

A HYBRID FACE RECOGNITION APPROACH USING LOCAL FUSION OF COMPLEX DUAL-TREE WAVELET COEFFICIENTS AND RIDGELET TRANSFORM

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Abstract

In this paper, we propose novel face recognition method based on local appearance feature extraction using hybrid mode of local ridgelet and fused dual-tree complex wavelet transform (DT-CWT). It provides a local multiscale description of images with good directional selectivity, effective edge representation and invariance to shifts and in-plane rotations. In the dual-tree implementation, two parallel discrete wavelet transform (DWT) with different lowpass and highpass filters in different scales are used. The linear combination of subbands generated by two parallel DWT is used to generate 6 different directional subbands with complex coefficients. It is insensitive to illumination variations and facial expression changes. 2-D dual-tree complex wavelet transform is less redundant and computationally efficient. The fusion of local DT-CWT coefficients of detail subbands and local Finite Ridgelet Transform (FRIT) coefficients of approximate subbands of DT-CWT are used to extract the facial features which improve the face recognition with small sample size in less computation. The local features based methods have been successfully applied to face recognition and achieved state-of-the-art performance. Normally most of the local appearance based methods the facial features are extracted from several local regions and concatenated into an enhanced feature vector as a face descriptor. In this approach we divide the face into several ($m \times m$) non-overlapped parallelogram blocks instead of square or rectangle blocks. The local mean and standard deviation of hybrid FRIT and fused DT-CWT coefficients are used to describe the face image. Experiments, on two well-known databases, namely, Yale and ORL databases, shows the Local hybrid FRIT and fused DT-CWT approach performs well on illumination, expression and perspective variant faces with single sample compared to PCA and global DT-CWT. Furthermore, in addition to the consistent and promising classification performances, our proposed Hybrid Local FRIT and fused DT-CWT based method has a really low computational complexity.

Keywords:

Gabor Wavelet Transform, Finite Ridgelet Transform, Dual Tree Discrete Wavelet Transform, Dual Tree Complex Wavelet Transform, PCA, Parallelogram Regions

1. INTRODUCTION

In recent years face recognition received more attention in the field of biometric authentication. This is due to increased concerns in security. However, the general problem of face recognition remains to be solved, since most of the systems to date can only successfully recognize faces when images are obtained under prescribed conditions. Their performance will degrade abruptly when face images are captured under varying pose, lighting, with accessories and expression. Another one of the most challenging problems for face recognition is the so-called Single Sample Problem (SSP) problem, i.e., a single face for a subject is used for training. Large training samples cannot be guaranteed in practice such as identity card verification, passport

verification, etc. Some face recognition algorithms have been proposed to solve the face recognition problem with only a single training image with various mode of process [1]. A face recognition system should, to a large extent, take into account all the above-mentioned natural constraints and cope with them in an effective manner. In order to achieve this, one must have efficient and effective representations for faces.

Many techniques have been proposed in the literature for representing face images. Some of these include Principal Components Analysis [2], Discrete Wavelet Transform [3], [4]. Gabor Wavelet-based representation provides an excellent solution when one considers all the above desirable properties. For this reason, Gabor wavelets have been extensively implemented in many face recognition approaches [5]-[8].

Even though Gabor wavelet-based face image representation is optimal in many respects, it has got two important drawbacks that shadow its success. First, it is computationally very complex. A full representation encompassing many directions (e.g., 8 directions), and many scales (e.g., 5 scales) requires the convolution of the face image with 40 Gabor wavelet kernels. Second, memory requirements for storing Gabor features are very high. The size of the Gabor feature vector for an input image of size 128x128 pixels is $128 \times 128 \times 40 = 655360$ pixels when the representation uses 8 directions and 5 scales.

Wavelets under multiresolution framework have been shown to help combat the detrimental effects of both noise and non-uniform illumination [9] in face recognition and facial feature extraction. The success of wavelets is mainly due to the good performance for piecewise smooth functions in one dimension. Unfortunately, such is not the case in two dimensions. In essence, wavelets are good at catching zero-dimensional or point singularities, but two-dimensional piecewise smooth signals resembling images have one-dimensional singularities. That is, smooth regions are separated by edges, and while edges are discontinuous across, they are typically smooth curves. Intuitively, wavelets in two dimensions are obtained by a tensor-product of one dimensional wavelets and they are thus good at isolating the discontinuity across an edge, but will not see the smoothness along the edge. To overcome the weakness of wavelets in higher dimensions, Candès and Donoho [14], [15] pioneered of representations named ridgelets which deal effectively with line singularities in 2-D. The idea is to map a line singularity into a point singularity using the Radon transform [16].

