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# FUZZY PROBABILISTIC AND FRACTAL DIMENSIONAL APPROACH FOR CHLORIDE INDUCED CORROSION TIME (CICT)

**R. Arun Balaji[1], M. Muruganandam[2] and J. Madhivadhanan[3]**
[1]*Department of Physics, Wollega University, Ethiopia*
[2]*Department of Electrical and Computer Engineering, Wollega University, Ethiopia*
[3]*Department of Informatics, Wollega University, Ethiopia*

***Abstract***

***An attempt for exertion is made to utilize the capacity of fuzzy arbitrariness in dealt with instabilities to develop a generic approach for strength based administration life plan of fortified cement basic individuals. In the proposed procedure, convection and surface chloride focus are handled as irregular variable and subsequently the diffusion coefficient is characterized as a fuzzy arbitrary variable. The present work, dealt about the service life in sea-side location that lacks region of support due to limited distinction plan is evaluated using fuzzy probabilistic approach and fractal analysis. A case issue is also discussed to outline the adequacy of the proposed approach.***

***Keywords:***

***Corrosion, Permeation, Fuzzy Set, Fuzzy Random Variable, Fractal Dimension***

## 1. INTRODUCTION

Chloride incited consumption of fortification is an issue of real sympathy toward strengthened solid structure situated in beach front zones. Consumption of fortification is an electrochemical procedure and different frameworks for checking the level of erosion in this field have been utilized by various analysts. It is intriguing to note that, Colubi et al. [4] have distinguished a wide class of fuzzy sets with some probabilistic ideas. Utilizing the ideas displayed in [4], it is conceivable to consider fuzzy valued random elements in decision making problems, as majority of the variables are probabilistic in nature. The fuzzy probabilistic approach exhibited will be valuable for outstanding life estimation and for settling on choice with respect to administration examinations [2]. A fuzzy arbitrary strategy has turned into an exceptionally intense apparatus in the examination of issues, which include unsupervised information. Investigation of these issues utilizing fuzzy irregular strategies yields preferable outcomes over any non-fuzzy arbitrary methods.

This paper examines scientifically the chloride transport problem using fuzzy random theory as a tool. The chloride transport issue is described by uncertainty and imprecision in data that is required to settle on choice which is typically inadequate. The essential information like diffusion, convection, surface chloride concentration etc. are required in modeling the above framework. The values are taken in [1], [7], [8], [18]. Owing to these uncertainties, fuzzy random theory tackles the problem by considering the issue of greater amount of basic mechanical nature and inclusion of inconvenient calculations, with the help of matlab programming. Fractal dimension is being used as an additional tool for the supportive analysis of Chloride Induced Corrosion Time which vouches the merit of the proposed work.

The organization of this research article is as follows: Section 2 provides the literature survey. Preliminaries on fuzzy set, fuzzy number, fuzzy random variables and fractal dimension are depicted in the section 3. Section 4 portrays on the transport condition and their coefficient terms that are taken on assumptions; Gauge of service life and the failure probability; and case issue for a transport condition of a reinforced concrete. Section 5 gives results and discussion along with supportive fractal analysis, while the conclusions are drawn in section 6.

## 2. LITERATURE SURVEY

Boddy et al. [1] developed service life prediction models, in order to ensure adequate durability of reinforced concrete structures in chloride environments. Different mathematical models were attempted by different researchers [2]-[6], [19]-[20]. The risk in the building based on critical values in risk evaluation strategies, parameter weights, and grey statistical clustering coefficients and the case study were presented to demonstrate the applicability and effectiveness of the proposed model [7]-[11]. The fractal approach is used to analyze the images in medical field [12-18].

The causes of concrete degradation can be divided into chemical and physical attacks separated by Dhir et al. [21], Young et al. [22], Mamlouk et al. [23], Secco et al. [24], Wu, et al. [25]. Tsai-Lung Weng adopted grey statistical clustering and multi-phase fuzzy statistics in the development of a risk assessment model for existing reinforced concrete buildings [26].The feasibility of using fuzzy logic for the development of models to estimate the chloride diffusion coefficient in concrete were discussed by Wellington Mazer et al [27]. The application of fuzzy logic technique (Gaussian and Triangular) for developing a model for predicting the compressive strength of high-strength concrete having supplementary cementitious materials were studied [28].

