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# PREDICTION OF COMPACTION PARAMETERS OF SOIL USING GA AND PSO OPTIMIZED RELEVANCE VECTOR MACHINE (RVM)

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***Abstract***

***The present research introduces the best architectural relevance vector machine (RVM) model for predicting the compaction parameters of soil. The two types of RVM models, i.e., single kernel function-based (SRVM) and dual kernels (parallel) function-based (DRVM), have been constructed in this study. However, the RVM is a kernel function-based approach. Therefore, linear, gaussian, laplacian, and polynomial kernel functions have been implemented in these models. Each model has been optimized by each Genetic algorithm (GA) and particle swarm optimization (PSO) algorithm. For this purpose, 59 soil samples have been collected from the literature. The root mean square error (RMSE), mean absolute error (MAE), and correlation coefficient (R) statistical tools have been used to measure the performance and accuracy of models. From the overall analysis, models MC10 and MD12 have predicted OMC (RMSE = 0.8194%, R = 0.9956, MAE = 0.7920%) and MDD (RMSE = 0.1310g/cc, R = 0.9941, MAE =0.0008g/cc) better than other RVM models. It has also been observed that the DRVM model predicts the compaction parameters better than the SRVM models. The GA algorithm is robust in predicting OMC prediction, and the PSO algorithm is robust in MDD prediction. The score analysis also confirms the robustness of the dual kernel function based DRVM models for predicting OMC and MDD of soil. The sensitivity analysis demonstrates that compaction parameter prediction is strongly influenced by the specific gravity, liquid limit, and plasticity index of soil.***

***Keywords:***

***Compaction Parameters, Hybrid Approach, Genetic Algorithm, Particle Swarm Optimization Algorithm, Relevance Vector Machine***

## 1. INTRODUCTION

The available soil on the earth has been classified into cohesive and non-cohesive soil based on their behaviour. Each soil has different engineering or geotechnical properties, representing the behaviour of the soil. These properties can be determined by in-situ and laboratory procedures. However, both procedures are time-consuming and arduous. The consistency limits, compaction parameters, strength parameters, and permeability are the geotechnical properties of soil. The laboratory procedures for determining these properties are time-consuming, requiring human resources and proper equipment maintenance, which is costly.

Moreover, the compaction parameters, such as optimum moisture content (OMC) and maximum dry density (MDD), are the principal parameter for determining permeability and strength properties, showing the necessity of the compaction parameters. Various types of equipment are available for determining the compaction parameters of soil. Still, the standard and modified proctor tests are famous among geotechnical engineers/ designers. Both proctor tests require several attempts to draw an inverted "V" shaped curve to compute the OMC and MDD of soil. Due to lengthy, arduous, and time-consuming procedures, it has been decided to introduce different techniques to compute the OMC and MDD of soil. Numerous researchers have used the statistical approach, i.e., regression analysis, to compute compaction parameters and map the relationship between the soil parameters. It has been observed that regression analysis computes the compaction parameters with considerable prediction error. As we know, modern problems require advanced solutions. Thus, several researchers and scientists have employed different computational approaches associated with artificial intelligence.

V Hohn et al. [1] have introduced empirical models to predict the OMC and MDD of soil using 169 soil samples. The authors have used LL, PL, G, S, FC, and $\gamma_s$ as input parameters for computing the compaction parameters of soil. The authors have concluded that proposed empirical models have predicted OMC and MDD with a correlation of 0.872 and 0.873, which is comparatively higher than the published empirical models of Sridharan and Nagaraj (2005), Nagaraj et al. (2015), Noor et al. (2011), Gunaydin (2009), and Sivrikaya (2008). Pentoś et al. [2] have mapped a comparative study between MLR and machine learning approaches in predicting soil compaction and shear stress using electrical parameters. The authors have reported that developed regression models have attained a lower 50% (R=0.5) accuracy in the prediction. Still, the machine learning approach, namely artificial neural network, has attained over 75% (R=0.75) accuracy, comparatively higher than regression models. Yousif and Mohamed [3] have predicted compaction parameters using soil index properties of 311 soil samples of sandy and sandy-silty soils. As a result, it has been observed that nonlinear regression models predict OMC and MDD of soil better than linear regression models. Verma and Kumar [4] have employed MLP_NN models to predict the modified compaction parameters of coarse and fine-grained soil. The authors have trained and tested MLP_NN models in the reported study using 179 and 69 databases, respectively. The authors have concluded that the developed NN models have computed MDD of coarse and fine-grained soil with ±4% and ±2% variation. In contrast, the models have predicted OMC within ±8% variations. Othman [5] has used gradational parameters and consistency limits to estimate the compaction parameters of soil. For this purpose, 240 artificial neural network models have been developed, trained, tested, and analyzed to find the optimum ANN model for computing each OMC and MDD of soil. The authors have mapped the following conclusions: (i) the tanh activation function is better than sigmoid and Relu functions; (ii) the performance of the ANN model deteriorates with the increasing number of hidden layers and neurons; (iii) the optimum performance ANN models have predicted OMC and MDD of soil with COD of 0.903 and 0.928, respectively. Othman and Abdelwahab [6] have estimated compaction parameters using the ANN approach. The authors have concluded that 3HL interconnected two neurons-based ANN models have estimated MDD and OMC of soil with COD of 0.864 and 0.924, respectively. Jalal et al. [7] have employed GEP and