The dual tree implementation of complex wavelet transform provides a directional multiresolution decomposition of a given image much like the Gabor wavelets however unlike the Gabor wavelets it has relatively less computational expense. Thus,

complex approximately analytic wavelets provide an excellent alternative to Gabor wavelets with the potential to overcome the above-mentioned shortcomings of the Gabor wavelets.

In this paper we propose a new hybrid approach to improve the robustness of multi resolution and multi directional approach for pose and expression invariant face recognition with small sample size. This approach is based on local magnitude of the DT-CWT detailed subbands and the Ridgelet coefficients of approximate image of DT-CWT. In order to capture multi-orientation information in face images better, each face image is described by a subset of band filtered images containing DT-CWT coefficients which characterize the face textures. The edge related features are properly gathered by the Ridgelet transform from the approximate image. We divide those sub-bands into small sub-blocks, from which we extract compact and meaningful feature vectors using simple statistical measures.

The rest of the paper is organized as follows: Section 2 briefly give an overview of Gabor wavelets, DT-CWT and Ridgelet. Section 3 describes the proposed method, and Section 4 which compare the performances of Local hybrid approach of Ridgelet and DT-CWT on parallelogram image structures approach with DT-CWT on the pose, illumination and expression variant faces of Yale and ORL datasets. Finally, conclusions are drawn in Section 5.

2. RELATED WORK

2.1 GABOR WAVELET TRANSFORM

A Gabor wavelet filter is a Gaussian kernel function modulated by a sinusoidal plane wave:

$$\begin{aligned} \psi_g(x,y) &= \frac{f^2}{\eta\gamma\pi} \exp(-\beta^2 y'^2 - \alpha^2 x'^2) \exp(2\pi j f x'), \\ x' &= x \cos\theta + y \sin\theta, \\ y' &= y \cos\theta - x \sin\theta, \end{aligned} \quad (1)$$

where f is the central frequency of the sinusoidal plane wave, θ is the anticlockwise rotation of the Gaussian and the envelope wave, α is the sharpness of the Gaussian along the major axis parallel to the wave, and β is the sharpness of the Gaussian minor axis perpendicular to the wave. $\gamma = f/\alpha$ and $\eta = f/\beta$ are defined to keep the ratio between frequency and sharpness constant. The 2D Gabor wavelet as defined in (1) has Fourier transform:

$$\begin{aligned} \psi_g(u,v) &= \exp\left(-\pi^2 \left(\frac{(u' - f')^2}{\alpha^2} + \frac{v'^2}{\beta^2} \right)\right) \\ u' &= u \cos\theta + v \sin\theta \\ v' &= v \cos\theta - u \sin\theta \end{aligned} \quad (2)$$

Fig.1(a) and 1(b) show, respectively, the real part and magnitude of the Gabor wavelets for 4 scales and 6 directions. Gabor wavelets possess many properties which make them attractive for many applications. Directional selectivity is one of the most important of these properties. The Gabor wavelets can be oriented to have excellent selectivity in any desired direction. They respond strongly to image features which are aligned in the same direction and their response to other feature directions is weak. Invariance properties to shifts and rotations also play an

important role in their success. In order to accurately capture local features in face images, a space frequency analysis is desirable.

Gabor functions provide the best tradeoff between spatial resolution and frequency resolution. The optimal frequency-space localization property allows Gabor wavelets to extract the maximum amount of information from local image regions. This optimal local representation of Gabor wavelets makes them insensitive and robust to facial expression changes in face recognition applications. The representation is also insensitive to illumination variations due to the fact that it lacks the DC component. Last but not least, there is a strong biological relevance of processing images by Gabor wavelets as they have similar shapes to the respective fields of simple cells in the primary visual cortex.

Despite many advantages of Gabor wavelet-based algorithms in face recognition, the high-computational complexity and high memory capacity requirement are important disadvantages. With a face image of size 128×128 , the dimension of the extracted Gabor features would be 655 360, when 40 wavelets are used. This feature is formed by concatenating the result of convolving the face image with all the 40 wavelets. Such vector dimensions are extremely large and, in most cases, down sampling are employed before further dimensionality reduction techniques such as PCA is applied. The computational complexity is high even when fast Fourier transform (FFT) is employed. Because of the above-mentioned shortcomings, one usually looks for other transforms that can preserve most of the desired properties of Gabor wavelets and at the same time reduces the computational complexity and memory requirement. Complex wavelet transforms provide a satisfactory alternative to this problem.