From literature survey, it is concluded that the fuzzy logic serves a fine prediction for service life of all structures. Hence it motivates to analyze the service period of concrete structures in sea bed and also in flat surfaces. This paper projects the analysis that helps to predict the penetration of chloride ions into concrete structures under sea environments.

## 3. PRELIMINARIES

### 3.1 FUZZY SET

The membership function $\tilde{\mu}_A(x)$ of a fuzzy set $\tilde{A}$ is a function so every element in $x$ in $X$ has membership degree [3]: $\mu_{\tilde{A}}(x) \in [0,1]$. $A$ is completely determined by the set of tuples:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) x \in X\} \tag{1}$$

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree $\alpha$ is called the $\alpha$ - cut set:

$$A_\alpha = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\} \tag{2}$$

$A_\alpha^{'} = \{x \in X / \mu_{\tilde{A}}(x) > \alpha\}$ is called strong α- cut[1].

If a fuzzy set is convex and normalized the membership function is defined in $R$ and piecewise continuous, it is called as fuzzy number. So fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy. The triangular fuzzy number $\tilde{N}$ is defined by three numbers $\alpha < m < \beta$ as follows, $\tilde{A}$ = ($\alpha$, $m$, $\beta$). This representation is interpreted as membership function [5] in Fig.1 and Fig.2.

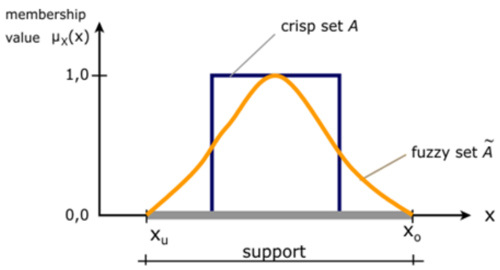


Fig.1. Fuzzy set

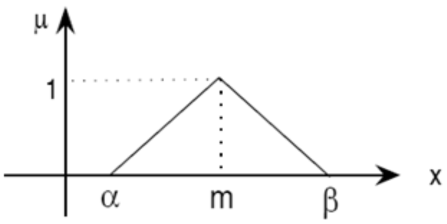


Fig.2. Fuzzy Membership Function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-\alpha}{m-\alpha} & \alpha \leq x < m \\ 1 & x = m \\ \dfrac{x-\beta}{m-\beta} & m < x \leq \beta \\ 0 & otherwise \end{cases} \tag{3}$$

### 3.2 FUZZY RANDOM VARIABLES (FRV)

Fuzzy randomness simultaneously describes objective and subjective information as a fuzzy set of possible probabilistic models over some range of imprecision. This generalized uncertainty model contains fuzziness and randomness as special cases. Objective uncertainty in the form of observed/measured data is modeled as randomness, whereas subjective uncertainty, e.g., due to a lack of trustworthiness or imprecision of measurement results, distribution parameters, environmental conditions, or the data sources are described as fuzziness [10], [11]. The model fuzzy randomness combines but not to mix objectivity and subjectivity that are separately visible at any time. It may be understood as an imprecise probabilistic model, which allows for simultaneously considering all possible probability models that are relevant to describing the problem [19] and [20].

The space of the random elementary events Ω and the fundamental set $X = R^n$ are introduced. Instead of a real-valued realization, a fuzzy realization $\tilde{x}(\omega) = (\tilde{x}_1 \tilde{x}_2, ..., \tilde{x}_n) \subset X$ is assigned to each elementary event $\omega \in \Omega$ see Fig.1. [11]

A fuzzy random variable or fuzzy random vector is the fuzzy result of the uncertain mapping $\Omega \tilde{\rightarrow} F(R^n)$, where $F(R^n)$ is the set of all fuzzy numbers in $R^n$. Each real valued random vector $x$ on $X$ that is completely contained in $\tilde{X}$ is referred to as an original $x_i$ of $\tilde{X}$. That is, the crisp realizations $x_i$ of an original $X_i$ are elements of the fuzzy realizations $\tilde{x}$ of the associated fuzzy random variable $\tilde{X}$ for all elementary events, $\omega \in \Omega$ see Fig.3. The potential for FRVs becomes more apparent when we explore probability distributions. An example is the distribution of a random variable $X \sim N(\mu, \sigma^2)$, with fuzzy $\mu$ and crisp $\sigma^2$ [6]. The situation is depicted in Fig.4.