MEP approaches for predicting compaction characteristics. For this purpose, the authors have used C, PL, PI, and SG parameters as input parameters. The authors have reported that the GEP approach gives the most promising results of compaction parameters of soil. Benbouras and Lefilef [8] have reported that the random forest approach predicts the compaction parameters with high performance and accuracy.

Wang and Yin [9] have constructed models based on the MEP approach using G, S, FC, LL, PL, and CE parameters to predict the OMC and MDD of soil. The authors have found that the proposed models have predicted the compaction parameters of soil with COD of more than 0.85. Özbeyaz, A., and Söylemez [10] have compared the efficiencies of MLS_SVR, KB_SVR, and DT regression approach in computing compaction parameters of soil. Also, the authors have computed OMC and MDD individually and combined. The performance analysis demonstrates that polynomial and gaussian kernel function-based SVR models (developed for combined property prediction) have attained higher performance than models developed for single property prediction. Kurnaz and Kaya [11] have conducted a study on the prediction of the compaction parameters of soil. In the published research, authors have developed ELM, BRNN, GMDH, and SVM models, and performance has been compared. The authors have reported that the ELM models have predicted OMC and MDD of soil with the performance of 0.9385 and 0.9521, respectively, comparatively higher than BRNN, GMDH, and SVM models. Ratnam and Prasad [12] have mapped the relationship between gradational parameters, consistency limits, and compaction parameters of soil. Hasnat et al. [13] have predicted OMC and MDD with R of 0.86 and 0.91, respectively, using the support vector regression approach. Bunyamin et al. [14] have computed compaction parameters of cement kiln dust stabilized cohesive soil using the ANN approach. The authors have observed that ANN models of OMC (10-5-1) and MDD (10-7-1) have predicted OMC and MDD of soil with R of 0.983 and 0.9884, respectively. Khalid and Rehman [15] have constructed empirical models for predicting compaction parameters of soil for standard and modified proctor tests. The authors have reported that MDD of standard and modified proctor tests have a correlation of only ±0.4% and OMC of standard and modified proctor tests have ±2.7% correlation. Karimpour-Fard et al. [16] have used the results of 728 soil samples to compute the compaction parameters of soil using ANN, and MLR approaches. The authors have found that ANN models have predicted compaction parameters of soil better than the MLR approach using gradational parameters and consistency limits. Ardakani and Kordnaeij [17] have employed GMDH neural network models and genetic algorithms to compute the compaction parameters of soil. For this purpose, the authors have used the LL, PL, FC, and SC of 212 soil samples to train and test the developed models. The authors have compared the performance (R=0.90 for MDD, R=0.92 for OMC) of GMDH neural network models with the published research. Finally, the authors have concluded that GMDH models are superior to published ones. Vinod and Sreelekshmy [18] have derived empirical equations for predicting compaction parameters of fine-grained soils. From the overall analysis, the authors have mapped the following conclusion (i) OMC decreases with an increase in compaction energy, (ii) maximum dry density has a good correlation with dry unit weight, and (iii) SG is the less influencing parameter for maximum dry density. Sreelekshmy and Vinod [19] have re-examined the compaction parameters of soil using computational methods.

