Then a fine search was conducted with a step size of  $\tau_{max}/100$  to look for the global optima. The optimum result was obtained as  $T_g = 2.76\mu$  secs. The corresponding power allocation profile is provided in Fig.7. The graph tapers towards the PU band because lesser power is allocated in the SU sub-carriers spectrally closer to the PU. We have also provided the interference profile to the 8 PU sub-bands on execution of the various power allocation schemes (Fig.8). The proposed algorithm maintains the interference to each PU sub-band under the threshold, while the interference from the others exceeds the threshold  $I_{th}$ . These results are reported for a single instance of the channel.

The simulation parameters for the multi-user scenario are the same as those of the single user case. 3 SUs have been assumed, which contend for the 5 MHz bandwidth, which is divided into 128 OFDM sub-carriers. Fig.9(a) depicts the variation in SU throughput wrt  $T_g$ , with the proposed algorithm (*Algorithm 2*). Fig.9(b) demonstrates the power allocation profile and the assignment of sub-carriers to the users, with a total power budget  $P_t=1W$ . Due to the PU interference constraint, the profile tapers towards the PU band.

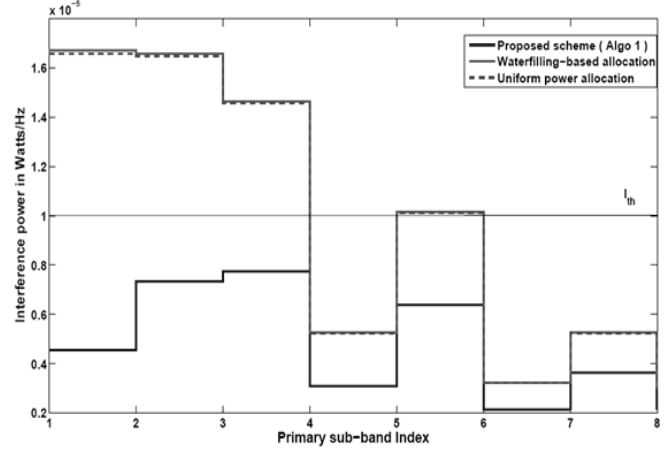
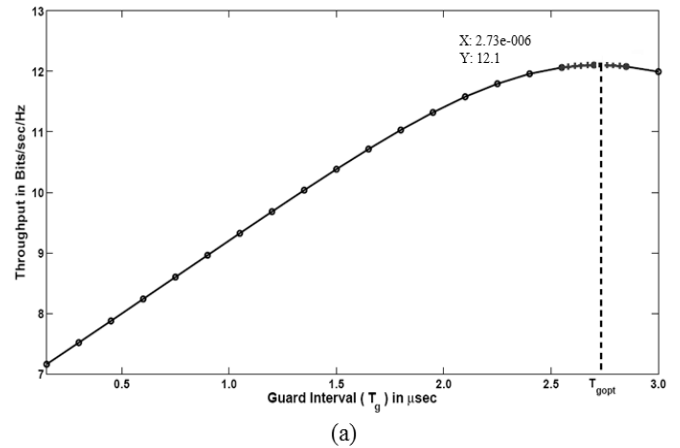
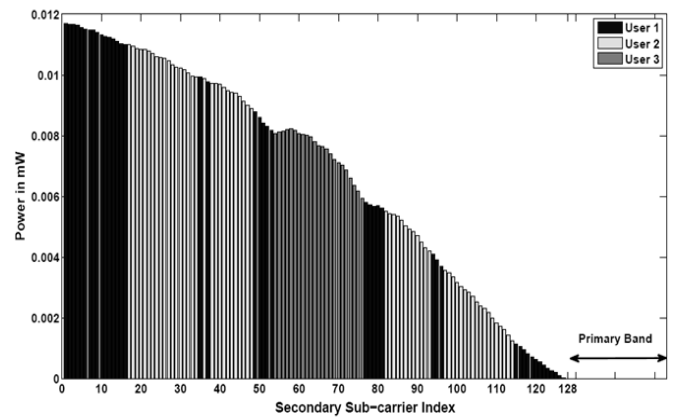


Fig.8. PU Interference profile



(a)



(b)

Fig.9. Execution of Algorithm 2: (a) Throughput vs. GI (b) Sub-carrier allocation and Power profile

## 8. CONCLUSION

The major contribution of the paper is towards joint determination of the channel adaptive GI along with sub-carrier power for an OFDM-based CR; the objective being-maximization of the SU's throughput, within a power budget and PU interference constraints. The PU spectral band is divided into sub-bands, and the proposed algorithms (for both single and multiple SUs) effectively mitigate the interference to each of them. Simulation results are provided, with rigorous interpretations of each of the graphs. The performance results are encouraging, and motivate the deployment of the suggested strategies in practical CR networks. While the proposed algorithms are mainly for stationary SUs and may be applicable to walking speeds, the resource allocation for medium/high speed mobile SUs, is an issue we intend to tackle in the near future. Along with bit loading and sub-carrier bandwidth sizing, determination of the optimum power allocation and GI, covers all the parameters that can be reconfigured by the RRM module to meet the system objectives of an OFDM-based CR.

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