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DESIGN FRACTIONAL ORDER PI$^\lambda$D$^\mu$ CONTROLLER FOR CSTR USING TLBO OPTIMIZATION ALGORITHM

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Abstract

Optimization techniques serve as significantly easier yet one of the best methods to tune PID controllers. Response of these techniques are unforeseeable and usually vary on the basis of different parameters. Fractional order controllers provide a more accurate control in comparison to traditional PID controllers. This paper deals with the Concentration control of an Isothermal CSTR using FOPID Controller, for which a comparative study of a newly developed algorithm, teaching learning based optimization (TLBO) algorithm with the very popular Particle swarm optimization (PSO) algorithm is performed. Both PSO and TLBO are population based algorithms where PSO was inspired by behavior of animal groups while TLBO got inspiration from the idea of learning of a group of students and the effect of teacher on them. A comparative analysis of different Performance Indices is also provided.

Keywords:

CSTR, FOPID Controller, Particle Swarm Optimisation, Teaching Learning Based Optimization

1. INTRODUCTION

If a system can be modeled by a set of fractional differential equations then it said to be a fractional order system. Similarly, fractional calculus means generalizing ordinary differentiation and integration to non-integer order. The topic of Fractional Calculus is three centuries old but unavailability of solution methods for fractional differential equations constrained its implementation in real world applications but now the availability of various approximation methods has empowered the researchers to dig deep into this field. Fractional Order Proportional Integral Derivative (FOPID) Controller is a PI$^\lambda$D$^\mu$ type controller, where λ and μ are fractions, which is in contrast to traditional PID controller where the order of λ and μ is unity. A FO controller can achieve similar robustness which otherwise is achievable using a very high order IO controller [1].

Particle Swarm Optimization (PSO) is an algorithm inspired by the animal groups. It is one of the most widely used optimization technique in world of control systems. Its simplicity and versatility makes is one of the best algorithms [2]. With the evolution of technology a simpler algorithm was required so that it could obtain better results with fewer parameters. A new algorithm Teaching-Learning-Based-Algorithm (TLBO) is another population based Algorithm. It considers a group of students eager to learn as population and it also depends on the influence of a teacher on the students. Teaching learning based optimization algorithm for multi constrained optimization has been discussed in [3]. Constraint and unconstraint teaching learning based optimization algorithm has been explained in [4].

Reactions are core of any chemical process in which basically some particular raw materials react together to transform into resultant products. The vessel in which reactions occur is known as a Chemical Reactor. Design of chemical reactors is a paramount subject and is endeavored to maximize the net value of the reaction. It is to be ensured that a high yield of desired products is obtained with minimum investment. There are various types of reactors: batch reactors, continuous-stirred-tank-reactors, plug flow reactors. Selection of a chemical reactor depends on many factors like, temperature and pressure of reaction, product delivery pattern (batch or continuous), need for addition and removal of reactants and products, rate, catalyst requirement etc. A Continuous Tank Stirred Reactor (CSTR) is a semi-batch type of reactor, it can be perceived as a tank with a stirrer/impeller. It is operated at steady-state with a continuous flow of reactants and products. The modeling of continuous stirred tank reactor has been explained in [5], [6]. This paper deals with the comparative study of concentration control of an isothermal CSTR using PSO-FOPID controller with TLBO-FOPID controller.

2. CONTINUOUS STIRRED TANK REACTOR (CSTR)

A CSTR is a very commonly used equipment for conducting reactions in Chemical industries. Here, we are using an Isothermal CSTR which is used for synthesis of Chalcone.

Chalcone is obtained when isothermal Aldol condensation occurs between Acetophenone and Benzaldehyde in the presence of Sodium Hydroxide/Ethanol (catalyst) at 80°C.

Chalcone is yellow colored α, β-unsaturated ketone which contains reactive Ketoethylenic group, it is used to make various lifesaving drugs. A particular concentration is necessary for better productivity of above reaction and therefore, our control objective is to keep the concentration at a certain desired value. The Fig.1 shows the pictorial representation of CSTR.