Özgan et al. [20] have used SPSS software to develop compaction parameters models using particle size and compaction parameters. Furthermore, Jyothirmayi et al. [21] have mapped the relationship between compaction parameters and the plastic limit of soil. Gurtug and Sridharan [22] have estimated the compaction parameters of soil using compaction energies of soil tested by standard, reduced-modified, and modified proctor tests. The authors have reported that plastic limit strongly correlates with the standard proctor energy level OMC. Also, the OMC has an excellent correlation with the MDD of soil. Farooq et al. [23] have predicted compaction parameters of fine-grained soil using consistency limits. The published research has been carried out using empirical models. Finally, the authors have reported that the proposed empirical models have predicted MDD and OMC with confidence intervals of ±2.5% and ±9.5%, respectively. Ören [24] has estimated the compaction parameters using a sediment volume test. For this purpose, the author has derived linear equations and has concluded that the plastic limit is the best parameter for predicting the compaction parameters of clay soil.

Khuntia et al. [25] have employed models based on the MARS approach to predict the compaction parameters of coarse-grained soil. In addition, the authors have employed ANN, and LSSVM approaches. The performance comparison shows that the MARS models have predicted OMC and MDD with a variation of ±13% and ±4%, respectively. Moreover, Sivrikaya et al. [26] have predicted compaction parameters of coarse-grained soil using MLR and GEP. Al-saffar and Khattab [27] have constructed an artificial neural network to compute the OMC and MDD of soil. Based on the statistical analysis, it has been found that the ANN models have the capability to predict the OMC and MDD of soil. Mujtaba et al. [28] have mapped the statistical relationship between gradational and compaction parameters of sandy soils. As a result, the authors have reported that the proposed relationships have computed OMC and MDD of sandy soil with the confidence interval of ±3% and ±5%, respectively. Similarly, several researchers and investigators have developed and employed various approaches for computing the compaction parameters of coarse and fine-grained soils [29]-[33].

## 1.1 GAPS IN THE LITERATURE SURVEY

The literature survey demonstrates that numerous researchers have developed models based on statistical and artificial intelligence approaches. In the reported study, the relevance vector machine approach has not been employed for predicting the compaction parameters of soil. Also, no hybrid soft computing approach has been developed and employed in predicting the OMC and MDD of soil. Again, the impact of the optimization technique has not been studied yet in compaction parameters and for the relevance vector machine approach.

## 1.2 OBJECTIVES OF THE PRESENT RESEARCH

Considering the outlines given in the gaps in the literature, the present study constructs the models based on the relevance vector machine (RVM) approach. However, the relevance vector machine is an advanced approach to the support vector machine. Both approaches are based on kernel functions. Therefore, linear, gaussian, laplacian, and polynomial kernel functions have been implemented in the RVM models. Single (SRVM) and dual (DRVM) kernel-based relevance vector machine models have been developed using the kernel functions. In addition, to study the effect of the optimization technique, the developed SRVM and DRVM models have been optimized by genetic algorithm (GA) and particle swarm optimization (PSO) algorithm.

Research Significance – It is well-known that determining soil compaction parameters using laboratory procedures is time-consuming and arduous. For this purpose, the present research introduces a robust computational tool for predicting the compaction parameters of soil. Also, this research helps geotechnical engineers/ designers decide the best optimization technique and kernel function of the relevance vector machine models for OMC and MDD prediction.

## 2. RESEARCH METHODOLOGY

The present research has been conducted to predict the compaction parameters of soil and introduce the robust relevance vector machine model. The relevance vector machine is a kernel function-based approach. Using the linear, gaussian, laplacian, and polynomial kernel functions, four SRVM (single kernel-based RVM) models have been employed to predict each OMC and MDD of soil. Each developed SRVM model has been optimized by GA and PSO optimization techniques for predicting each OMC and MDD of soil. For this purpose, fifty-nine soil samples have been collected from the literature study. Forty-one and eighteen soil samples have been randomly picked up to create the training and testing databases to train and test the developed models. To measure the performance of the models, root mean square error (RMSE), mean absolute error (MAE), correlation coefficient (R), and coefficient of determination (R2) have been used.

