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APPLICATION OF INTERVAL VALUED INTUITIONISTIC FUZZY SOFT SETS OF ROOT TYPE IN DECISION MAKING

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Abstract

In this paper we introduce the concentration and dilation operators on interval valued intuitionistic fuzzy soft sets of root type which is a generalization of fuzzy set, intuitionistic fuzzy set, interval valued intuitionistic fuzzy set and intuitionistic fuzzy set of root type and establish some properties of these operators. We define Hamming distance between two interval valued intuitionistic fuzzy soft sets of root type and it is shown that it is a metric. A similarity measure based on this Hamming distance is defined and some properties are established. We also develop a decision making method based on the similarity measure of Hamming distance between interval valued intuitionistic fuzzy soft sets of root type. We develop an algorithm for the decision making problem and illustrate its working by means of examples.

Keywords:

Interval Valued Intuitionistic Fuzzy Soft Set of Roottype, Hamming Distance, Similarity Measure, Decision Making Technique

1. INTRODUCTION

Uncertainty forms a vital part in our day to day life. While handling real life problems which involve uncertainty such as medical fields, economics, engineering, industry and so on, the conventional tools may not be sufficient and convenient. The introduction of fuzzy set theory by Zadeh [24] came as a boon for studying some type's uncertainties, whenever traditional tools fail. Fuzzy theory and its generalizations contributed to some remarkable applications of mathematics in a variety of real life problems involving certain types of uncertainties. Several generalizations and modifications of fuzzy set theory such as theory of intuitionistic fuzzy sets, vague sets, rough sets, soft sets, and interval mathematics have also been developed for handling different types of uncertainties. This paper deals with another generalization of fuzzy set, namely interval valued intuitionistic fuzzy soft set of root type, some operations, on this set and an application of the set in decision making problem.

Measuring the similarity between fuzzy sets, an important tool in fuzzy environment, has gained attention of researchers for their wide applications in real life problems. Similarity measures are very much useful in many application areas, such as pattern recognition, machine learning, decision making and market prediction. Different measures of similarities between fuzzy sets have been proposed and applied by researchers in recent years.

In this paper we define two new operators namely Concentration and Dilation on interval valued intuitionistic fuzzy soft set of root type (IVIFSSRT) and discuss some theoretical properties of these operators. A similarity measure base on Hamming distance between IVIFSSRT is introduced and some basic properties investigated. Finally, we develop a decision making method based on similarity measure between IVIFSSRT and construct an algorithm for this purpose. Examples are provided to illustrate the working of this algorithm.

The rest of this paper is organized as follows: In section 2, a brief review of literature relating to research work carried out in this paper is presented. In section 3, the preliminaries on IVIFS and the need for IVIFSSRT are explained. It is shown that IVIFSRT is a generalization of IVIFS. Two new operations Concentration and Dilation based on IVIFSSRT are defined and some of its interesting theoretical properties are also established in section 4. In section 5, Hamming distance between two IVIFSSRT is defined and shown that it is a metric. A similarity measure based on Hamming distance of IVIFSSRT is defined and some of its properties are also established. In this section, a new method for tackling decision making problems in fuzzy environment is explained and an algorithm is developed for this purpose. The working of the algorithm is illustrated with suitable examples. A brief conclusion is presented in the last section.

2. LITERATURE REVIEW

Atanassov [2, 5] initiated the study of intuitionistic fuzzy sets. They [3, 4] also contributed to the development of interval valued intuitionistic fuzzy set theory. Some basic operations on intuitionistic fuzzy sets of root type are introduced by Palaniappan et al. [18, 19]. The notion of soft sets was introduced and studied by Molodtsov [17]. Maji et al. [12, 13, 14] developed the notions of fuzzy soft sets and intuitionistic fuzzy soft sets. Gorzalczany [8] developed a method of inference in approximate reasoning based on interval valued fuzzy sets. Deschriver and Kerre [7] explained the relationship between some extensions of fuzzy set theory. Yang et al. [23] combined the notions of interval valued fuzzy set and soft set. The ideas of fuzzy soft sets are applied to decision making problems by Roy and Maji [20] and Kong et al. [10].

Similarity measure is a tool useful in the investigation of proximity between intuitionistic fuzzy sets. Different types of similarity measures are first introduced by Atanassov [5]. Similarity measures are also used in pattern recognition problems by Li and Cheng [11]. Similarity measure based on Hausdorff distance was used in pattern recognition problems by Hung and Yang [9]. Use of similarity measure between interval valued intuitionistic fuzzy set in pattern recognition was established by Xu [21]. Majumdar and Samanta [15, 16] introduced a similarity measure based on distance between soft sets and fuzzy soft sets. Xu [22] proposed a distance measure between intervals valued intuitionistic fuzzy sets for group decision making problems. Recently Deli and Cagman [6] developed a similarity measure based on distance between intuitionistic fuzzy soft sets.

3. PRELIMANARIES

Some definitions and results essential for this study are recalled in this section.

Definition 3.1 [2] Let X be a non-empty set. An intuitionistic fuzzy set A is an object of the form,

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},$$

where the functions $\mu_A:X\rightarrow[0,1]$ and $\nu_A:X\rightarrow[0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively, and for every $x \in X$, $0 < \mu_A(x) + \nu_A(x) \leq 1$.