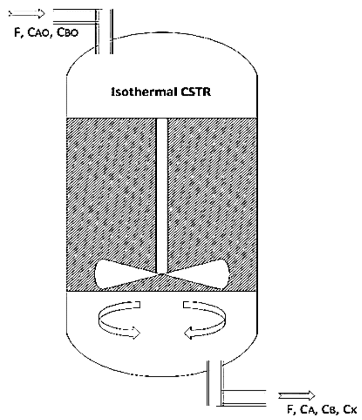

Fig.1. Pictorial representation of CSTR

The modeling of this system can be done by the help of mass and heat balance equations as explained in [8]. If the reactants are *P*, *Q* and *Z*, mixing is considered perfect and volume is constant inside reactor, then mass balance reactions can be written as follows:

$$\frac{dC_P}{dt} = \frac{F}{V}\left(C_{P0} - C_P\right) - K_0 C_P C_Q \quad (1)$$

$$\frac{dC_Q}{dt} = \frac{F}{V}\left(C_{Q0} - C_Q\right) - K_0 C_P C_Q \quad (2)$$

$$\frac{dC_Z}{dt} = -\frac{F}{V} C_Z - K_0 C_P C_Q \quad (3)$$

where, C_{P0}, C_{Q0} and C_Z are concentrations of Benzaldehyde, Acetophenone and Chalcone respectively, *F* is volumetric flow rate and *V* is the volume of reactor.

A steady-state analysis of the system is done. The linear state space model is represented as:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \quad (4)$$

where, state and input variables are defined in deviation variable form. The dynamic functional equations can be represented as follows:

$$f_1\left(C_p, C_Q, C_Z, \frac{F}{V}\right) = \frac{dC_p}{dt} \quad (5)$$

$$f_2\left(C_p, C_Q, C_Z, \frac{F}{V}\right) = \frac{dC_Q}{dt} \quad (6)$$

$$f_3\left(C_p, C_Q, C_Z, \frac{F}{V}\right) = \frac{dC_Z}{dt} \quad (7)$$

The state or system matrix is found by:

$$A = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} & \frac{df_1}{dx_3} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} & \frac{df_2}{dx_3} \\ \frac{df_3}{dx_1} & \frac{df_3}{dx_2} & \frac{df_3}{dx_3} \end{bmatrix} \quad (8)$$

The input matrix is found by:

$$B = \begin{bmatrix} \frac{df_1}{du} \\ \frac{df_2}{du} \\ \frac{df_3}{du} \end{bmatrix} \quad (9)$$

The output matrix:

$$c = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad (10)$$

The feed forward matrix:

$$D = null\ matrix \quad (11)$$

By using the parameter values from the Table.1, we get:

$$A = \begin{bmatrix} -7.5 & -6.05 & 0 \\ -6.5 & -7.05 & 0 \\ 6.5 & 6.05 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2.45 \\ 3.40 \\ -1.0 \end{bmatrix}$$

State space equations can be converted into transfer function as follows:

$$G(s) = C(sI - A)^{-1} B + D$$

$$G(s) = \frac{-s^2 + 21.95s + 22.95}{s^3 + 15.55s^2 + 28.1s + 13.55} \quad (12)$$

Transfer functions of input flow disturbance due to reactants A and B:

$$G_{dA}(s) = \frac{6.5s + 6.5}{s^3 + 15.55s^2 + 28.1s + 13.55} \quad (13)$$

$$G_{dB}(s) = \frac{6.05s + 6.05}{s^3 + 15.55s^2 + 28.1s + 13.55} \quad (14)$$

Table.1. Reactor Parameters

Parameter	Value
K_0	1 mol^{-1} L min^{-1}
C_{P0}	8.5 g mol^{-1} L
C_{Q0}	9.9 g mol^{-1} L
C_P	6.05 g mol^{-1} L
C_Q	6.5 g mol^{-1} L
F/V	1 mol^{-1}
C_Z	1 g mol^{-1} L

3. PERFORMANCE INDICES

Controller which we aim to design is based on time domain. So, for evaluating its performance a quantitative analysis is required and performance indices are benchmark for it. Aim of the controller will be to minimize the following performance indices:

Integral Square Error (ISE):

$$ISE = \int_0^T e(t)^2\, dt \quad (15)$$

Integral Absolute Error (IAE):

$$IAE = \int_0^T \left|e(t)\right| dt \quad (16)$$

Integral Time Square Error (ITSE):

$$ITSE = \int_0^T te(t)^2\, dt \quad (17)$$

Integral Time Absolute Error (ITAE):

$$ITAE = \int_0^T t\left|e(t)\right| dt \quad (18)$$

4. FRACTIONAL ORDER PID CONTROLLER (FOPID)

Real world systems are generally fractional ordered but order of fraction of many of them is very low. An Integer ordered description of a fractional ordered system can cause many differences with real system. Several researchers have demonstrated that fractional derivative and integral based models are more adequate than integer order models. Description of Memory and Hereditary effects can be provided by the use of Fractional order derivatives and integrals making them significantly advantageous. But the main reason of using an Integer ordered description was unavailability of solution methods for fractional-ordered differential equations. For such systems a Fractional Order PID (FOPID) controller is more appropriate, a fractional order PID controller can be conceptualized by involving a fractional-order integrator and a fractional-order differentiator. Concentration control of CSTR through fractional order PID controller by using soft techniques has been discussed in [7]. A generalized form of PID controller having λ order integrator and μ order differentiator is proposed in [7]. The transfer function of fractional $PI^{\lambda}D^{\mu}$ controller can be represented as:

$$G(s)=\frac{U(s)}{E(s)}=K_P+K_Is^{-\lambda}+K_Ds^{\mu}\left(\lambda,\mu>0\right) \tag{19}$$

where, $G(s)$ is the transfer function of the controller, $U(s)$ is the input of controller and $E(s)$ is the error.

Equation of the output of $PI^{\lambda}D^{\mu}$ controller in time domain can be shown as:

$$u(t)=K_Pe(t)+K_ID^{-\lambda}e(t)+K_DD^{\mu}e(t) \tag{20}$$

If we take $\lambda=\mu=1$, then we obtain a classical PID controller. Whereas, if $\lambda=1$ and $\mu=0$ we get a PI controller. For $\lambda=0$ and $\mu=1$, we get a PD controller while $\lambda=\mu=0$ will result in just gain.

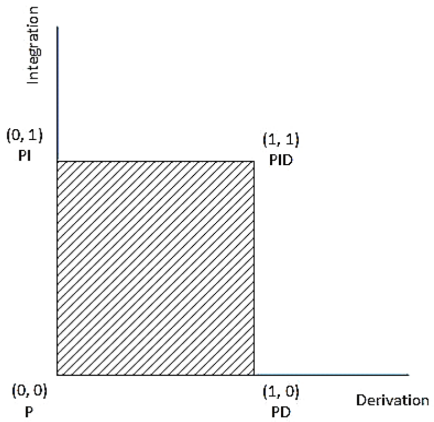

Fig.2. Point form to planar form expansion of PID controller

We can see that above shown classical controllers are particular cases of $PI^{\lambda}D^{\mu}$ controller. Also, the two extra degrees of freedom in the FOPID controller makes it more flexible and appropriate to control the dynamical properties of the fractional-order control system. A fractional order fuzzy PID controller design using optimization algorithm has been explained in [8] [9].

5. PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization is a social behavior based algorithm designed on the basis of interaction of individuals of a group of animals such as birds, fishes, insects etc. for performing a group task. The tuning of fractional PID controller using PSO has been explained in [10] [11]. The entire workflow of the Particle Swarm Optimization technique can be more clearly explained using a flow chart as shown below in Fig.3.