Based on the performance comparison, one better performing SRVM model and kernel function have been identified. Thus, one GA-optimized SRVM model for OMC, one PSO-optimized SRVM model for OMC, one GA-optimized SRVM model for MDD, and one PSO-optimized SRVM model for MDD have been identified as better-performing models. The kernel function of the respective better-performing models has been used to develop various combinations with the rest of the kernel functions and to develop the DRVM (dual kernel function-based RVM) models. For example, the laplacian kernel has been recognized as a better-performing kernel function for the SRVM model. The kernel combinations are laplacian + linear, laplacian + gaussian, and laplacian + polynomial.

As a result, three DRVM models have been optimized by GA and PSO optimization techniques for predicting each OMC and MDD of soil. Based on the performance comparison, one GA-optimized DRVM model for OMC, one PSO-optimized DRVM model for OMC, one GA-optimized DRVM model for MDD, and one PSO-optimized DRVM model for MDD have been recognized as better-performing models.

Finally, four RVM models (GA-optimized SRVM, PSO-optimized SRVM, GA-optimized DRVM and PSO-optimized DRVM) for predicting each OMC and MDD have been identified as the better-performing models. Furthermore, the performance of the better-performing SRVM and DRVM models has been compared to identify the best architecture models for predicting each OMC and MDD of soil.

In addition, the score analysis has been performed to recognize the best architecture model. On the other hand, sensitivity analysis has been performed to determine the most influencing input parameter in predicting the compaction parameters of soil.

## 3. DATA ANALYSIS

In the present research, fifty-nine soil samples have been collected from the literature [34]-[37]. The database contains gravel content (G in %), sand content (S in %), silt content (M in %), clay (C in %), specific gravity (SG), liquid limit (LL in %), plasticity index (PI in %), optimum moisture content (OMC in %), and maximum dry density (MDD in g/cc) of fifty-nine soils samples. The descriptive statistics of the database are given in Table.1, demonstrating that the database consists of 0.0-74.0% gravel content, 0.0-100.0% sand content, 0.0-82.0% silt content, 0.0-87.0% clay content, 2.52-2.80 specific gravity, 25.57-54.18% liquid limit, 9.02-30.9% plasticity index, 7.61-24.72% optimum moisture content, and 1.53-2.01g/cc maximum dry density. In addition, the Pearson product-moment correlation coefficient method has mapped the relationship, as shown in Fig.1.

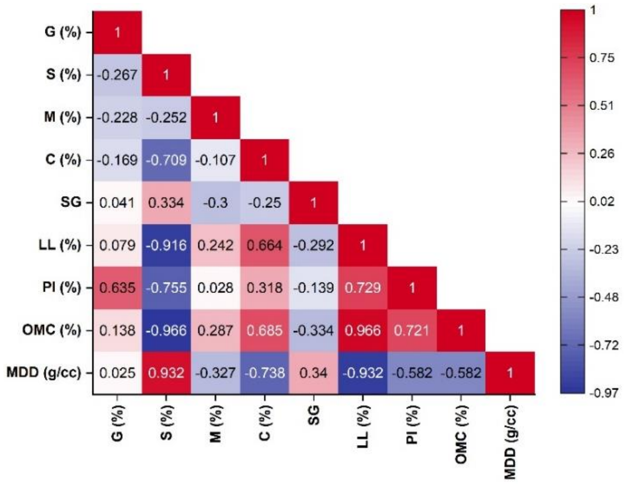



Fig.1. Relationship between input and output parameters

The Pearson product-moment correlation coefficient values ±0.81 to ±1.0, ±0.61 to ±0.80, ±0.41 to ±0.60, ±0.21 to ±0.40, and ±0.0 to ±0.20 show very strong, strong, moderate, weak and no relationship between the two variables [38]. Fig.1 illustrates that sand content (-0.966) and liquid limit (0.966) very strongly correlate with OMC. The clay content (0.685) and plasticity index (0.721) strongly correlate with OMC. The gravel content (0.138) has no relationship with the OMC of the soil. On the other hand, MDD is very strongly with sand content (0.932) and liquid limit (0.932). The clay content and plasticity index very strongly and moderately correlate with maximum dry density.