2.2 DUAL-TREE COMPLEX WAVELET TRANSFORM

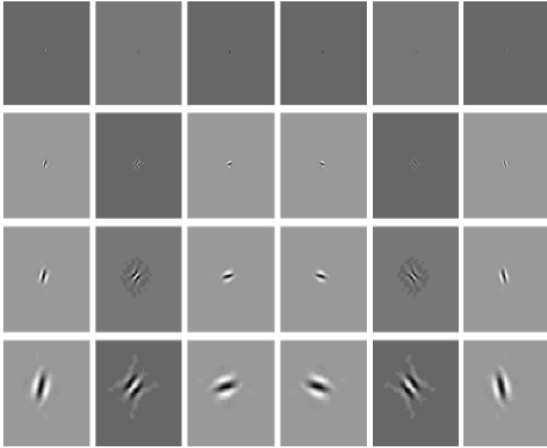
One of the most promising decompositions that remove the above drawbacks satisfactorily is the dual-tree complex wavelet transform (DT-CWT) [10], [11]. Two classical wavelet trees (with real filters) are developed in parallel, with the wavelets forming (approximate) Hilbert pairs. One can then interpret the wavelets in the two trees of the DT-CWT as the real and imaginary parts of some complex wavelet $\Psi_c(t)$.

The requirement for the dual tree setting for forming Hilbert transform pairs is the well-known half sample delay condition. The resulting complex wavelet is then approximately analytic (i.e., approximately one sided in the frequency domain). The properties of the DT-CWT can be summarized as,

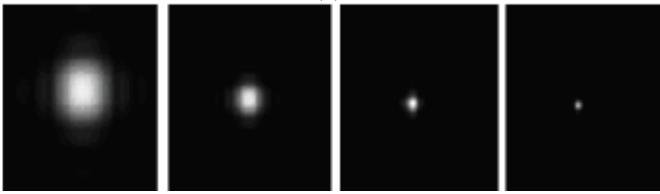
- approximate shift invariance;
- good directional selectivity in 2 dimensions;
- phase information;
- perfect reconstruction using short linear-phase filters;
- limited redundancy, independent of the number of scales, 2 : 1 for 1D (2m : 1 for mD);
- efficient order-N computation—only twice the simple DWT for 1D (2m times for mD).

It has the ability to differentiate positive and negative frequencies and produces six subbands oriented in $\pm 15^\circ, \pm 45^\circ, \pm 75^\circ$.

However, these directions are fixed unlike the Gabor case, where the wavelets can be oriented in any desired direction. Fig.1 shows the impulse responses of the dual-tree complex wavelets. It is evident that the transform is selective in 6 directions in all of the scales except the first. The DT_CWT based face representation [12]-[15] improves the recognition performance significantly. Fig.2 shows the magnitude and real part of a face image processed using the DT-CWT.



(a)

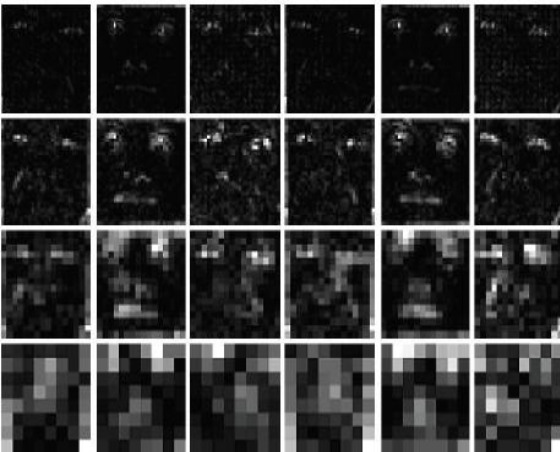


(b)