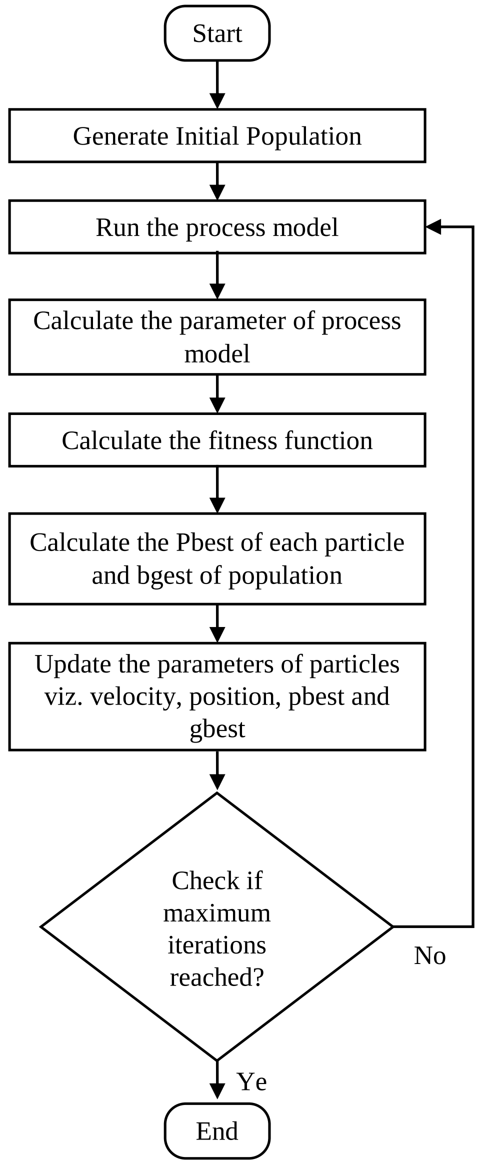

Fig.3. PSO Flowchart

In PSO a group of particles are allocated random positions x and velocities v in a given search space. A function is evaluated for every iteration using particle's coordinates as input. These positions are updated and the function is re-evaluated when particles discover a better pattern, these patterns are recorded as variable P_{best}. Then difference between the best positions till now i.e. P_{best} and the current position is stochastically added to current velocity. Since, each particle's position will also depend on its topological neighbor's position therefore, a stochastically weighted difference of the current position and best position in the neighborhood is added to its velocity. In this manner at every iteration the group reaches closer to optimized result.

Assume a swarm of N particles moving in d-dimensional search space. Initially, each particle possess some random position and velocity. For every iteration, each particle will update these parameters on the basis of its own best experience and on the basis of best experience of others. The i^{th} particle can be as follows:

$$x_i = \left(x_{i1}, x_{i2}, ..., x_{id}\right) \tag{21}$$

The local best position of the i^{th} particle will be represented as:

$$Pbest_i = \left(Pbest_{i1}, Pbest_{i2}, ..., Pbest_{id}\right) \tag{22}$$

After every iteration the position and velocity of the particle will be updated according to the following formula:

$$\begin{aligned} V_{id}\left(t+1\right) = W \cdot V_{id}\left(t\right) + C_1 \cdot R_1 \cdot \left(Pbest_{id} - x_{id}\right) \\ + C_2 \cdot R_2 \cdot \left(gbest_{id} - x_{id}\right) \end{aligned} \tag{23}$$

$$x_{id}\left(t+1\right) = x_{id}\left(t\right) + V_{id}\left(t+1\right) \tag{24}$$

where, W is the inertia weight factor, R_1 and R_2 are random numbers, C_1 and C_2 are acceleration constant of local best position and global best position respectively.

6. TEACHING LEARNING BASED OPTIMIZATION

Teaching Learning Based Optimization (TLBO) is also a population based algorithm. It also uses a population of solutions to reach the global solution, here group of students are considered as population. TLBO is inspired from the process of learning of students in a class which is dependent on learning through a teacher and the mutual learning among the students. Output is measured in the terms of result. The quality of teacher effects the outcome of students making their grades better. The basic concept of TLBO has been discussed in [12] [13].

The TLBO algorithm works in two phases: Teacher Phase and Learner Phase. Teacher phase means the learning from a teacher while Learner phase means learning by interaction with other learners. It has been proved that TLBO provides global solutions to non-linear functions with a lesser computation and higher consistency making it more effective than other optimization algorithms in following parameters viz. best solution, average solution, convergence rate and computational effort. Other optimization algorithms are too dependent on the algorithm parameters making their effectiveness vulnerable.

6.1 CONCEPT OF TLBO

Assume there are two different classes of students having students of almost similar merit level and are taught by two different teachers.