Table.1. Descriptive Statistics of fifty-nine soil samples

| Variables | Symbols | Units | Category | Min | Max | Mean | Median | StDev | CL | Kurtosis | Skewness |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gravel Content | G | % | Input | 0.00 | 74.00 | 12.07 | 1.74 | 17.81 | 4.64 | 2.00 | 1.62 |
| Sand Content | S | % | Input | 0.00 | 100.00 | 54.50 | 60.88 | 31.09 | 8.10 | -1.30 | -0.23 |
| Silt Content | M | % | Input | 0.00 | 82.00 | 10.66 | 7.00 | 15.52 | 4.04 | 13.19 | 3.56 |
| Clay Content | C | % | Input | 0.00 | 87.00 | 17.79 | 8.40 | 21.38 | 5.57 | 1.24 | 1.42 |
| Specific Gravity | SG | % | Input | 2.52 | 2.80 | 2.67 | 2.69 | 0.05 | 0.01 | 1.57 | -0.70 |
| Liquid Limit | LL | % | Input | 25.57 | 54.18 | 33.10 | 28.88 | 8.65 | 2.26 | 0.47 | 1.25 |
| Plasticity Index | PI | % | Input | 9.02 | 30.90 | 15.06 | 13.91 | 5.01 | 1.30 | 0.87 | 1.00 |
| Optimum Moisture Content | OMC | % | Output | 7.61 | 24.72 | 13.86 | 12.09 | 5.20 | 1.35 | -0.43 | 0.81 |
| Maximum Dry Density | MDD | g/cc | Output | 1.53 | 2.01 | 1.83 | 1.88 | 0.14 | 0.04 | -0.49 | -0.80 |

## 4. METHOD AND METHODOLOGY

This section discusses the relevance vector machine approach, genetic algorithm, and particle swarm optimization technique. Also, the model designations of the developed SRVM and DRVM models are given in this section.

### 4.1 RELEVANCE VECTOR MACHINE

The relevance vector machine (RVM) was introduced by Tipping. The RVM is a probabilistic extended linear model [39]. The RVM has an identical function to the SVM but provides primary classification. The RVM is equivalent to a Gaussian process approach model with a covariance function.

$$K(x, x') = \sum_{j=1}^{N} \frac{1}{\alpha_j} \varphi(x, x_j) \varphi(x', x_j) \quad (1)$$

where $\varphi$ is the kernel function (default Gaussian), $\alpha_j$ are the variance, $x_1$, $x_2$,..,$x_N$ are the input vector of the training datasets [40]. The flowchart of the relevance vector machine is shown in Fig.2.

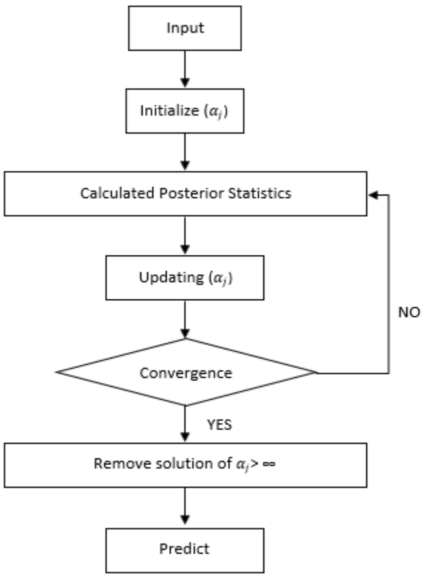



Fig.2. Flowchart of relevance vector machine

The relevance vector machine is based on kernel function, and kernel functions are mathematical algorithms. The mathematical formulation of Gaussian, Linear, Laplacian, and Polynomial, etc. kernels are:

Polynomial Kernel

$$K(x_i,x_j) = (x_i.x_j+1)^d \quad (2)$$

Gaussian Kernel

$$K(x,y) = \exp(-\|x-y\|^2/(2\sigma^2)) \quad (3)$$

Gaussian Radial Basis Function (RBF)

$$K(x_i,x_j) = \exp(-\gamma\|x_i,x_j\|^2) \quad (4)$$

Laplace RBF Kernel

$$K(x,y)=\exp(-\|x-y\|/\sigma) \quad (5)$$

Hyperbolic Tangent Kernel

$$K(x_i,x_j)=\tanh(kx_i.x_j+c) \quad (6)$$

Sigmoid Kernel

$$K(x,y)=\tanh(\alpha x^T y+c) \quad (7)$$

Numerous researchers have solved engineering issues using the relevance vector machine approach [41-47]. Based on the outcomes, Gaussian, Linear, Laplacian, and Polynomial kernel functions have been used to develop the SRVM and DRVM models to predict the compaction parameters of the soil in this study. The hyperparameters of the SRVM and DRVM are given in Table.2.