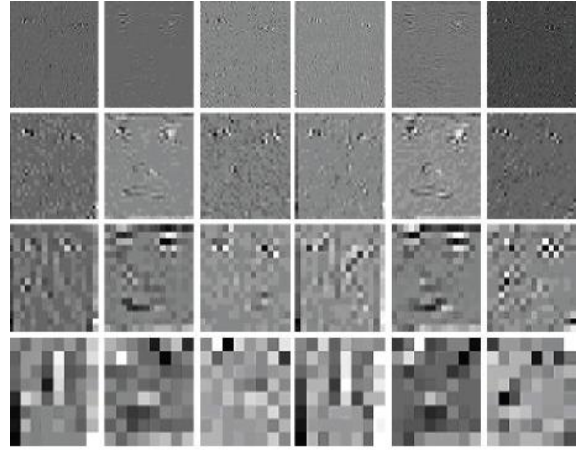
Fig.1. Impulse response of dual-tree complex wavelets at 4 levels and 6 directions (a) Real part (b) Magnitude



(a)



(b)



(c)

Fig.2. (a) Sample image for transformation (b) The magnitude of the transformation (c) The real part of the transformation

2.3 RIDGELET TRANSFORM

Ridgelet analysis was developed by E.J. Candes and D.L. Donoho [16], [17] for solving important problems such as constructing neural networks or approximating and estimating multivariate functions by linear combinations of ridge functions. Ridgelet transform is a multiscale representation for functions on continuous spaces that have some discontinuities along lines. Orthonormal version of the ridgelet transform for discrete and finite-size images was proposed by Minh Do and M. Vetterli, [18], [19]. It uses the finite Radon transform proposed in [20] and [21] as a basic building block. This leads to a set of directional and Orthonormal bases for images.

The continuous ridgelet transform of an integrable bivariate function $f(x)$ is represented by

$$CRT_f(a, b, \theta) = \int_{\mathbb{R}^2} \psi_{a,b,\theta}(x) f(x) dx, \quad (3)$$

where, ridgelet $x \cos \theta + y \sin \theta = \text{constant}$ in 2-D, are defined from a wavelet type function in $\psi(x)$ in 1-D, as

$$\psi_{a,b,\theta}(x) = \sqrt{a} \psi((x \cos \theta + y \sin \theta - b)/a), \quad (4)$$

Wavelets are good to represent the point singularities and ridgelet represent line singularities. In image processing applications it is important to detect singularities along edges. Ridgelet can be thought of a way of juxtaposing 1-D wavelets along lines. This results in a very effective representation of objects with singularities along lines. In 2-D, points and lines are related through the Radon transform, thus the wavelet and ridgelet transforms are linked through the Radon transform. The Radon transform is denoted as

$$R_f(\theta, t) = \int_{\mathbb{R}^2} f(x, y) \delta(x \cos \theta + y \sin \theta - t) dx dy, \quad (5)$$

where \mathbb{R} denotes the real line. Then the ridgelet transform is the application of a 1-D wavelet transform to the slices (also referred to as projections) of the Radon transform, and is denoted as

$$CRT_f(a, b, \theta) = \int_{\mathbb{R}} \psi_{a,b}(t) R_f(\theta, t) dt. \quad (6)$$

Finite Radon Transform (FRAT) is redundant and not orthogonal, this redundancy can be removed by applying 1-D DWT on the projections of the Finite Radon Transform (FRAT), we can obtain an orthonormal transform. For a more general setting, let us assume that we have a collection of $(p+1)$ 1-D orthonormal transforms on (which can be the same), one for each projection k of FRAT, that have bases as $\{W_m^{(k)}: m \in Z_p\}$ $k = 0, 1, \dots, p$, where $(p \times p)$ is the size of input image. By definition, the Finite Ridgelet Transform (FRIT) can be indicated as

$$FRIT_f[k, m] = FRAT_f[k, \cdot], w_m^{(k)}[\cdot]. \quad (7)$$

To overcome the weakness of wavelets in higher dimensions, ridgelets can be used effectively to deal with line singularities in 2-D. The idea is to map a line singularity into a point singularity using the Radon transform. Then, the wavelet transform can be used to effectively handle the point singularity. In short, the Ridgelet transform is the application of 1-D wavelet transform to the slices of the radon transform.

3. LOCAL FUSION OF DUAL TREE COMPLEX WAVELET AND RIDGELET TRANSFORM BASED FACE REPRESENTATION

First the directional multiscale decomposition of the gray level face image is performed up to level 4. The approximate image is fed into ridgelet transform to get edge oriented features. The Local Fusion of Dual Tree Complex Wavelet (Local Fusion DT-CWT) [23] face representation performs well than the Local Dual Tree Complex Wavelet Transform [22] representation. The six detailed subbands ($\pm 15, \pm 45, \pm 75$) at each level are fused to make a single representation of the face. The fused representation of a face is shown in the Fig.3. In order to extract the most discriminative features each fused image is partitioned into a set of equally-sized blocks as well as parallelogram blocks in a non-overlapping way. The statistical measures such as mean and variance of the magnitude of the complex coefficients for each fused bands at each decomposition level are used to represent the facial description.