Definition 3.2 [3] An interval valued intuitionistic fuzzy set on anon empty set X is an object of the form,

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},$$

where, $\mu_A(x)$ and $\nu_A(x):A \rightarrow D([0,1])$. Here $D([0,1])$ stands for the set of all closed subintervals of [0, 1] and $\mu_A(x)$, $\nu_A(x) \in D([0,1])$. For each $x \in X$, $\mu_A(x)=\left[\underline{\mu}_A(x),\bar{\mu}_A(x)\right]$ and $\nu_A(x)=\left[\underline{\nu}_A(x),\bar{\nu}_A(x)\right]$ satisfying the condition $0<\bar{\mu}_A(x)+\bar{\nu}_A(x)\leq 1$.

Remark 3.1 Atanassov‘s [2] definition of intuitionistic fuzzy set imposes a condition on $\mu_A(x)$ and $\nu_A(x)$ as $0 < \mu_A(x) + \nu_A(x) \leq 1$, which in turn implies that $\nu_A(x) \leq 1 - \mu_A(x)$ for each $x \in X$. This goes against the spirit that $\mu_A(x)$ and $\nu_A(x)$ are assigned independently. As this independence criteriais more important, we relax the condition $0 < \mu_A(x) + \nu_A(x) \leq 1$ and hence there is possibility for $\mu_A(x) + \nu_A(x) \geq 1$. To make the assignments of membership $\mu_A(x)$ and non-membership $\nu_A(x)$ more realistic, we impose a new condition $\sqrt{\mu_A(x)}+\sqrt{\nu_A(x)}\leq 2$ and consider the more generalized intuitionistic fuzzy set namely the intuitionistic fuzzy set of root type.

Definition 3.3 [18]. Let X be a non-empty set. An intuitionistic fuzzy set of root type A is an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ where the functions μ_A: X $\rightarrow$ [0,1] and ν_A: X $\rightarrow$ [0,1] define the degree of membership and degree of non-membership of the element $x \in X$ respectively, and for every $x \in X$, $0<\frac{1}{2}\sqrt{\mu_A(x)}+\frac{1}{2}\sqrt{\nu_A(x)}\leq 1$.

Definition 3.4 [18] The degree of non-determinacy (uncertainty or hesitancy) of an element $x \in X$ to the intuitionistic fuzzy set of root type A is defined as,

$$\pi_A(x)=\left(1-\sqrt{\mu_A(x)}-\sqrt{\nu_A(x)}\right)^2.$$

Definition 3.5 [19] The concentration of an intuitionistic fuzzy set of root type A of X denoted by *concrt*, is defined as,

$$concrt(A)=\left\{\left\langle x,\mu_A(x),1-\left(1-\sqrt{\nu_A(x)}\right)^2\right\rangle x\in X\right\}.$$

Definition 3.6 [19] The dilation of an intuitionistic fuzzy set of root type A over X denoted by *dilt*, is defined as,

$$dilt(A)=\left\{\left\langle x,\left(\mu_A(x)\right)^{1/4},1-\sqrt{1-\sqrt{\nu_A(x)}}\right\rangle x\in X\right\}.$$

Let U be the universe of objects and E the set of parameters in relation to objects in U. Parameters are often attributes, characteristics or properties of objects.

Definition 3.7 [12] Let $F(U)$ be the set of all fuzzy subsets of U and $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F:A \rightarrow F(U)$.

For any parameter $\alpha \in A$, $F(\alpha)$ is a fuzzy subset of U and it is called fuzzy value set of the parameter $\alpha \in A$, $F(\alpha)=\left\{\left(x,\mu_{F(\alpha)}(x)\right):x\in U\right\}\mu_{F(\alpha)}(x)$ denotes the membership degree that an object x holds on the parameter α.

Definition 3.8 [10] Let $P(U)$ denote the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. A pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F:A\rightarrow P(U)$.

Definition 3.9 [6] Let $U = \{a_1,a_2,\ldots a_n\}$ be an universal set, $E = \{e_1,e_2,\ldots,e_n\}$ be a set of parameters and (F, A), (G, B) be two intuitionistic fuzzy soft sets on U. Then the Hamming distance between (F, A) and (G, B) is defined as,

$$H_d\left\langle(F,A),(G,B)\right\rangle=\frac{1}{2m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left|\mu_{F(e_i)}(a_j)-\mu_{G(e_i)}(a_j)\right|+\left|\nu_{F(e_i)}(a_j)-\nu_{G(e_i)}(a_j)\right|\right)\right\}$$

Definition 3.10 [1] Let U be an universe and E a set of parameters. Let *IVIFSSRT*(U) denote the set of all interval valued intuitionistic fuzzy sets of root type of U and $A \subseteq E$. A pair is called an *IVIFSSRT* over U, where F is a mapping given by $F : A \rightarrow$ *IVIFSSRT*(U) and

$$(F,A)=\left\{\left\langle x,\left[\underline{\mu}_{F(e)}(x),\bar{\mu}_{F(e)}(x)\right],\left[\underline{\nu}_{F(e)}(x),\bar{\nu}_{F(e)}(x)\right]\right\rangle : x\in U,e\in A\right\}$$

For any parameter $e \in A$, $F(e)$ is an *IVIFSSRT*.