Table.2. Hyperparameters of the SRVM and DRVM models

| Hyperparameters | SRVM | DRVM |
|---|---|---|
| Free Basis | Enable | |
| Kernel Functions | Gaussian, Linear, Laplacian, Polynomial | |
| Max. Iterations | 1000 | |
| Kernels | Single | Two |
| Methods | GA & PSO | |
| Target | Single Kernel | Two Kernels |
| Ib | $2^{-6}$ | $2^{-5}$, $10^{-2}$, $10^{-3}$, $10^{-3}$ |
| uB | $2^{-6}$ | $2^{-5}$, $10^{0}$, $10^{3}$, $10^{3}$ |
| Num. Variable | 1 | 4 |
| Max. Iterations | 100 | |
| Kfolds | 5 | |

## 4.2 OPTIMIZATION TECHNIQUES

The genetic algorithm (GA) and particle swarm optimization (PSO) algorithm have been implemented with SRVM and DRVM models to predict each OMC and soil MDD. The GA and PSO optimization techniques are discussed as follows:

### *4.2.1 Genetic Algorithm (GA):*

The genetic algorithm is a metaheuristic that is inspired by the process of natural selection. It is a family member of a larger class of evolutionary algorithms. The genetic algorithm is used to evaluate the high-quality solution to optimization by relying on biologically inspired mutation, crossover, and selection operators [48]. The genetic algorithm enhances the performance of models based on the AI approaches [49]-[53].

### *4.2.2 Particle Swarm Optimization (PSO) Algorithm:*

Particle swarm optimization is a computational method to optimize problems and give high-quality solutions. The particles move around in the search space in particle swarm optimization based on the simple mathematical formula. Every particle moves due to its local best-known position, which is also guided by the most prominent positions in the search space. One of the bio-inspired algorithms, particle swarm optimization (PSO), is straightforward in its search for the best solution in the problem area. PSO algorithm enhances the performance and accuracy of AI models in solving geotechnical issues [54]-[60].

### *4.2.3 Model Designations:*

In this work, fourteen RVM models have been developed using MATLAB R2020a to predict each OMC and MDD of soil. The developed SRVM and DRVM models have been configured with hyperparameters, as mentioned in Table.2. The designation of the developed RVM models is given in Table.3.

Table.3. Hyperparameters of the SRVM and DRVM models

| RVM Type | Kernel Functions | OMC Models | MDD Models |
|---|---|---|---|
| GA – SRVM | Gaussian | MC1 | MD1 |
| | Linear | MC2 | MD2 |
| | Laplacian | MC3 | MD3 |
| | Polynomial | MC4 | MD4 |
| PSO – SRVM | Gaussian | MC5 | MD5 |
| | Linear | MC6 | MD6 |
| | Laplacian | MC7 | MD7 |
| | Polynomial | MC8 | MD8 |
| GA – DRVM | K1+K2 | MC9 | MD9 |
| | K1+K3 | MC10 | MD10 |
| | K1+K4 | MC11 | MD11 |
| GA – DRVM | K1+K2 | MC12 | MD12 |
| | K1+K3 | MC13 | MD13 |
| | K1+K4 | MC14 | MD14 |

*$K1+K_n$ are presenting combined kernel functions for DRVM

## 4.3 PERFORMANCE EVALUATION

The performance of the developed RVM models has been measured by using statistical tools, such as root mean square error (RMSE), mean absolute error (MAE), correlation coefficient (R), and coefficient of determination ($R^2$). The ideal values of the RMSE, MAE, $R$, and $R^2$ are 0, 0, 1, and 1, respectively. The best architectural model always has a performance of more than 0.8, demonstrating high prediction accuracy. In this study, the best architecture RVM model has been selected if the model attains over 95% ($R$=0.95) accuracy in predicting each OMC and MDD of soil. The mathematical formula of RMSE, MAE, R, and R2 is:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\alpha-\beta)^2} \tag{8}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|\omega-\alpha| \tag{9}$$

$$R^2 = \frac{\sum_{i=1}^{r}(\alpha-\beta)^2 - \sum_{i=1}^{r}(\alpha-\omega)^2}{\sum_{i=1}^{r}(\alpha-\beta)^2} \tag{10}$$

$$r = \frac{\sum(\alpha_i-\bar{\beta})(\omega_i-\bar{\omega})}{\sqrt{\sum(\alpha_i-\bar{\beta})^2(\omega_i-\bar{\omega})^2}} \tag{11}$$