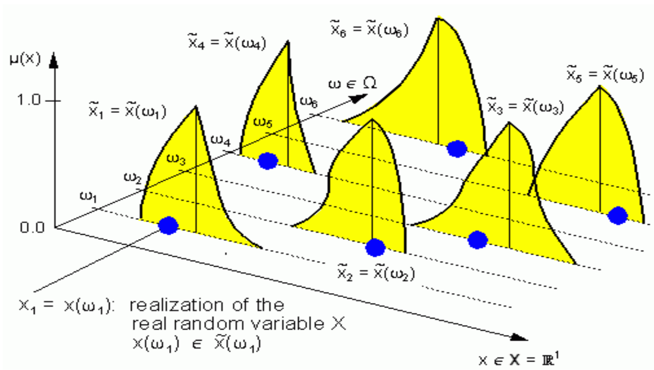


Fig.3. Model of a fuzzy random variable

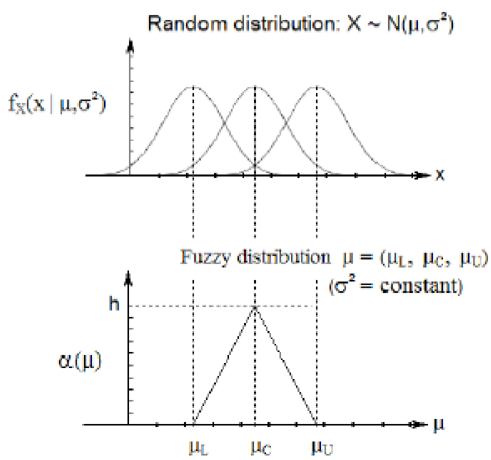


Fig.4. Fuzzy random variable

As indicated in the Fig.4, the mean of the distribution of $X$ is the triangular – shaped fuzzy number $(\mu_L, \mu_C, \mu_U)$. As a consequence, $\mu$ may fall as for to the left as $\mu_L$ resulting in the left–hand distribution for $X$, and as for to the right as $\mu_U$, resulting in the right hand distribution for $x$, although the highest possibility is associated with $\mu_C$.

### 3.3 FRACTAL DIMENSION

Mandelbrot introduced Fractal Geometry to the world of research in 1982, which has gained due weightage over the years in the field of research due to its broad spectrum of application domains. The self-similar and irregular geometric primitives are referred to as Fractals. Self-similarity is a property, where a subset is indistinguishable from the whole, upon different scales of magnification [12], [13].

Fractal objects are described by their Fractal dimension (FD), which materialize their geometric structure. In the fractal world, the Fractal dimension of an object, need not be an integer number and is normally greater than its topological dimension. In Euclidean n-space, the bounded set $A$ is said to be self-similar when $A$ is the union of $X_t$ distinct non-overlapping copies of itself, each of which is similar to $A$ scaled down by a ratio $t$. Fractal Dimension FD of A can be derived from the relation [14], as,

$$FD = \frac{\log(X_t)}{\log\left(\frac{1}{t}\right)} \tag{5}$$

The potential of fractal dimension has been broadly explored and considered as a supportive tool for the analysis of chloride induced corrosion factor in terms of detection of service life in this present study.