Definition 3.11 [1] The necessity operator on an *IVIFSSRT*(F, A) is denoted by $\Theta(F, A)$ and is defined as

$$\Theta(F,A)=\left\{\left\langle x,\left[\underline{\mu}_{F(e)}(x),\bar{\mu}_{F(e)}(x)\right],\left[\left(1-\sqrt{\bar{\mu}_{F(e)}(x)}^2\right),\left(1-\sqrt{\underline{\mu}_{F(e)}(x)}^2\right)\right]\right\rangle : x\in U,e\in A\right\}$$

Definition 3.12 [1] The possibility operator on an *IVIFSSRT*(F, A) is denoted by $\Delta(F, A)$ and is defined as

$$\Delta(F,A)=\left\{\left\langle x,\left[\left(1-\sqrt{\bar{\nu}_{F(e)}(x)}^2\right),\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}^2\right)\right],\left[\underline{\nu}_{F(e)}(x),\bar{\nu}_{F(e)}(x)\right]\right\rangle : x\in U,e\in A\right\}$$

4. CONCENTRATION AND DILATION OPERATORS ON IVIFSSRT

In this section, we define the operators' concentration (*Con*) and dilation (*Dil*) on *IVIFSSRT* and study some of their properties.

Definition 4.1. Let A, $B \subseteq E$.(F, A) is an interval valued intuitionistic fuzzy soft subset of root type of (G, B) denoted by $(F, A) \subset (G, B)$ if and only if,

i) $A \subseteq B$;

ii) $\forall e \in A$, $F(e)$ is an interval valued intuitionistic fuzzy soft subset of root type of $G(e)$ i.e, $\forall x \in U$, $e \in A$,

$$\underline{\mu}_{F(e)}(x) \le \underline{\mu}_{G(e)}(x), \bar{\mu}_{F(e)}(x) \le \bar{\mu}_{G(e)}(x),$$
$$\underline{\nu}_{F(e)}(x) \ge \underline{\nu}_{G(e)}(x), \bar{\nu}_{F(e)}(x) \ge \bar{\nu}_{G(e)}(x).$$

Further (G, B) is called an interval valued intuitionistic fuzzy superset of root type for (F, A) and is denoted by $(G, B) \supset (F, A)$.

Definition 4.2. The degree of non-determinacy (hesitancy) of an element $x \in U$, $e \in A$ to the *IVIFSSRT*(F, A) is defined as,

$$\underline{\pi}_{F(e)}(x) = \left(1 - \sqrt{\underline{\mu}_{F(e)}(x)} - \sqrt{\underline{\nu}_{F(e)}(x)}\right)^2 \text{ and}$$

$$\bar{\pi}_{F(e)}(x) = \left(1 - \sqrt{\bar{\mu}_{F(e)}(x)} - \sqrt{\bar{\nu}_{F(e)}(x)}\right)^2$$

Definition 4.3. The concentration of an *IVIFSSRT*(F, A) over U denoted by $Con(F, A)$ is defined as,

$$Con(F, A) = \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] \left[1 - \left(1 - \sqrt{\underline{\nu}_{F(e)}(x)}\right)^2, 1 - \left(1 - \sqrt{\bar{\nu}_{F(e)}(x)}\right)^2\right]\right\rangle : x \in U, e \in A\right\}$$

Proposition 4.1. Let U be a non-empty set and let (F, A) be an *IVIFSSRT* over U. Then the following are true:

i) If $\pi_{F(e)}(x) = 0$. Then $\pi_{ConF(e)}(x) = 0$, if and only if $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [0,0]$ or $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [1,1]$.

ii) $\Theta[Con(F, A)] = Con[\Theta(F, A)]$, if and only if $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [0,0]$ or $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [1,1]$.

iii) $\Delta[Con(F, A)] = Con[\Delta(F, A)]$, if and only if $\left[\underline{\nu}_{F(e)}(x), \bar{\nu}_{F(e)}(x)\right] = [0,0]$ or $\left[\underline{\nu}_{F(e)}(x), \bar{\nu}_{F(e)}(x)\right] = [1,1]$.

Proof:

i) We have $\left[\underline{\pi}_{F(e)}(x), \bar{\pi}_{F(e)}(x)\right] = [0,0]$

$$\Rightarrow \left(1 - \sqrt{\underline{\mu}_{F(e)}(x)} - \sqrt{\underline{\nu}_{F(e)}(x)}\right)^2 = 0 \quad \text{and}$$

$$\left(1 - \sqrt{\bar{\mu}_{F(e)}(x)} - \sqrt{\bar{\nu}_{F(e)}(x)}\right)^2 = 0$$

$$\Rightarrow \sqrt{\underline{\mu}_{F(e)}(x)} + \sqrt{\underline{\nu}_{F(e)}(x)} = 1 \text{ and}$$

$$\sqrt{\bar{\mu}_{F(e)}(x)} + \sqrt{\bar{\nu}_{F(e)}(x)} = 1.$$

$$Con(F, A) = \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right], \left[1 - \left(1 - \sqrt{\underline{\nu}_{F(e)}(x)}\right)^2, 1 - \left(1 - \sqrt{\bar{\nu}_{F(e)}(x)}\right)^2\right]\right\rangle : x \in U, e \in A\right\}$$

$$= \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right], \left[1 - \underline{\mu}_{F(e)}(x), 1 - \bar{\mu}_{F(e)}(x)\right]\right\rangle : x \in U, e \in A\right\}$$

$$\underline{\pi}_{Con\,F(e)}(x) = 0$$

$$\Leftrightarrow \left(1 - \sqrt{\underline{\mu}_{F(e)}(x)} - \sqrt{1 - \underline{\mu}_{F(e)}(x)}\right)^2 = 0$$

