

SOLVING FUZZY SEQUENCING PROBLEMS BASED ON THE GEOMETRIC MEAN OF THE HORIZONTAL POINTS ON THE REFERENCE FUNCTIONS

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Abstract

Jobs sequencing on machines is quite common in industries, such as Manufacturing, Logistics, Project Management, Healthcare, etc., and in Military organizations where imprecision in processing and elapsed times is prevalent. In practice, the imprecision in times can be represented by fuzzy intervals or by a fuzzy number more significantly. The fuzzy set theory provides scope to address the imprecision using fuzzy numbers. This paper deals with the sequencing problems with imprecise processing times for jobs, represented by trapezoidal fuzzy numbers. The geometric mean of the horizontal points on the reference functions of the fuzzy numbers is used to order the fuzzy numbers and suggest solutions of the sequencing problems. Finally, the suggested technique is illustrated by numerical examples and comparative reviews.

Keywords:

Fuzzy Sequencing Problems, Geometric Mean, Horizontal Points, Reference Functions

1. INTRODUCTION

Sequencing problems are concerned with an appropriate order (sequence) for a set of jobs to be done on a finite number of service facilities (like machines) in some well-determined technological order so as to optimize the efficiency such as total elapsed time of the jobs and idle time of the available facilities (machine). In such cases, the effectiveness is a function of the order or sequence in which the jobs are to be performed. The effectiveness may be measured in terms like cost, time, etc. A sequencing problem could involve jobs in a manufacturing plant, aircraft waiting for landing and clearance, maintenance scheduling in a factory, programmers to be run on a computer centre, customers in a bank, and so forth.

In real-life scenarios, the processing times in job sequencing problems are not always precise. The fuzzy set theory presents a foundation in which the processing times of the jobs are represented by fuzzy numbers to capture imprecision and vagueness in data. Many authors presented their studies of job sequencing problems with imprecise processing times represented by fuzzy numbers. To overcome the problem of vagueness, Zadeh [1] introduced the concept of fuzzy numbers as fuzzy subsets of real line, and Dubois [2] demonstrated the operations on fuzzy numbers, whereas Yagar [3] presented the ranking of fuzzy sets over the unit intervals. Cahon [4, 5] used fuzzy numbers in inventory control and presented fuzzy job sequencing for a job shop. Ishii and Tada [6] solved single machine scheduling with fuzzy precedence relation. Ishubuchi and Lee [7] also worked in scheduling problems under a fuzzy environment. Yuan et al. [8] solved scheduling problems with fuzzy due dates. Jadhav and Bajaj [9] used an average high-ranking method for defuzzification in scheduling problems. Nirmala and Anju [10] presented an application of fuzzy quantifier in sequencing problem with fuzzy

ranking method. Kripa and Govindarajan [11] solved the job sequencing problem using trapezoidal fuzzy numbers. Gupta et al. [12] proposed a new method for obtaining optimal sequence and compared it with Johnson's method. Sahoo [13] used Yagers' ranking index method for defuzzification of fuzzy numbers. Selvakumari and Santhi [15, 16] worked with pentagonal and octagonal fuzzy numbers and Selvakumari and Sowmiya [14] used hexagonal fuzzy numbers for solving Sequencing problems. Kanchana et al. [17] has taken processing times as triangular fuzzy numbers and by using ranking method for solving job sequencing problems. Sundari [18] presented fuzzy sequencing problem: A Novel Approach. Arunadevi et al. [19] demonstrated an Approach for solving fuzzy sequencing problems with trapezoidal fuzzy numbers using the Robust ranking method and Johnson's algorithm. Radhakrishnan and Saikethana [21] presented a new method for solving fuzzy sequencing problem. Senthil and E. Baskarprabhu (2022) presented graphical method for solving fuzzy sequencing problems. Kaur [20, 23] presented an algorithm for solving fuzzy job sequencing problems using Pythagorean numbers and studied Sequencing Problems using different ranking techniques in a Fuzzy Environment. Thirupathi and Kirubhashankar [24] suggested new ranking method for Linguistic triangular fuzzy sequencing problem,

Apart from the above introduction in Section 1, the forward task of the article is planned into the following five following sections. Section 2 comprises a brief appraisal of the basic notion of fuzzy number, geometric mean of the horizontal points on the reference functions and the ordering of the fuzzy numbers under the heading "Preliminaries". Section 3 briefs fuzzy sequencing problems and the modus of problem-solving. Section 4 comprises the numerical illustrations of the suggested method and comparative reviews. The concluding remarks are furnished at last in section 5.