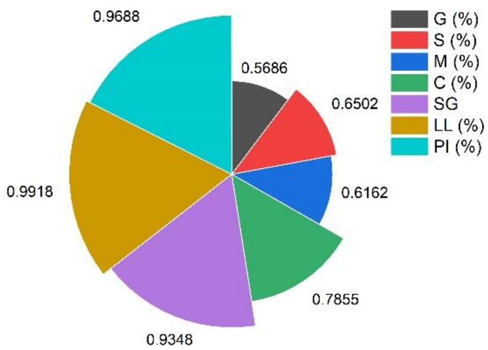
where $\alpha$ and $\omega$ are the actual and predicted $i^{th}$ value, $n$ presents the total number of data, $\beta$ is the mean of the actual values, $\bar{\omega}$ is the mean of the predicted value.

## 4.4 SENSITIVITY ANALYSIS

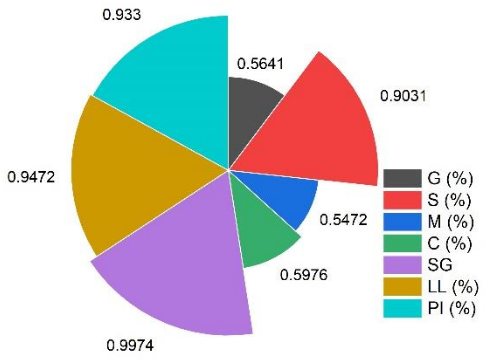
To understand the behaviour of the input soil variable on the predicted compaction parameters, the sensitivity analysis on the comprehensive databases has been conducted. For a particular input variable $x$, the sensitivity $SS$ can be calculated as [9],[17],[61]-[64]:

$$SS = \frac{\sum_{i=1}^{N}x_i y_i}{\sqrt{\sum_{i=1}^{N}x_i^2\sum_{i=1}^{N}y_i^2}} \tag{12}$$

where $y_i$ is the predicted output variable, and $N$ is the number of data points (in this study, $N$=59). The results of the analysis are graphically presented in Fig.3(a) and Fig.3(b).




(a)




(b)

Fig.3. Illustration of sensitivity analysis for (a) OMC and (b) MDD

The sensitivity SS varies from 0 to 1, showing the related strength between each input and predicted output variables. The value of SS close to 1 demonstrates the specific input variable is the most influencing variable in the prediction. Fig.3 (a) and (b) demonstrate that the prediction of compaction parameters is highly influenced by the specific gravity, liquid limit, and plasticity index of soil. Also, the maximum dry density is highly influenced by sand content. The other input parameters moderately influence the compaction parameter prediction.

## 5. RESULTS AND DISCUSSION

### 5.1 SIMULATION OF RVM MODELS

The present uses a relevance vector machine hybrid approach to predict the compaction parameters of the soil. For this purpose, fifty-nine soil samples have been collected from the literature. Forty-one and eighteen soil samples have been randomly picked up from 59 databases to create the training and testing databases. The SRVM and DRVM models have been trained and tested by the 41 and 18 soil samples. The details of the performance of the developed models have been discussed as follows:

#### *5.1.1 GA-SRVM Models:*

The gaussian, linear, laplacian and polynomial kernel functions have been implemented in the developed SRVM models to predict each OMC and MDD of soil. Each SRVM model has been optimized by the GA technique. The performance of GA-optimized SRVM models in predicting OMC has been calculated, as shown in Table.4.