$$\Leftrightarrow 1 - \sqrt{\underline{\mu}_{F(e)}(x)} = \sqrt{1 - \underline{\mu}_{F(e)}(x)}$$

$$\Leftrightarrow \left(1 - \sqrt{\underline{\mu}_{F(e)}(x)}\right)^2 = 1 - \underline{\mu}_{F(e)}(x)$$

$$\Leftrightarrow \sqrt{\underline{\mu}_{F(e)}(x)} = \underline{\mu}_{F(e)}(x)$$

$$\Leftrightarrow \sqrt{\underline{\mu}_{F(e)}(x)}\left(1 - \sqrt{\underline{\mu}_{F(e)}(x)}\right) = 0$$

$$\Leftrightarrow \underline{\mu}_{F(e)}(x) = 0 \text{ or } \underline{\mu}_{F(e)}(x) = 1.$$

Similarly, $\bar{\pi}_{Con\,F(e)}(x) = 0 \Leftrightarrow \bar{\mu}_{F(e)}(x) = 0$ or $\bar{\mu}_{F(e)}(x) = 1$.

ii)

$$\Theta(F, A) = \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right], \left[\left(1 - \sqrt{\bar{\mu}_{F(e)}(x)}^2\right), \left(1 - \sqrt{\underline{\mu}_{F(e)}(x)}^2\right)\right] : x \in U, e \in A\right\rangle\right\}$$

Also,

$$Con(F, A) = \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right], \left[1 - \left(1 - \sqrt{\underline{\nu}_{F(e)}(x)}\right)^2, 1 - \left(1 - \sqrt{\bar{\nu}_{F(e)}(x)}\right)^2\right]\right\rangle : x \in U, e \in A\right\}$$

$$= \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right], \left[1 - \underline{\mu}_{F(e)}(x), 1 - \bar{\mu}_{F(e)}(x)\right]\right\rangle : x \in U, e \in A\right\}$$

$\Theta[Con(F, A)] = Con[\Theta(F, A)]$

$$\Leftrightarrow \left(1 - \sqrt{\bar{\mu}_{F(e)}(x)}\right)^2 = 1 - \bar{\mu}_{F(e)}(x)$$

$$\Leftrightarrow 2\sqrt{\bar{\mu}_{F(e)}(x)} = 2\bar{\mu}_{F(e)}(x)$$

$$\Leftrightarrow \bar{\mu}_{F(e)}(x) = \left(\bar{\mu}_{F(e)}(x)\right)^2$$

$$\Leftrightarrow \bar{\mu}_{F(e)}(x)\left(1-\bar{\mu}_{F(e)}(x)\right) = 0$$

$$\Leftrightarrow \bar{\mu}_{F(e)}(x) = 0 \text{ or } \bar{\mu}_{F(e)}(x) = 1.$$

Similarly, $\underline{\mu}_{F(e)}(x) = 0$ or $\underline{\mu}_{F(e)}(x) = 1$.

iii) $\Delta[Con(F, A)]$ =

$$\left\{\left\langle x, \left[\begin{array}{c} 1-\sqrt{1-\left(1-\sqrt{\bar{\nu}_{F(e)}(x)}\right)^2}^{\,2}, \\ \sqrt{1-\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2}^{\,2} \end{array}\right], \left[1-\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2, 1-\left(1-\sqrt{\bar{\nu}_{F(e)}(x)}\right)^2\right]\right\rangle : x \in U, e \in A\right\}$$

Also,

$$Con(F,A) = \left\{\left\langle x, \left[\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2, \left(1-\sqrt{\bar{\nu}_{F(e)}(x)}\right)^2\right], \left[1-\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2, 1-\left(1-\sqrt{\bar{\nu}_{F(e)}(x)}\right)^2\right]\right\rangle : x \in U, e \in A\right\}$$

$\Delta[Con(F, A)] = Con[\Delta(F, A)]$

$$\Leftrightarrow \left(1-\sqrt{1-\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2}\right)^2 = \left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2$$

$$\Leftrightarrow \sqrt{1-\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2} = \sqrt{\underline{\nu}_{F(e)}(x)}$$

$$\Leftrightarrow \left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2 = 1-\underline{\nu}_{F(e)}(x)$$

$$\Leftrightarrow \underline{\nu}_{F(e)}(x) - \sqrt{\underline{\nu}_{F(e)}(x)} = 0$$

$$\Leftrightarrow \sqrt{\underline{\nu}_{F(e)}(x)}\left(\sqrt{\underline{\nu}_{F(e)}(x)}-1\right) = 0$$

$$\Leftrightarrow \underline{\nu}_{F(e)}(x) = 0 \text{ or } \underline{\nu}_{F(e)}(x) = 1.$$

Similarly, $\underline{\nu}_{F(e)}(x) = 0$ or $\bar{\nu}_{F(e)}(x) = 1$.