2. PRELIMINARIES

2.1 FUZZY NUMBER

A fuzzy subset $\tilde{A}$ of the real line R is known as a fuzzy number if its membership function $f_{\tilde{A}}(x)$ which satisfies the following conditions for $a, b, c, d \in \tilde{A}$, ($a \leq b \leq c \leq d$),

$f_{\tilde{A}}(x)$ is a piece-wise continuous function of R to the closed interval $[0, 1]$, $f_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$, $f_{\tilde{A}}(x) = 1$, for all $x \in [a, b]$, $f_{\tilde{A}}(x)$ is strictly decreasing on $[c, d]$, and $f_{\tilde{A}}(x) = 0$, for all $x \in]-\infty, a] \cup [d, \infty[$. The fuzzy number in Def. 2.1 is conveniently represented as $A = (a, b, c, d)$, and its membership function $f_{\tilde{A}}(x)$ is expressed as

$$\tilde{f}_A(x) = \begin{cases} \tilde{f}_A^L(x), & x \in [a, b], \\ 1, & x \in [b, c], \\ \tilde{f}_A^R(x), & x \in [c, d], \\ 0, & \text{Otherwise.} \end{cases} \quad (1)$$

where $\tilde{f}_A^L(x) : [a, b] \rightarrow [0, 1]$ and $\tilde{f}_A^R(x) : [c, d] \rightarrow [0, 1]$ are known as the left and the right membership functions of the fuzzy number $\tilde{A}$, respectively. $\tilde{f}_A^L(x)$ is continuous and strictly increasing on $[a, b]$, whereas $\tilde{f}_A^R(x)$ is continuous and strictly decreasing on $[c, d]$.

2.2 GEOMETRIC MEAN OF THE HORIZONTAL POINTS AS A RANKING FUNCTION OF FUZZY NUMBERS

Let $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ are the points on the left and the right membership functions of a fuzzy number $\tilde{A} = (a, b, c, d)$, respectively. The visual depictions of these points are illustrated in Fig.1.

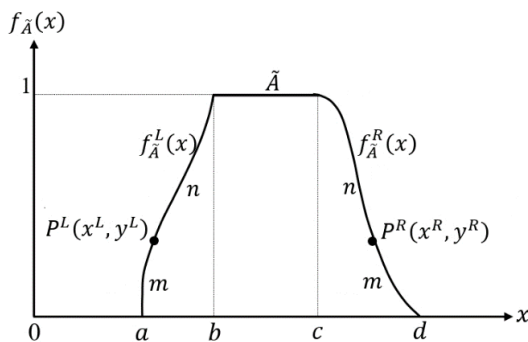


Fig.1. Visual depictions of the horizontal points on the left-right membership functions.

The points divide the corresponding membership functions in the same ratio $m:n$. The coordinates of the points $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ are obtained by solving the following equations.

$$n \int_a^{x^L} \sqrt{1 + \left(\frac{d\tilde{f}_A^L(x)}{dx} \right)^2} dx = m \int_{x^L}^b \sqrt{1 + \left(\frac{d\tilde{f}_A^L(x)}{dx} \right)^2} dx, \quad (2)$$

$$n \int_0^{y^L} \sqrt{1 + \left(\frac{d\tilde{g}_A^L(y)}{dy} \right)^2} dy = m \int_{y^L}^1 \sqrt{1 + \left(\frac{d\tilde{g}_A^L(y)}{dy} \right)^2} dy, \quad (3)$$

$$m \int_c^{x^R} \sqrt{1 + \left(\frac{d\tilde{f}_A^R(x)}{dx} \right)^2} dx = n \int_{x^R}^d \sqrt{1 + \left(\frac{d\tilde{f}_A^R(x)}{dx} \right)^2} dx, \quad (4)$$