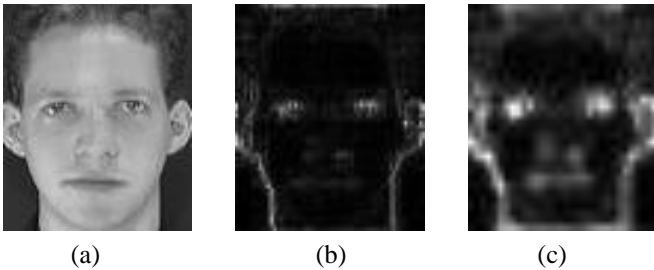


Fig.3. (a) Sample image for transformation (b) Fusion of DT-CWT (detailed) at level one (c) Fusion of DT-CWT (detailed) at level two

Let $I_{ij}(x, y)$ be the image at the specific block j of sub-band i , the resulting feature vector $v_{ij} = \{\mu_{ij}, \sigma^2_{ij}\}$, where $\mu_{ij} = \text{mean}$, $\sigma^2_{ij} = \text{variance}$

$$\mu_{ij} = \frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N |I_{ij}(x, y)|$$

$$\sigma^2_{ij} = \frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N |I_{ij}(x, y) - \mu_{ij}|^2. \quad (8)$$

The feature vector of a face is then constructed by concatenating each block measure to one big feature vectors. This approach extracts the most discriminative features as well as reduces the feature vector size(1020) from normal block based representation for all twenty four subbands into only four fused subbands which is of the size(198) one ninety eight feature vector if the image size 128×128 . The Euclidean distance metric is applied for classification.

In this paper in order to enhance the performance of this hybrid approach of local fused complex wavelet and ridgelet based face representation, the feature vector comparison is done on the basis of local parallelogram non overlapping windows of the subbands face structure instead of rectangular or square windows. The arrangement of a parallelogram window is different from a rectangular window as seen in Fig.4.



Fig.4. Squared and Parallelogram Grid

The perspective variation can be solved with the concept of parallelogram window based feature vectors. The combined feature vector from right and left direction parallelogram window provides better performance than the traditional square or rectangular window for the perspective variation.

4. EXPERIMENTAL RESULTS

This section evaluates the performance of local fusion of complex dual tree wavelet coefficients and ridgelet transform based face recognition on single sample problem. Two standard databases are used for evaluation of these results. The first database is ORL [24] database that contains a set of faces taken between April 1992 and April 1994 at the Olivetti Research Laboratory in Cambridge, UK. One of the persons is shown in Fig.5.

There are 10 different images of 40 distinct subjects. For some of the subjects, the images were taken at different times, varying lighting slightly, facial expressions (open/closed eyes, smiling/non-smiling) and facial details (glasses/no-glasses).



Fig.5. One person in the ORL database

All the images are taken against a dark homogeneous background and the subjects are in up-right, frontal position (with tolerance for some side movement). The size of each image is 92×112 , 8-bit grey levels. In this paper experiments were performed with one normal training image and 9 test images for each person.

The Yale face [25] database consists of 15 individuals, where for each individual, there are 11 face images containing variations in illumination and facial expression. From these 11 face images, we use frontal single face for training and the remaining 10 images are used for testing. Fig.6 depicts some sample images from the Yale database.



Fig.6. Faces from the YALE Face Database

The experimental results presented in this section are divided into two parts. We make a comparison of Local Fusion DT-CWT and Ridgelet on square image structure as well as Local Fusion DT-CWT and Ridgelet on parallelogram image structure with local DT-CWT, Local Fusion DT-CWT, PCA and GW+PCA. The respective performances of the above described methods based on the expression, illumination and perspective variant faces with single training image are shown in Table.1.