Definition 3.4. The dilation of an *IVIFSSRT*(F, A) over U denoted by $Dil(F, A)$ is defined as,

$$Dil(F,A) = \left\{\left\langle x, \left[\begin{array}{c} \left(\underline{\mu}_{F(e)}(x)\right)^{1/4}, \\ \left(\bar{\mu}_{F(e)}(x)\right)^{1/4} \end{array}\right], \left[1-\sqrt{1-\sqrt{\bar{\nu}_{F(e)F(e)}(x)}}, 1-\sqrt{1-\sqrt{\underline{\nu}_{F(e)F(e)}(x)}}\right]\right\rangle : x \in U, e \in A\right\}.$$

Proposition 4.2 Let U be a non-empty set and let (F, A) be an *IVIFSSRT* over U. Then the following are true:

i) If $\pi_{F(e)}(x) = 0$, then $\pi_{Dil\,F(e)}(x) = 0$, if and only if $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [0,0]$ or $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [1,1]$.

ii) $\Theta[Dil(F,A)] = Dil[\Theta(F,A)]$ if and only if $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [0,0]$ or $\left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right] = [1,1]$.

iii) $\Delta[Dil(F,A)] = Dil[\Delta(F,A)]$ if and only if $\left[\underline{\nu}_{F(e)}(x), \bar{\nu}_{F(e)}(x)\right] = [0,0]$ or $\left[\underline{\nu}_{F(e)}(x), \bar{\nu}_{F(e)}(x)\right] = [1,1]$.

Proof: The proof is similar to Proposition 3.1.

Proposition 4.3. For any *IVIFSSRT* $Con(F,A) \subset (F,A) \subset Dil(F,A)$.

Proof:

Consider, $$(F,A) = \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right], \left[\underline{\nu}_{F(e)}(x), \bar{\nu}_{F(e)}(x)\right]\right\rangle\right\}$$

$$Con(F,A) = \left\{\left\langle x, \left[\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x)\right], \left[\begin{array}{c} 1-\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2, \\ 1-\left(1-\sqrt{\bar{\nu}_{F(e)}(x)}\right)^2 \end{array}\right]\right\rangle : x \in U, e \in A\right\}$$

Since, $\underline{\nu}_{F(e)}(x), \bar{\nu}_{F(e)}(x) \in [0,1]$,

$$1-\left(1-\sqrt{\underline{\nu}_{F(e)}(x)}\right)^2 \geq \underline{\nu}_{F(e)}(x) \text{ and } 1-\left(1-\sqrt{\bar{\nu}_{F(e)}(x)}\right)^2 \geq \bar{\nu}_{F(e)}(x)$$

Hence $Con(F,A) \subset (F,A)$.

$$Dil(F,A) = \left\{\left\langle x, \left[\begin{array}{c} \left(\underline{\mu}_{F(e)}(x)\right)^{\frac{1}{4}}, \\ \left(\bar{\mu}_{F(e)}(x)\right)^{\frac{1}{4}} \end{array}\right], \left[\begin{array}{c} 1-\sqrt{1-\sqrt{\bar{\nu}_{F(e)}(x)}}, \\ 1-\sqrt{1-\sqrt{\underline{\nu}_{F(e)}(x)}} \end{array}\right]\right\rangle : x \in U, e \in A\right\}$$

Since, $\underline{\mu}_{F(e)}(x), \bar{\mu}_{F(e)}(x), \underline{\nu}_{F(e)}(x), \bar{\nu}_{F(e)}(x) \in [0,1]$,

$$\underline{\mu}_{F(e)}(x) \leq \left(\underline{\mu}_{F(e)}(x)\right)^{1/4}, \bar{\mu}_{F(e)}(x) \leq \left(\bar{\mu}_{F(e)}(x)\right)^{\frac{1}{4}},$$

$$\underline{\nu}_{F(e)}(x) \geq 1-\sqrt{1-\sqrt{\underline{\nu}_{F(e)}(x)}}$$

and $\bar{\nu}_{F(e)}(x) \geq 1-\sqrt{1-\sqrt{\bar{\nu}_{F(e)}(x)}}$.

Hence $(F,A) \subset Dil(F,A)$.

Therefore, $Con(F,A) \subset (F,A) \subset Dil(F,A)$.

5. DECISION MAKING METHOD BASED ON SIMILARITY MEASURE OF IVIFSSRT

In this section we define a similarity measure on IVIFSSRT and develop a decision making method based on this similarity measure.

Definition 5.1. Let $U = \{x_1, x_2,\ldots, x_n\}$ be an universal set, $E = \{e_1, e_2,\ldots e_m\}$ be a set of parameters and (F, A), (G, B) are two *IVIFSSRT* on U. Then the Hamming distance between (F, A) and (G, B) is defined as,

$$\xi\left\langle (F,A),(G,B)\right\rangle = \frac{1}{4m}\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(|\underline{\mu}_{F(e_i)}(x_j)-\underline{\mu}_{G(e_i)}(x_j)| + |\overline{\mu}_{F(e_i)}(x_j)-\overline{\mu}_{G(e_i)}(x_j)| + |\underline{\nu}_{F(e_i)}(x_j)-\underline{\nu}_{G(e_i)}(x_j)| + |\overline{\nu}_{F(e_i)}(x_j)-\overline{\nu}_{G(e_i)}(x_j)| + |\underline{\pi}_{F(e_i)}(x_j)-\underline{\pi}_{G(e_i)}(x_j)| + |\overline{\pi}_{F(e_i)}(x_j)-\overline{\pi}_{G(e_i)}(x_j)|\Big).$$

Theorem 5.1. Let $U = \{x_1, x_2,\ldots, x_n\}$ be a finite non-empty universal set, $E = \{e_1, e_2,\ldots e_m\}$ be the set of parameters and *IVIFSSRT*(U) denote the set of all *IVIFSRT* over U. The distance function ξ from *IVIFSSRT*(U) to the set of non negative real numbers is a metric.