If the results of both the classes are assumed to be skewed normal distribution curves as shown in Fig.4, then it is observed that class-B having marks designated by curve-2 shows better results than that of class-A having marks designated by curve-1 because mean of class-B is greater than that of class-A. Therefore, it can be said that teacher-2 is better than teacher-1 i.e. a good teacher produces a better mean for results of the learners. Learners will also learn from mutual interaction resulting in further improvement of their result.

Teacher is considered as one of the most knowledgeable person of society, therefore it will be on the rightmost of the curve. Also, a teacher will always try to increase the mean towards him according to his or her capability. At one level when mean approaches the level of teacher, a new teacher with higher level of knowledge will be required for further improvement of students, which can be observed in Fig.4. We can also see that the mean of class-b is more than class-A, because knowledge of teacher TB is more than teacher TA.

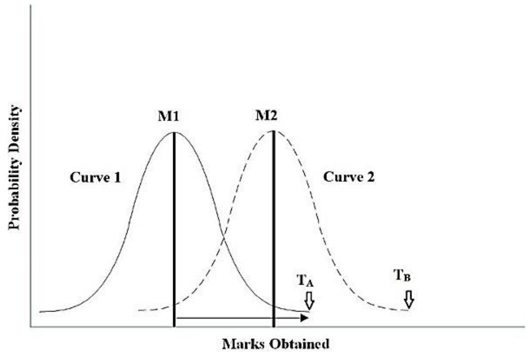

Fig.4. Students Obtained Marks distribution Model

TLBO works in two phases:

- Teacher Phase (Learning from teacher)
- Learner Phase (Learning through mutual interaction of learners)

Teacher Phase: Ideally a teacher should be able to raise the level of students up to his own level but in reality it does not happen because it depends on the capabilities of the Class too. Let for the i^{th} iteration, M_i be the mean and T_i be the teacher. T_i will try to move M_i towards its own level. Let the new mean be M_{new}, then the solution which will be updated depending on the difference of current and new mean can be expressed as:

$$Diff\_Mean_i = r_i\left(M_{new} - T_F M_i\right) \tag{25}$$

where, r_i is a random number between [0,1] and TF is teaching factor which is the deciding factor of the value of mean to be changed. Value of TF can be either 1 or 2 and is calculated heuristically with the equal probability as:

$$T_F = round\left[1 + rand\left(0,1\right)\left\{2-1\right\}\right] \tag{26}$$

Therefore, existing solution is modified as:

$$X_{new,i} = X_{old,i} + Diff\_Mean_i \tag{27}$$

Learner Phase: In learner phase a student learns with mutual interaction with other students via communication, presentations and discussions. But he learns only if his buddy has more knowledge than him or her. We can express this process as follows:

For $i = 1{:}P_n$

Random learners X_i and X_j, where $X_i \neq X_j$

If $f(X_i) < f(X_j)$

$$X_{new,I} = X_{old,i} + r_i(X_i - X_j) \tag{28}$$

End For

X_{new} is accepted if it gives a better value.