Table.1. Results on Expression, Illumination and Perspective Invariant Face Recognition with Single Sample per Class

Methods	Experimental Datasets	
	ORL	Yale
PCA	71.00%	80.00%
GW+PCA	74.20%	87.30%
DT-CWT	76.59%	88.59%
Local DT-CWT	78.44%	90.33%
Local Fusion DT-CWT	82.22%	93.33%
Local DT-CWT and FRIT	82.22%	92.66%
Local DT-CWT and FRIT (PS)	83.61%	94.00%
Local Fusion DT-CWT and FRIT	84.44%	94.66%
Local Fusion DT-CWT and FRIT (PS)	84.72%	95.33%

From the above result we can see that the proposed Local Fusion of DT-CWT coefficients and FRIT on Square structure method outperforms DT-CWT with 7.85% for ORL dataset and 6.07% for Yale dataset. The proposed hybrid approach on parallelogram structure method outperforms DT-CWT with 8.13% for ORL dataset and 6.74% for Yale Dataset. We can see from the Table.1, that our enhanced hybrid approach of local fusion of DT-CWT and FRIT based low level statistical feature based

description has improved the DT-CWT performance greatly in this experiment.

Computational complexity: The face recognition program is performed with an Intel Pentium(R) D 2.40GHz CPU and 512GB RAM with Matlab7.5. Table.2 shows the results of the average time for feature extraction of one face image and matching between an image pair as well as overall recognition system.

Table.2. Comparison of computational complexity

	Average Time(ms)	
	Gabor	Local fusion of DT-CWT and FRIT
Feature extraction	32	4.47
Matching	0.44	0.042
Recognition System	144.12	21.56

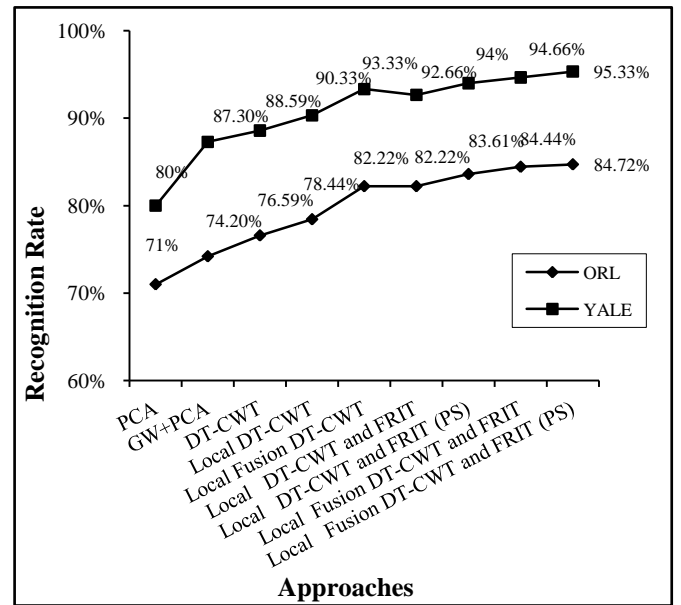


Fig.7. Recognition Rate Analysis on ORL and YALE using Hybrid Approaches of DT-CWT and FRIT with Single Sample per Class

From the Fig.7, it is found out that the hybrid approach of local fusion of complex dual tree wavelet coefficients and Ridgelet Transform based face recognition has improved the Local DT-CWT performance significantly in this experiment.

5. CONCLUSION

In this paper, we have proposed a novel hybrid approach of local fusion of dual tree complex wavelet and ridgelet based facial feature description for human face recognition. Yale and ORL datasets are used, which contains the face images with different orientations, expressions and illumination. In our approach, the input face image is decomposed using dual tree complex wavelet transform and finite fusion of the detailed subbands as well as the finite ridgelet transform on approximate coefficients at each level are used to represent the face. The recognition is done on the basis of euclidean distance measure on parallelogram image structure. Compared to the traditional Ga-

bor and local dual tree complex wavelet methods, the Hybrid approach of local fusion of DT-CWT and Ridgelet Transform based method not only performs better, but also inherits low computational complexity.

This paper has also evaluated the performances of the proposed hybrid approach on parallelogram image structure in terms of normal and changes in illumination, perspective and facial expressions faces of Yale face dataset and ORL datasets. Compared to the proposed approach defined on square structure, the hybrid approach on parallelogram structure has more potential for perspective invariant face recognition. We observe the results of face recognition using this approach are very good and encouraging. Furthermore, only one image per person is used for training which makes it useful for practical face recognition applications.

ACKNOWLEDGEMENT

The comments and suggestions from the anonymous reviewers greatly improve this paper.

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