Proof: Let (F, A), (G, B) and (H, C) be three *IVIFSRT*s over U.

i) $\xi\langle(F, A), (G, B)\rangle > 0$ follows from Definition.

ii) $\xi\langle(F, A), (G, B)\rangle = 0$

$$\Leftrightarrow |\underline{\mu}_{F(e_i)}(x_j)-\underline{\mu}_{G(e_i)}(x_j)|+|\overline{\mu}_{F(e_i)}(x_j)-\overline{\mu}_{G(e_i)}(x_j)| + |\underline{\nu}_{F(e_i)}(x_j)-\underline{\nu}_{G(e_i)}(x_j)|+|\overline{\nu}_{F(e_i)}(x_j)-\overline{\nu}_{G(e_i)}(x_j)| + |\underline{\pi}_{F(e_i)}(x_j)-\underline{\pi}_{G(e_i)}(x_j)|+|\overline{\pi}_{F(e_i)}(x_j)-\overline{\pi}_{G(e_i)}(x_j)|$$

$$\Leftrightarrow \underline{\mu}_{F(e_i)}(x_j)=\underline{\mu}_{G(e_i)}(x_j), \overline{\mu}_{F(e_i)}(x_j)=\overline{\mu}_{G(e_i)}(x_j), \underline{\nu}_{F(e_i)}(x_j)=\underline{\nu}_{G(e_i)}(x_j), \overline{\nu}_{F(e_i)}(x_j)=\overline{\nu}_{G(e_i)}(x_j), \underline{\pi}_{F(e_i)}(x_j)=\underline{\pi}_{G(e_i)}(x_j), \overline{\pi}_{F(e_i)}(x_j)=\overline{\pi}_{G(e_i)}(x_j)$$

$$\Leftrightarrow (F,A)=(G,B).$$

iii) Clearly, $\xi\langle(F, A), (G, B)\rangle = \xi\langle(G, B), (F, A)\rangle$

iv) Next we prove that, $\xi\left\langle (F,A).(G,B)\right\rangle = \xi\left\langle (F,A),(H,C)\right\rangle+\xi\left\langle (H,C).(G,B)\right\rangle$.
For all $i \in \{1, 2, \ldots, m\}$, $j \in \{1, 2, \ldots, n\}$, $\xi\langle(F, A), (G, B)\rangle$

$$=|\underline{\mu}_{F(e_i)}(x_j)-\underline{\mu}_{G(e_i)}(x_j)|+|\overline{\mu}_{F(e_i)}(x_j)-\overline{\mu}_{G(e_i)}(x_j)| + |\underline{\nu}_{F(e_i)}(x_j)-\underline{\nu}_{G(e_i)}(x_j)|+|\overline{\nu}_{F(e_i)}(x_j)-\overline{\nu}_{G(e_i)}(x_j)| + |\underline{\pi}_{F(e_i)}(x_j)-\underline{\pi}_{G(e_i)}(x_j)|+|\overline{\pi}_{F(e_i)}(x_j)-\overline{\pi}_{G(e_i)}(x_j)|$$

$$=|\underline{\mu}_{F(e_i)}(x_j)-\underline{\mu}_{H(e_i)}(x_j)+\underline{\mu}_{H(e_i)}(x_j)-\underline{\mu}_{G(e_i)}(x_j)| + |\overline{\mu}_{F(e_i)}(x_j)-\overline{\mu}_{H(e_i)}(x_j)+\overline{\mu}_{H(e_i)}(x_j)-\overline{\mu}_{G(e_i)}(x_j)| + |\underline{\nu}_{F(e_i)}(x_j)-\underline{\nu}_{H(e_i)}(x_j)+\underline{\nu}_{H(e_i)}(x_j)-\underline{\nu}_{G(e_i)}(x_j)| + |\overline{\nu}_{F(e_i)}(x_j)-\overline{\nu}_{H(e_i)}(x_j)+\overline{\nu}_{H(e_i)}(x_j)-\overline{\nu}_{G(e_i)}(x_j)| + |\underline{\pi}_{F(e_i)}(x_j)-\underline{\pi}_{H(e_i)}(x_j)-\underline{\pi}_{H(e_i)}(x_j)-\underline{\pi}_{G(e_i)}(x_j)| + |\overline{\pi}_{F(e_i)}(x_j)-\overline{\pi}_{H(e_i)}(x_j)-\overline{\pi}_{H(e_i)}(x_j)-\overline{\pi}_{G(e_i)}(x_j)|$$

$$\le |\underline{\mu}_{F(e_i)}(x_j)-\underline{\mu}_{H(e_i)}(x_j)|+|\underline{\mu}_{H(e_i)}(x_j)-\underline{\mu}_{G(e_i)}(x_j)| + |\overline{\mu}_{F(e_i)}(x_j)-\overline{\mu}_{H(e_i)}(x_j)|+|\overline{\mu}_{H(e_i)}(x_j)-\overline{\mu}_{G(e_i)}(x_j)| + |\underline{\nu}_{F(e_i)}(x_j)-\underline{\nu}_{H(e_i)}(x_j)|+|\underline{\nu}_{H(e_i)}(x_j)-\underline{\nu}_{G(e_i)}(x_j)| + |\overline{\nu}_{F(e_i)}(x_j)-\overline{\nu}_{H(e_i)}(x_j)|+|\overline{\nu}_{H(e_i)}(x_j)-\overline{\nu}_{G(e_i)}(x_j)| + |\underline{\pi}_{H(e_i)}(x_j)-\underline{\pi}_{H(e_i)}(x_j)|+|\underline{\pi}_{H(e_i)}(x_j)-\underline{\pi}_{G(e_i)}(x_j)| + |\overline{\pi}_{F(e_i)}(x_j)-\overline{\pi}_{H(e_i)}(x_j)|+|\overline{\pi}_{H(e_i)}(x_j)-\overline{\pi}_{G(e_i)}(x_j)|$$

$$=\xi\left\langle (F,A),(H,C)\right\rangle+\xi\left\langle (H,C).(G,B)\right\rangle.$$

Hence ξ is a metric.