## 4. CHLORIDE TRANSPORT MODEL

The transportation of chloride ions into concrete is a convoluted procedure which includes dissemination, slender suction, pervasion and convective move through the pore framework and miniaturized scale splitting system, joined by physical adsorption and synthetic imposing. Whenever the chloride gets intruded, cement gets dispersed and infused. Hence, in the present study, two mechanisms are considered for modeling the chloride ingress through concrete. The chloride transport equation, given by [8] is shown below,

$$\frac{\partial c}{\partial t} = D \times \frac{\partial^2 c}{\partial x^2} - V \times \frac{\partial c}{\partial x} \tag{6}$$

where,

$C$ - free chloride in solution at depth $x$ after time $t$,

$D$ - Diffusion co-efficient

$V$ - Average linear velocity = $\frac{Q}{nA} = -\frac{k}{n}\frac{dh}{dx}$

$Q$ - Flow rate

$A$ - Cross – sectional area

$k$ - Permeability co-efficient (hydraulic conductivity)

$h$ - Hydraulic head

$n$ - Porosity

The left-hand side of Eq.(1) indicates the rate of change in chloride concentration with time. Two different mechanisms are represented by two terms on right-hand side of the equation. The first term, the diffusion term comes from Fick’s second law for one-dimensional non-stationary flow in a semi-infinite medium. The second term describes the change in chloride concentration occurred due to permeation.

### 4.1 DIFFUSION

By diffusion, the chloride transport in saturated concrete does occur in the presence of a chloride concentration gradient. The value of the diffusion coefficient $D$ depends on the age of concrete ($t$) as well as the exposure temperature ($T$) and can be modeled [12] as,

$$D(t,T) = D_{ref} \cdot \left(\frac{t_{ref}}{t}\right)^m \cdot \exp\left[\frac{U}{R} \cdot \left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right] \tag{7}$$

where,

$D(t,T)$ - diffusion co-efficient at time $t$ and temperature $T$

$D_{ref}$ - diffusion co-efficient at some reference time $t_{ref}$ and reference temperature $T_{ref}$

$m$ - Constant (depending on mix proportions)

$U$ - Activation energy of the diffusion process

$R$ - Gas constant

$T$ - Exposure Temperature (°$K$)

The value of constant $m$ is mainly dependent on the type of cement, which can be taken as 0.2 for ordinary Portland cement concrete.

### 4.2 PERMEATION

Permeation is the rate of fluid ingress driven by a pressure gradient. It is generally known that the permeability co-efficient varies with time, due to continued hydration of cement paste with temperature, influence on viscosity and density of penetrating fluid. In the present study, the time dependent permeability co-efficient is modeled as [7],

$$k(t,T) = \frac{k_{ref}}{Z} \cdot \left(\frac{t_{ref}}{t}\right)^n \tag{8}$$

where,

$k$ - Permeability co-efficient at time $t$ and temperature $T$

$k_{ref}$ - Permeability co-efficient at some reference time $tref$

$Z$ - Viscosity temperature correction factor

$n$ - Porosity

The chloride transport model is capable of assimilation for the following considerations into calculations,

- Initial diffusion value ($D$) for concrete,
- Time ($t$) - dependent reduction of $D$,
- Temperature ($T$) dependent variation of $D$,
- Varying surface chloride concentration with time
- Variation in temperature with time.

- Time ($t$) is variation of permeation.

The following assumptions are made, in the present study for the analysis.

- The analysis is one – dimensional
- The concrete is homogeneous and isotropic (no cracking),
- The concrete is fully saturated in the modeling region
- The liquid carrying the chlorides is incompressible,
- Diffusion co-efficient is constant with depth
- The concrete is under isothermal conditions
- There is an infinite supply of chloride to ingress into the concrete.

## 4.3 ESTIMATION OF SERVICE LIFE

In order to devolve the service life, proposed methodology is applied as follows. The initiation of fuzzy random for chloride induced corrosion of reinforcement in concrete is considered as the end of service life. The time for corrosion which defines the service life of the structural behavior, can be estimated as the time at which the chloride concentration at the level of reinforcement becomes equal to the critical chloride concentration ($C_{cr}$). The input parameters are defined as follows.