$$m \int_1^{y^R} \sqrt{1 + \left(\frac{d\tilde{g}_A^R(y)}{dy} \right)^2} dy = n \int_{y^R}^0 \sqrt{1 + \left(\frac{d\tilde{g}_A^R(y)}{dy} \right)^2} dy, \quad (5)$$

where $\tilde{g}_A^L(y)$ and $\tilde{g}_A^R(y)$ are the inverse of the left and the right membership functions $\tilde{f}_A^L(x)$ and $\tilde{f}_A^R(x)$, respectively. The geometric mean of the two points $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ on the left and the right membership functions, respectively of the fuzzy number $\tilde{A} = (a, b, c, d)$, is denoted by $G(\tilde{A})$ and defined as:

$$G(\tilde{A}) = \sqrt{x^L \cdot x^R} \quad (6)$$

2.3 ORDERING ALGORITHM OF FUZZY NUMBERS

Using the geometric mean value of the horizontal points from Eq.(6), the ordering of the two fuzzy numbers $\tilde{A}_i = (a_i, b_i, c_i, d_i)$, and $\tilde{A}_j = (a_j, b_j, c_j, d_j)$, ; $i, j = 1, 2, \dots, n$, is specified as follows:

If $G(\tilde{A}_i) > G(\tilde{A}_j)$, then $\tilde{A}_i \succ \tilde{A}_j$,

If $G(\tilde{A}_i) < G(\tilde{A}_j)$, then $\tilde{A}_i \prec \tilde{A}_j$, (7)

If $G(\tilde{A}_i) = G(\tilde{A}_j)$, then $\tilde{A}_i \sim \tilde{A}_j$.

2.4 TRAPEZOIDAL FUZZY NUMBERS

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number, if its membership function $\tilde{f}_A(x)$ is given by

$$\tilde{f}_A(x) = \begin{cases} \tilde{f}_A^L(x) = \frac{x-a}{b-a}, & x \in [a, b], \\ 1, & x \in [b, c], \\ \tilde{f}_A^R(x) = \frac{x-d}{c-d}, & x \in [c, d], \\ 0, & \text{Otherwise.} \end{cases} \quad (8)$$

For convenience, the trapezoidal fuzzy number is also denoted similarly as $\tilde{A} = (a, b, c, d)$. In the case of trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, the points $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ on the respective left and right membership functions which divide them in the same ratio $m:n$ are obtained as follows:

$$x^L = \frac{mb + na}{m + n}, \quad y^L = \frac{m}{m + n}, \quad (9)$$

$$x^R = \frac{mc + nd}{m + n}, \quad y^R = \frac{m}{m + n}. \quad (10)$$

Hence, from Eq.(6), the mean value $G(\tilde{A})$ for the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is given by

$$G(\tilde{A}) = \sqrt{\left(\frac{mb + na}{m + n} \times \frac{mc + nd}{m + n} \right)}. \quad (11)$$

2.5 ARITHMETIC OPERATIONS OF FUZZY NUMBER

The arithmetic operations of any two fuzzy numbers $\tilde{A}_i = (a_i, b_i, c_i, d_i)$, and $\tilde{A}_j = (a_j, b_j, c_j, d_j)$, are defined as follows:

Addition: $\tilde{A}_i \oplus \tilde{A}_j = (a_i, b_i, c_i, d_i) \oplus (a_j, b_j, c_j, d_j)$
 $= (a_i + a_j, b_i + b_j, c_i + c_j, d_i + d_j)$.

Subtraction: $\tilde{A}_i \ominus \tilde{A}_j = (a_i, b_i, c_i, d_i) \ominus (a_j, b_j, c_j, d_j)$
 $= (a_i - d_j, b_i - c_j, c_i - b_j, d_i - a_j)$.