Definition 5.2. For any two *IVIFSRT*(F, A) and (G, B) over U, the similarity measure between (F, A) and (G, B) based on Hamming distance denoted by $S\langle(F, A), (G, B)\rangle$ is defined as,

$$S\left\langle (F,A),(G,B)\right\rangle = \frac{1}{1+\xi\left\langle (F,A),(G,B)\right\rangle}.$$

Definition 5.3. Let (F, A) and (G, B) be two *IVIFSSRTs* on U. Then (F, A) and (G, B) are said to be α - similar, denoted as $(F, A) \approx \alpha(G, B)$, if and only if $S\langle(F, A), (G, B)\rangle \ge \alpha$ for $\alpha \in (0, 1)$. We call the two *IVIFSSRTs* significantly similar if $S\langle(F, A), (G, B)\rangle \ge 1/2$.

Theorem 5.2. For any two IVIFSRTs (F, A) and (G, B) over U. The following are true:

i) $0 \le S\langle(F, A), (G, B)\rangle \le 1$;

ii) $S\langle(F, A), (G, B)\rangle = S\langle(G, B), (F, A)\rangle$;

iii) $S\langle(F, A), (G, B)\rangle = 1$ if and only if $(F, A) = (G, B)$.

Proof: Proof is obvious.

5.1 ALGORITHM FOR DECISION MAKING METHOD

Now we develop a decision making method based on similarity measure of *IVIFSSRT*.

Step 1: Construct an *IVIFSSRT*(F, A) over U based on the evaluation by an expert.

Step 2: Construct an *IVIFSSRT*(G, B) over U based on the available data.

Step 3: Calculate the Hamming distance between (F, A) and (G, B).

Step 4: Calculate the similarity measure between (F, A) and (G, B).

Step 5: Conclude using the value of the similarity measure.

Example 5.1: Let us suppose that an earthquake occurs in deep ocean. A team of members from Tsunami Warning Center (TWC) has to decide on the region which is in danger zone. The attributes taken into consideration for finding the regions in the danger zone are $E = \{e_1, e_2, e_3, e_4\}$, where e_1 = wave length, e_2 = amplification, e_3 = ocean depth and e_4 = split in different areas. U = {Area-1, Area-2, Area-3} of a region. Based on these attributes the TWC has to alert the region that is in danger of possible tsunami threat.

Step 1: *IVIFSSRT*(F, E) gives the values prepared by TWC based on the previous records of tsunami occurrence.

Table.1. An *IVIFSSRT*(F, E) based on the previous records of tsunami occurrence

U	**Area-1**	**Area-2**	**Area-3**
e_1	[0.35,0.39],[0.59,0.63]	[0.39,0.45],[0.64,0.68]	[0.35,0.41],[0.67,0.71]
e_2	[0.61,0.63],[0.34,0.38]	[0.52,0.55],[0.39,0.46]	[0.55,0.59],[0.35,0.43]
e_3	[0.55,0.57],[0.41,0.44]	[0.49,0.52],[0.44,0.47]	[0.58,0.61],[0.39,0.41]
e_4	[0.69,0.71],[0.26,0.29]	[0.65,0.68],[0.27,0.31]	[0.69,0.74],[0.23,0.25]
e_5	[0.69,0.71],[0.34,0.37]	[0.66,0.75],[0.29,0.33]	[0.65,0.69],[0.33,0.35]

Step 2:

Table.2. *IVIFSSRT*(G, E) based on measurements recorded after an earthquake in Region-1

U	**Area-1**	**Area-2**	**Area-3**
e_1	[0.31,0.36],[0.54,0.56]	[0.34,0.39],[0.63,0.67]	[0.34,0.40],[0.62,0.68]
e_2	[0.55,0.58],[0.36,0.39]	[0.47,0.50],[0.43,0.49]	[0.51,0.53],[0.36,0.38]
e_3	[0.49,0.54],[0.43,0.46]	[0.41,0.48],[0.36,0.39]	[0.55,0.66],[0.38,0.46]
e_4	[0.63,0.73],[0.28,0.30]	[0.67,0.72],[0.24,0.35]	[0.65,0.76],[0.25,0.28]
e_5	[0.65,0.70],[0.32,0.38]	[0.65,0.74],[0.27,0.34]	[0.63,0.71],[0.30,0.36]

Table.3. *IVIFSSRT*(H, E) based on measurements recorded after an earthquake in Region-2