If $m$ is a constant dependent on mix proportions, which controls the rate of reduction of diffusivity. $\tilde{T}$ is absolute temperature of exposure for the structure defined as fuzzy set and $D_{ref}$ is diffusion coefficient at some reference time $t_{ref}$ and reference temperature $T_{ref}$, which is used in Eq.(2) to get fuzzy random coefficient of diffusion ( $\tilde{D}$ ) and also given other constant parameter values in Table.1. The CoV of fuzzy random diffusion is derived from fifteen percentage of $D_{ref}$. $C_{cr}$ is threshold chloride concentration or critical chloride level required to initiate corrosion of steel, it has taken the value 0.05% by weight of concrete.

If $K_{ref}$ is a permeability coefficient at some reference time $t_{ref}$ and this value use in Eq.(3) and Eq.(1). Convection value is random. ($C_s$) in surface chloride concentration which defined as random variable in three different zones. The CoV of surface chloride concentration and convection are derived from Eq.(9). Then by knowing the initial and boundary condition, and having through of fuzzy random coefficient of diffusion $\tilde{D}$ and random coefficient of V one can determines the chloride concentration at any depth $x$ at any time $t$ using finite difference method. The finite difference method is implemented in Matlab, which will be useful in determines the time to corrosion initiation reinforcement concrete structure member affected by chloride induced corrosion.

In the present study, $\alpha$ - cut levels are taken as 0, 0.2, 0.4, 0.6, 0.8 and 1.0. From values of failure probability for a specific α - cut level, interval of fuzzy random set of failure probability is determined. In this way, fuzzy random set of failure probability at any specific time is determined. An example is provided in the next section to illustrate the proposed method.

## 4.4 CASE ISSUE

An attempt is made to determine the predicted service life of reinforce concrete is proposed. The chloride transport equation is given as,

$$\frac{\partial \tilde{C}}{\partial t} = \tilde{D}.\frac{\partial^2 \tilde{C}}{\partial x^2} - V.\frac{\partial \tilde{C}}{\partial x}, 0 < x < 1, t > 0 \tag{9}$$

$BC$: $\tilde{C}$ (0, $t$) = $C_s(t)$,

$\tilde{C}$ (1, $t$) = 0 , $0 < x < 1, t > 0$

$IC$: $\tilde{C}$ ($x$, 0) = 0

A reinforcement concrete bridge deck, located in a marine environment is considered in the present study. The service life of the bridge deck, which is made up of concrete namely typical bridge deck is estimated bridge deck to be exposed to different concentration condition.

Table.1. Input Parameters

| Parameters | Values |
|---|---|
| $M$ | 0.2 |
| $C_{cr}$ | 0.05% by weight of concrete [1] |
| $D_{ref}$ | $0.33 \times 10^{-12}$m$^2$/s (23°C, 180 days) [1] |
| $k_{ref}$ | $1.00 \times 10^{-13}$m/s [1] |
| Thickness of cover | 50 mm |
| $C_s$ | 0.7820 year (atmospheric zone) |

The effect of temperature variation is also included in analysis by considering the fuzzy set in temperature for the different exposure condition. If $x$ is thickness of cover to reinforcement, $\tilde{D}$ is the diffusion coefficient for chloride in concrete, $C_s$ is the surface chloride concentration and $\tilde{C}$ is the critical chloride concentration. As outlined before, fuzzy random theory allows a quantification of the parameters in Eq.(4) based on the available input parameter data show in Table.1.

The value of $U$, the activation energy of the diffusion process from the diffusion co-efficient equation is taken to be 35000J/mol. The viscosity temperature correction factor $Z$ included in the permeability co-efficient equation is set to be 1.0. A porosity '$n$' of 8% is used. A hydraulic gradient '$h$' of 25m/m is used while investigating the effects of pressure-driven flow. The value of the gas constant ($R$) is 8.314472 J K$^{-1}$mol$^{-1}$ [7].