Multiplication: $\tilde{A}_i \otimes \tilde{A}_j = (a_i, b_i, c_i, d_i) \otimes (a_j, b_j, c_j, d_j)$

$$= (a_i \times a_j, b_i \times b_j, c_i \times c_j, d_i \times d_j).$$

$$\text{Division: } \tilde{A} \% \tilde{A}_j = (a_i, b_i, c_i, d_i) \% (a_j, b_j, c_j, d_j)$$

$$= \left(\frac{a_i}{d_j}, \frac{b_i}{c_j}, \frac{c_i}{b_j}, \frac{d_i}{a_j} \right).$$

$$\text{Multiplication by a scalar } k : \tilde{A} = \begin{cases} (ka_i, kb_i, kc_i, kd_i), & k \geq 0, \\ (kd_i, kc_i, kb_i, ka_i), & k < 0. \end{cases}$$

3. FUZZY SEQUENCING PROBLEM $n \geq 1$ JOBS AND TWO MACHINES

Considering $n \geq 1$ jobs ($J_1, J_2, \dots, J_n$), each of which must be processed one at a time on each of the two machines (M_1, M_2). Each job must be processed once on each machine. The order of processing each job on machines (M_1, M_2) is known, and the time required to process each job on each machine is also given. The problem is to find the appropriate sequence among $(n!)^2$ number of possible sequences for processing the jobs so that the total elapsed time for all the jobs must be minimal. The processing time T_{ij} , $i=1, 2, \dots, n$ of each job J_i on the machines $M_{jj}=1, 2$ are represented by trapezoidal fuzzy numbers. Thus, the problem can be addressed in tabular form in Table.1 as follows:

Table.1. Problem

Job	Machine	
	M_1	M_2
J_1	$\tilde{T}_{11}$	$\tilde{T}_{12}$
J_2	$\tilde{T}_{21}$	$\tilde{T}_{22}$
...	...	...
J_n	$\tilde{T}_{n1}$	$\tilde{T}_{n2}$

where $\tilde{T}_{ij}$ is a trapezoidal fuzzy number that represents the processing time of the job J_i on the machine M_j ($j=1, 2$).

3.1 JOHNSON'S ALGORITHM

Johnson's rule in sequencing problems is as follows:

Step 1: Find the job of shortest processing time on the Machine M_1 and Machine M_2 .

Step 2: a) If the shortest value is on the Machine M_1 process that job first.

b) If the shortest value is on the Machine M_2 process that job last.

Step 3: a) If the shortest times for both the Machines M_1 and M_2 are equal, then perform the job on the Machine M_1 first and then on the Machine M_2 .

b) If there is a tie in shortest time among two or more jobs on the Machine M_1 then, select the job corresponding to the shortest time on the Machine M_2 and process it first.

c) If there is a tie in shortest time among two or more jobs on the Machine M_2 then, select the job corresponding to the shortest time on the Machine 1 and process it last.

Step 4: Find the total elapsed time (the time in between starting the first job and completing the last job) and the idle times of the machines M_1 and M_2 (the time during which the machine remains idle during the total elapsed time).

4. NUMERICAL ILLUSTRATION

This section comprises the numerical illustrations of the suggested method and some comparative reviews using examples taken from the literature that are common for comparative studies. Conveniently, for the geometric mean value computations, the ratio $m:n$ is taken as 2:3 throughout the numerical studies.

4.1 EXAMPLE

Consider five jobs J_1, J_2, J_3, J_4 , and J_5 which must go through two machines M_1 and M_2 . The fuzzy processing times for all the jobs on the two machines are given in Table.2.

Table.2. Fuzzy processing times

Job	Machine	
	M_1	M_2
J_1	(4, 4.5, 5.5, 6)	(1, 1.5, 2.5, 3)
J_2	(0, 0.5, 1.5, 2)	(5, 5.5, 6.5, 7)
J_3	(8, 8.5, 9.5, 10)	(6, 6.5, 7.5, 8)
J_4	(2, 2.5, 3.5, 4)	(7, 7.5, 8.5, 9)
J_5	(9, 9.5, 10.5, 11)	(3, 3.5, 4.5, 5)

Using Eq.(11), the geometric mean of the horizontal points on the membership functions of trapezoidal fuzzy numbers representing processing times of the jobs are displayed in Table.3.

Table.3. Processing times of the jobs

Job	Machine	
	M_1	M_2
J_1	4.936	1.833
J_2	0.6	5.946
J_3	8.964	6.954
J_4	2.891	7.96
J_5	9.968	3.919

The optimal sequence of jobs is obtained in Table.4 by using Johnson's algorithm given in subsection 3.1.

Table.4. Johnson's algorithm

J_2	J_4	J_3	J_5	J_1
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The elapsed time for the jobs is obtained as in Table.5.