U	**Area-1**	**Area-2**	**Area-3**
e_1	[0.59,0.67],[0.27,0.38]	[0.68,0.73],[0.22,0.29]	[0.65,0.71],[0.29,0.32]
e_2	[0.31,0.39],[0.61,0.66]	[0.42,0.48],[0.54,0.61]	[0.41,0.45],[0.51,0.65]
e_3	[0.25,0.28],[0.71,0.79]	[0.21,0.26],[0.73,0.76]	[0.24,0.28],[0.69,0.75]
e_4	[0.29,0.31],[0.71,0.74]	[0.32,0.35],[0.69,0.73]	[0.26,0.31],[0.75,0.77]
e_5	[0.26,0.33],[0.63,0.69]	[0.25,0.29],[0.66,0.77]	[0.27,0.35],[0.65,0.67]

Step 3: Hamming distance is calculated using Definition 5.1 as

$$\xi\langle(F, E), (G, E)\rangle = 0.1415$$

$$\xi\langle(F, E), (H, E)\rangle = 1.023$$

Step 4: Similarity measure of (G, E) and (H, E) calculated using Definition 5.2 is,

$$S\langle(F, E), (G, E)\rangle = 0.876$$

$$S\langle(F, E), (H, E)\rangle = 0.4943$$

Step 5: The similarity measure between (F, E) and (G, E) is > 1/2. Hence the two IVIFSSRT are significantly similar. We conclude that the Region-1 is in danger zone and is likely to be severely affected by tsunami. In the case of Region-2, the similarity measure between (F, E) and (H, E) is < 1/2. Hence the two IVIFSSRT are not significantly similar and we conclude that Region-2 do not have a tsunami threat.

Example 5.2: Let us suppose that Country-A has to choose a launch vehicle for launching its satellite from two different countries B and C. The attributes considered for choosing the launch vehicle are $E = \{e_1, e_2, e_3, e_4\}$, where e_1 = thrust, e_2 = exhaust speed, e_3 = burn time and e_4 = fuel ratio on each stage. U = {first stage, second stage, upperstage} of the launch vehicle.

Step 1: *IVIFSSRT*(F, E) over U gives the standard values of the launch vehicle.

Table.4. *IVIFSSRT*(F, E) over U for the launch vehicle

U	**First stage**	**Second stage**	**Upper stage**
e_1	[0.37,0.41],[0.57,0.62]	[0.34,0.39],[0.61,0.68]	[0.39,0.46],[0.54,0.57]
e_2	[0.69,0.72],[0.32,0.36]	[0.61,0.63],[0.38,0.41]	[0.69,0.74],[0.32,0.38]
e_3	[0.58,0.63],[0.36,0.42]	[0.59,0.61],[0.37,0.44]	[0.51,0.53],[0.44,0.49]
e_4	[0.57,0.63],[0.36,0.41]	[0.58,0.64],[0.31,0.37]	[0.62,0.65],[0.38,0.42]

Step 2:

Table.5. *IVIFSSRT*(G, E) over U for the launch vehicle provided by Country-B

U	**First stage**	**Second stage**	**Upper stage**
e_1	[0.65,0.71],[0.25,0.27]	[0.59,0.69],[0.29,0.31]	[0.67,0.75],[0.22,0.27]
e_2	[0.31,0.38],[0.68,0.76]	[0.42,0.45],[0.65,0.69]	[0.36,0.40],[0.59,0.72]
e_3	[0.77,0.82],[0.15,0.19]	[0.79,0.84],[0.14,0.17]	[0.72,0.87],[0.11,0.16]
e_4	[0.21,0.25],[0.69,0.79]	[0.16,0.22],[0.77,0.85]	[0.24,0.29],[0.71,0.76]

Table.6. *IVIFSSRT*(H, E) over U for launch vehicle provided by Country-C

U	**First stage**	**Second stage**	**Upper stage**
e_1	[0.29,0.39],[0.48,0.63]	[0.68,0.75],[0.18,0.29]	[0.33,0.55],[0.28,0.48]
e_2	[0.25,0.38],[0.43,0.64]	[0.15,0.32],[0.63,0.74]	[0.15,0.43],[0.26,0.59]
e_3	[0.63,0.72],[0.17,0.23]	[0.51,0.70],[0.18,0.39]	[0.45,0.62],[0.29,0.37]
e_4	[0.40,0.62],[0.25,0.39]	[0.47,0.63],[0.30,0.40]	[0.51,0.66],[0.34,0.43]

Step 3: Hamming distance is calculated using Definition 5.1 as,

$$\xi\langle(F, E), (G, E)\rangle = 1.006$$

$$\xi\langle(F, E), (H, E)\rangle = 0.5854$$

Step 4: Similarity measure is calculated using Definition 5.2 as,

$$S\langle(F, E), (G, E)\rangle = 0.498$$

$$S\langle(F, E), (H, E)\rangle = 0.6307$$

Step 5: Similarity measure of (F, E) and (G, E) is < 1/2. Since the two IVIFSSRTs are not significantly similar, we conclude that the launch vehicle of Country-B is not suitable for launching the satellite of Country-A. For the case of Country-C, the similarity measure of (F, E) and (H, E) is > 1/2. Since the two IVIFSSRTs are significantly similar, we conclude that the launch vehicle of Country-C is suitable for launching the satellite of Country-A.

6. CONCLUSION

In this paper we have defined two new operators on interval valued intuitionistic fuzzy soft sets of root type and established some properties of these operators. The hamming distance between interval valued intuitionistic fuzzy soft sets of root type is defined and is proved to be a metric. A similarity measure based on hamming distance between interval valued intuitionistic fuzzy soft sets root type is defined and a new decision making technique using this similarity measure is developed. We have constructed an algorithm for the decision making problems and provided examples to illustrate the working of this algorithm.

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