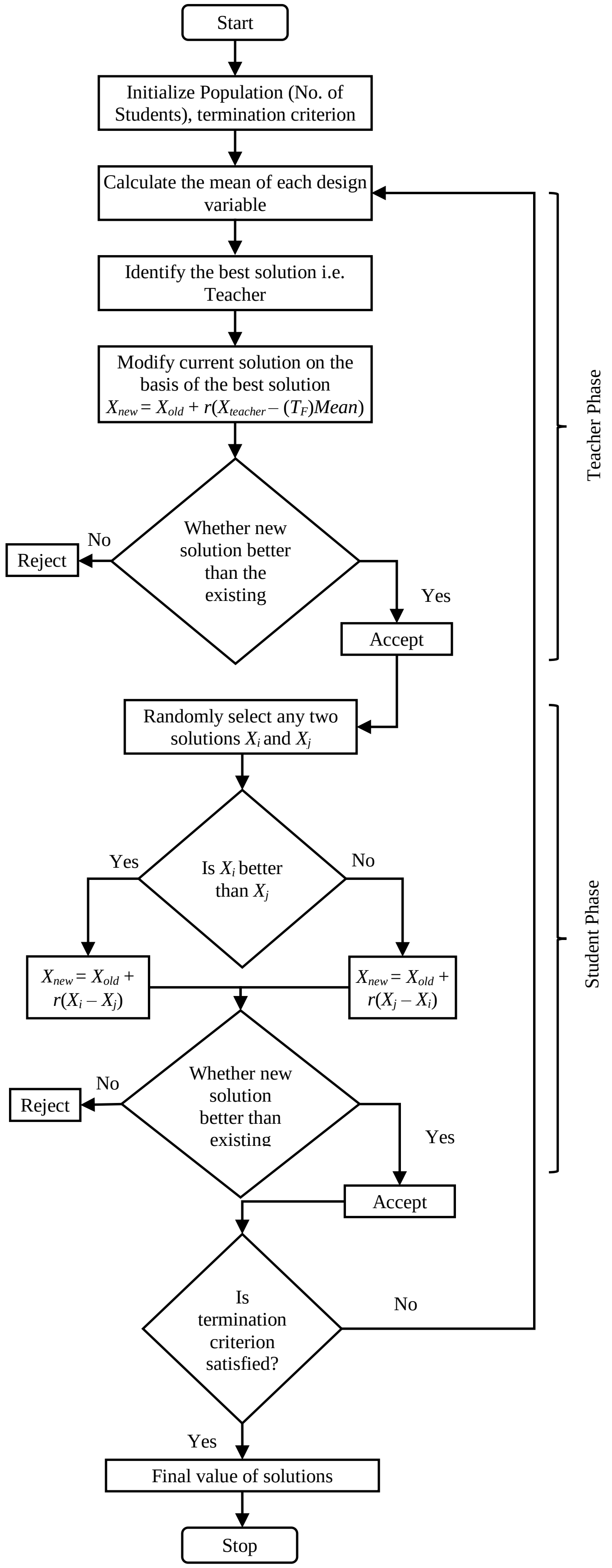

Fig.5. Flowchart of TLBO

6.2 FLOWCHART AND METHODOLOGY OF TLBO

The flowchart of Teaching-Learning-Based Optimization is shown in Fig.5. The PID controller tuning using TLBO has been explained in [14]. TLBO is analogous to other algorithms because firstly, it is also population based, here group of learners is the population. Secondly, it also has design variables, here different subjects offered to learners are the design variables.

7. RESULTS AND DISCUSSION

This section shows all the obtained results using simulation. The Table.2 shows all the obtained results for each performance index. Both the optimization techniques were used with a population count or swarm size of 50. For a given number of iterations following results were obtained. The Fig.6 and Fig.7 shows the plots when ITAE and IAE were used as fitness functions respectively.