Table.5. Elapsed time

Jobs Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
J_2	00	0.6	0.6	6.546
J_3	0.6	2.951	5.546	14.506

J_4	2.951	11.915	14.506	21.460
J_5	11.915	21.883	21.460	25.379
J_1	21.883	26.819	26.819	28.652

Hence, the elapsed time for processing the jobs = 28.652 hrs. Idle time of the machine M_1 = 1.833 hrs and idle time of the machine M_2 = 2.04 hrs.

Table.6. Comparative results

	Robust Ranking Method [2023]	Suggested ranking method
Total elapsed Time	30 hrs.	28.652 hrs.
Idle time for machine M_1	2 hrs.	1.833 hrs.
Idle time for machine M_2	3 hrs.	2.04 hrs.

Comparative results shows that the Elapsed time as well as Idle time for machine M_1 and M_2 are much less in compared to Robust Ranking method.

4.2 EXAMPLE

Consider the fuzzy sequencing problem with nine jobs $J_1, J_2, J_3, J_4, J_5, J_6, J_7, J_8$ and J_9 which must go through two machines M_1 and M_2 . The fuzzy processing times for all the jobs on the two machines are given in Table.7.

Table.7. Fuzzy processing times for all the jobs on the two machines

Job	M_1	M_2
J_1	(0, 2, 3, 4)	(11, 12, 13, 14)
J_2	(10, 12, 14, 16)	(16, 18, 20, 23)
J_3	(8, 10, 12, 14)	(14, 17, 20, 23)
J_4	(18, 20, 22, 25)	(8, 10, 12, 14)
J_5	(11, 12, 13, 14)	(6, 8, 10, 12)
J_6	(16, 18, 20, 23)	(18, 20, 22, 25)
J_7	(14, 17, 20, 23)	(6, 8, 10, 12)
J_8	(10, 12, 14, 16)	(18, 20, 22, 25)
J_9	(8, 10, 12, 14)	(22, 25, 28, 30)

Solution: Using Eq.(11), the geometric mean of the horizontal points on the membership functions of trapezoidal fuzzy numbers representing processing times of the jobs are obtained and displayed in Table.8.

Table.8. Processing times of the Jobs

Job	M_1	M_2
J_1	1.697	12.452
J_2	12.812	19.137
J_3	10.778	18.203
J_4	21.153	10.778
J_5	12.452	8.727
J_6	19.137	21.153
J_7	18.203	8.727
J_8	12.812	21.153

J_9	10.778	26.028
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The optimal sequence of jobs is obtained in Table.9 by using Johnson's algorithm given in subsection 3.1.

Table.9. Optimal sequence of jobs

J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
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The elapsed time for the jobs is obtained as in Table.10.

Table.10. Elapsed time for the jobs

Job	M_1		M_2	
J_1	00	1.697	1.697	14.149
J_3	1.697	13.2	14.149	32.352
J_9	13.2	24.2	32.352	58.380
J_2	24.2	37.2	58.380	77.517
J_8	37.2	50.2	77.517	98.670
J_6	50.2	69.5	98.670	119.823
J_4	69.5	90.8	119.823	130.601
J_7	90.8	109.3	130.601	139.328
J_5	109.3	121.8	139.328	148.055

Hence, the elapsed time for processing the jobs = 148.055 hrs. Idle time of the machine M_1 = 26.255 hrs and idle time of the machine M_2 = 1.697 hrs.

Table.11. Comparative results

	Sundari (2019) method	Suggested ranking method
Total elapsed Time	147.25 hrs.	148.055 hrs.
Idle time for machine M_1	27.5 hrs.	26.255 hrs.
Idle time for machine M_2	2.25 hrs.	1.697 hrs.

Comparative results shows that the Idle time for machine M_1 and M_2 are much less in compared to Sundari (2019).

5. CONCLUSIONS

Ranking of fuzzy numbers is hindered by inconsistency of the methods and counter intuitiveness of the results. This paper defines the geometric mean value for the fuzzy number as a ranking tool to obtain the optimal sequence of the sequencing problems. The comparative study in Ex. 4.1 and Ex. 4.2 acknowledges that the suggested method is noticeable and advantageous over others.

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