Table.2. Parameters for consideration of Fuzzy random, random variables and fuzzy set

| Parameter | Dimension | Type of Uncertainty | Mean | Standard deviation | Reference |
|---|---|---|---|---|---|
| ~ | [m$^2$/s] | Fuzzy Random | $7.0863\times10^{-6}$ | $1.5766\times10^{-6}$ | [1],[9] |
| $V$ | [m/s ] | Random | $489.5013\times10^{-6}$ | $7.4159\times10^{-5}$ | [1],[9] |
| $C_s$ | [years] | Random | 0.7820 | 0.0457 | [1],[9] |
| $\tilde{T}$ | [C] | Fuzzy Set | [15°, 20°, 25°] | | Proposed |

## 5. RESULTS AND DISCUSSIONS

Fuzzy random theory allows a quantification of the parameters in Eq.(8) and using finite difference method. The fundamental solution is obtained in each case by a direct Monte Carlo simulation based on 10000 sample elements, from which the expected value is estimated. The approximated membership function of the fuzzy expected value and the empirical fuzzy probability cumulative distribution function of the displacement are shown in Fig.5.

$$\tilde{C}(x,t+\Delta t)=\tilde{C}(x,t) + \frac{\tilde{D}\cdot\Delta t}{\Delta x^2}\left\lfloor(\tilde{C}(x-\Delta x,t)-2\tilde{C}(x,t)+\tilde{C}(x+\Delta x,t))\right\rfloor - \frac{V\cdot\Delta t}{\Delta x}\begin{bmatrix}(\tilde{C}(x+\Delta x,t)\\ -\tilde{C}(x,t))\end{bmatrix} \quad (10)$$

*BC*: $\tilde{C}$ (0, *t*) = $C_s(t)$, $\tilde{C}$ (1,*t*) = 0,

*IC*: $\tilde{C}$ (*x*,0)=0

The failure probability values obtained from fuzzy random analysis are more rational since appropriate representations of uncertainty are used for the different variables. Though, it is possible to obtain bounds on failure probability taking computationally expensive part into consideration, probability analysis is carried out to obtain probability distribution for failure probability.

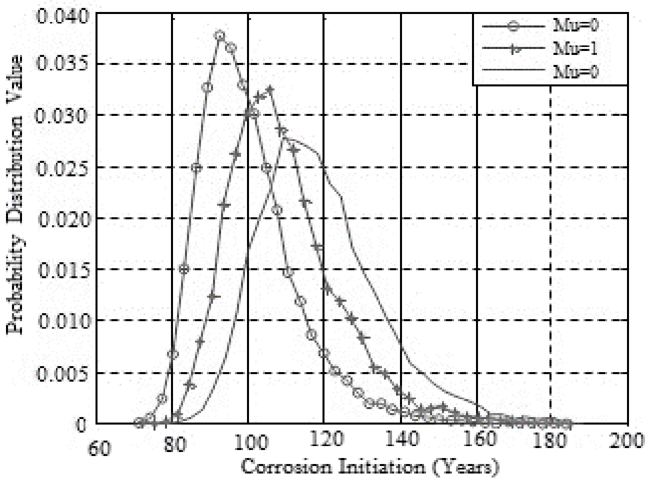


Fig.5. Fuzzy probability density function

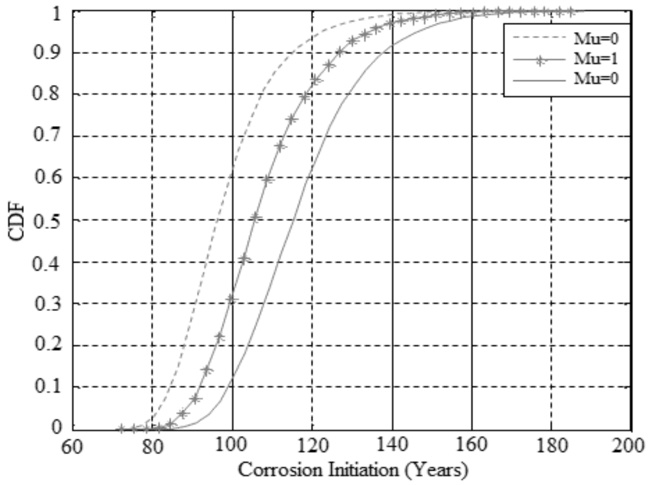


Fig.6. Fuzzy probability CDF

By the resulting fuzzy random set of failure probability obtained using the proposed procedure, one can obtain not only the possibility distribution of failure probability, but also the bounds for characteristic values of failure probability corresponding to the specified quantizes points that are taken at regular intervals from the cumulative distribution function (CDF) of random variable as shown in Fig.6. The bound for characteristic values of failure probability is 95% which is obtained from fuzzy random set at different times and it can be used to estimate fuzzy random failure probability at 95% where cut is zero. This is depicted in Fig.7.