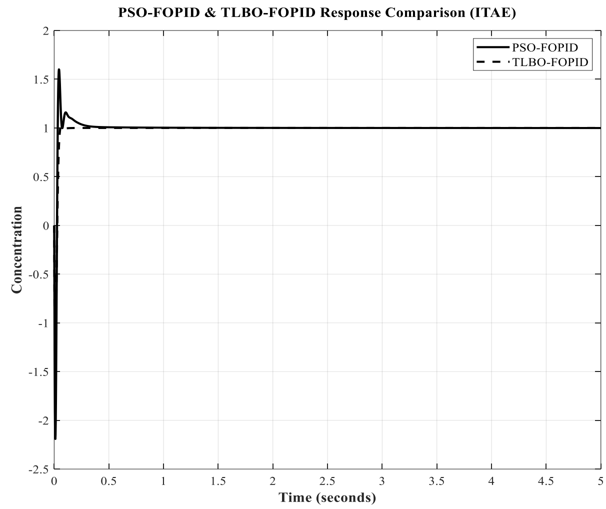

Fig.6. PSO-FOPID vs. TLBO-FOPID comparison with ITAE fitness function

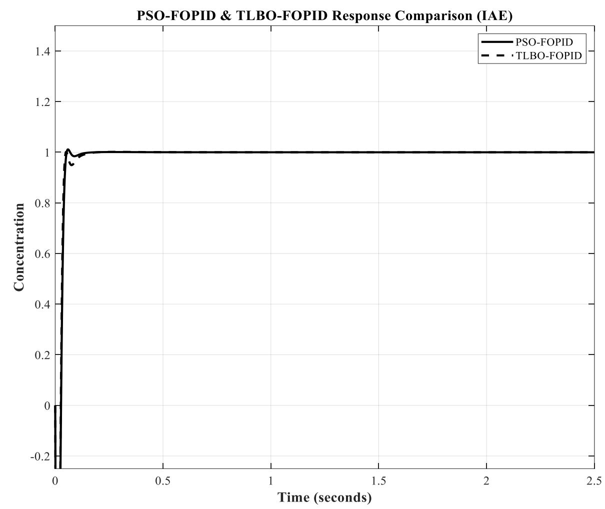

Fig.7. PSO-FOPID vs. TLBO-FOPID comparison with IAE fitness function

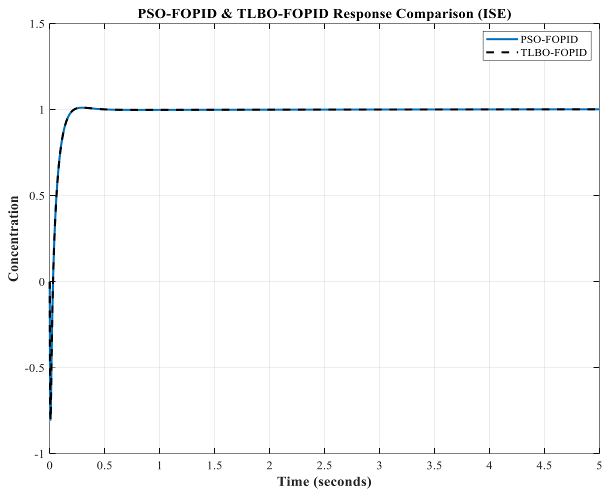

Fig.8. PSO-FOPID vs. TLBO-FOPID comparison with ISE fitness function

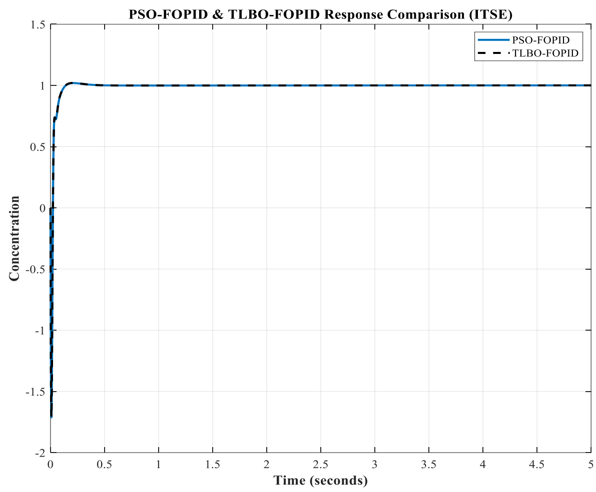

Fig.9. PSO-FOPID vs. TLBO-FOPID comparison with ITSE fitness function

It can be observed that TLBO is giving significantly better results than PSO in terms of settling time and peak overshoot when ITAE or IAE were used as fitness function. However, Fig.8 and Fig.9 shows almost similar results for both the techniques when ISE and ITSE were used as fitness functions respectively.

Table.2. Step Response analysis of various techniques

Technique	Rise Time (min)	Peak Time (min)	Settling Time (min)	Peak Overshoot (%)
PSO-FOPID (ITAE)	0.0053	0.0107	0.2006	60.2676
TLBO-FOPID (ITAE)	0.0181	1.4298	0.0507	1.8653
PSO-FOPID (IAE)	0.0160	0.0093	0.0473	1.1430
TLBO-FOPID (IAE)	0.0135	0.0092	0.0429	0.2251
PSO-FOPID (ISE)	0.1023	0.2948	0.1744	0.9459
TLBO-FOPID (ISE)	0.1025	0.2987	0.1742	1.0000
PSO-FOPID (ITSE)	0.0607	0.0088	0.1022	1.8972
TLBO-FOPID (ITSE)	0.0601	0.0088	0.1013	1.9814

8. CONCLUSIONS

This paper shows the design of a FOPID controller for an Isothermal CSTR. The controller was tuned using PSO and TLBO algorithm and a comparative analysis was performed using each performance index as fitness functions one by one. It was observed that however, TLBO was able to achieve little bit better results than PSO for ITAE and IAE fitness functions but for the other two performance indices viz. ISE and ITSE, almost similar results were obtained. Hence, by using a FOPID controller and tuning with either of the two algorithms will result in getting a quick control over the concentration of reactants and obtaining good productivity.

REFERENCES

[1] Y. Chen, I. Petras and D. Xue, "Fractional Order Control-A Tutorial", *Proceedings of American Control Conference*, pp. 1397-1411, 2009.

[2] R. Eberhart and J. Kennedy, "A New Optimizer using Particle Swarm Theory", *Proceedings of 6th International Symposium on Micro Machine and Human Science*, pp. 39-43, 1995.

[3] R.V. Rao, V.J. Savsani and D.P. Vakharia, "Teaching-Learning-based Optimization: A Novel Method for Constrained Mechanical Design Optimization Problems", *Computer-Aided Design*, Vol. 43, No. 3, pp. 303-315, 2011.

[4] R.V. Rao, "*Teaching Learning Based Optimization Algorithm and its Engineering Applications*", Springer, 2016.

[5] L. Gao, "Modeling and Dynamics Analyses of Immobilized CSTR Bioreactor using Transfer Function Model", *Proceedings of International Symposium on Information Technologies in Medicine and Education*, pp. 692-695, 2012.

[6] N. Kumar and N. Khanduja, "Mathematical Modelling and Simulation of CSTR using MIT Rule", *Proceedings of IEEE 5th India International Conference on Power Electronics*, pp. 1-5, 2012.

[7] A. Singh and V. Sharma, "Concentration Control of CSTR through Fractional Order PID Controller by using Soft Techniques", *Proceedings of 4th International Conference on Computing, Communications and Networking Technologies*, pp. 1-6, 2013.

[8] Indranil Pan and Saptarshi Das, "Fractional Order Fuzzy Control of Hybrid Power System with Renewable Generation using Chaotic PSO", *ISA Transactions*, Vol. 62, pp. 19-29, 2016.

[9] Saptarshi Das, Indranil Pan, Shantanu Das and Amitava Gupta, "A Novel Fractional Order Fuzzy PID Controller and its Optimal Time Domain Tuning based on Integral Performance Indices", *Engineering Applications of Artificial Intelligence*, Vol. 25, No. 2, pp. 430-442, 2012.

[10] Z. Bingul and O. Karahan, "Tuning of Fractional PID Controllers using PSO Algorithm for Robot Trajectory Control", *Proceedings of IEEE International Conference on Mechatronics*, pp. 955-960, 2011.

[11] Yinggan Tang, Mingyong Cui, Changchun Hua, Lixiang Li and Yixian Yang, "Optimum Design of Fractional Order PI$^{\lambda}$D$^{\mu}$ Controller for AVR System using Chaotic Ant

Swarm", *Expert Systems with Applications*, Vol. 39, No. 8, pp. 6887-6896, 2012.

[12] Banaja Mohanty, "TLBO Optimized Sliding Mode Controller for Multi-Area Multi-Source Nonlinear Interconnected AGC System", *International Journal of Electrical Power and Energy Systems*, Vol. 73, pp. 872-881, 2015.

[13] Rabindra Kumar Sahu, Sidhartha Panda, Umesh Kumar Rout and Dillip Kumar Sahoo, "Teaching Learning based Optimization Algorithm for Automatic Generation Control of Power System using 2-DOF PID Controller", *International Journal of Electrical Power and Energy Systems*, Vol. 77, pp. 287-301, 2016.

[14] Shamik Chatterjee and V. Mukherjee, "PID Controller for Automatic Voltage Regulator using Teaching Learning based Optimization Technique", *International Journal of Electrical Power and Energy* Systems, Vol. 77, pp. 418-429, 2016.