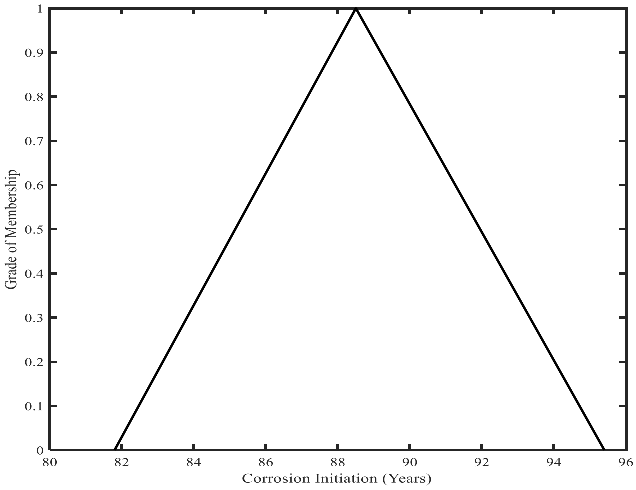


Fig.7. Fuzzy random of failure probability

## 5.1 FRACTAL ANALYSIS

The present study tries to evaluate the characterization of corrosion for predicting the service life of chloride reinforced concrete using Fractal dimension bound temporal analysis. This research work makes use of two consecutive images projecting different levels of corrosion and analyzes the extent to which corrosion has taken place in the concrete structures. By visual perception, the more corrosion levels from the structures have been identified and those levels are ascertained mathematically using the values of Fractal dimension. The fractal geometry based study, thereby justifies the fact that the level of fractal dimension values is to be maintained in ascertaining the corrosion induction. Temporal property in terms of roughness measure the fractal dimension values between the less corroded and the more corroded images of the respective images.

The structure behavior of corrosion in terms of roughness is measured through Fractal Dimension value, which is tabulated in Table.3. It is observed that for the image 1 the fractal dimension of the less corroded is 1.6152 and 1.7062 for the more corroded image. Similarly, for the image 2, the fractal dimension of the less corroded is 1.6517 and 1.7136 for the more corroded

From the Table.3, it can be observed that there is an increment in the values of fractal dimension, whenever the level of corrosion gets increased, which is ascertained from the image through human visual perception. Hence, it is clearly noticed that the fractal dimension value correlates very well with the roughness of the concrete structures, thereby would become an imperative parameter in the decision regarding the service life of concrete structures.

Table.3. Analysis of Increment in Fractal Dimension

| Images | Level of corrosion | Fractal Dimension | Increment in FD | Inference |
|---|---|---|---|---|
| Image 1 | Less corroded  | 1.6152 | 0.0910 | Confirmed Increased Corrosion |
| | More corroded  | 1.7062 | | |
| Image 2 | Less corroded  | 1.6517 | 0.0643 | Confirmed Increased Corrosion |
| | More corroded  | 1.7136 | | |

## 6. CONCLUSION

In this paper, the level of corrosion induced reinforcement in concrete is considered and analyzed. Non-fuzzy probabilistic approach could show the predicted service life of reinforce concrete as a mean value only, whereas the Fuzzy probabilistic approach is useful for estimating predicted service life of reinforced concrete from fuzzy random set. While comparing those values of two approaches, it is observed that more flexible solution is achieved by fuzzy random set as it shows a range of predicted service life. It is also been illustrated that the bounds for characteristic value of failure probability are determined from the resulting fuzzy random set with minimal computational effort. Further, the proposed temporal analysis of corrosion level using fractal dimension, that uses the measure of roughness, is proved to have the potential to assist in the detection of service life of concrete structures.

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