Table.4. Performance of GA-SRVM models in predicting OMC

| Model ID | Kernel | Phase | RMSE | R | MAE |
|---|---|---|---|---|---|
| **MC1** | **Gaussian** | Train | 0.1802 | 0.9987 | 0.1359 |
| | | **Test** | **1.4680** | 0.9883 | **0.8550** |
| MC2 | Linear | Train | 0.4646 | 0.9912 | 0.3845 |
| | | Test | 1.9858 | 0.9686 | 0.9538 |
| MC3 | Laplacian | Train | 0.0209 | 1.0000 | 0.0142 |
| | | Test | 2.1742 | 0.9828 | 0.8643 |
| MC4 | Polynomial | Train | 0.4402 | 0.9921 | 0.3685 |
| | | Test | 1.5566 | 0.9749 | 0.9068 |

*Bold values correspond to the better-performing model

The Table.4 illustrates that model MC1 (Gaussian kernel-based) has attained over 98% accuracy (training = 0.9987, testing = 0.9883) in predicting the OMC of soil. The performance comparison shows that model MC1 has predicted the OMC of soil with the least prediction error, i.e., test RMSE = 1.4680% and test MAE = 0.8550%, comparatively less than other GA-SRVM models. Also, Table.4 presents that model MC2 (linear kernel-based) has gained the least accuracy in the testing phase than other GA-SRVM models. The Fig.4 shows the relationship between actual and predicted OMC using model MC1.

Furthermore, four models using gaussian, linear, laplacian, and polynomial kernel functions have been developed and optimized by the GA algorithm for predicting the MDD of soil. The performance achieved by the GA-SRVM models in predicting the MDD of soil is given in Table.5.

Table.5. Performance of GA-SRVM models in predicting MDD

| Model ID | Kernel | Phase | MAE | R | RMSE |
|---|---|---|---|---|---|
| MD1 | Gaussian | Train | 0.0137 | 0.9902 | 0.0108 |
| | | Test | 0.1738 | 0.9084 | 0.0045 |
| **MD2** | **Linear** | Train | 0.0233 | 0.9715 | 0.0185 |
| | | **Test** | **0.1549** | **0.9470** | **0.0024** |
| MD3 | Laplacian | Train | 0.0000 | 1.0000 | 0.0000 |
| | | Test | 0.2133 | 0.9410 | 0.0049 |
| MD4 | Polynomial | Train | 0.0266 | 0.9626 | 0.0204 |
| | | Test | 0.1984 | 0.8245 | 0.0082 |

*Bold values correspond to the better-performing model

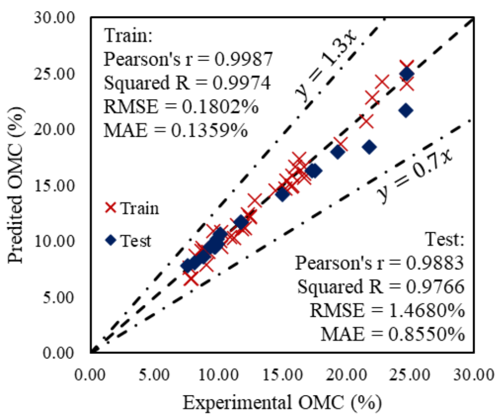



Fig.4. Actual vs predicted plot for OMC using model MC1

The Table.5 shows that model MD2 has achieved higher performance in the testing phase than other GA-SRVM models. Model MD2 has predicted MDD of eighteen soils with the RMSE of 0.1549g/cc, MAE of 0.0024g/cc, and R of 0.9470. Fig.5 presents the regression relationship between actual and predicted MDD of soil using model MD2.

The fitness curve of the models MC1 and MD2 has been calculated in the training phase and graphically presented in Figs. A and B, respectively (refer appendix).

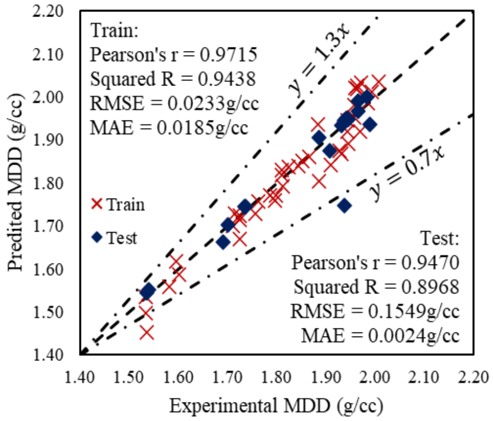



Fig.5. Actual vs predicted plot for MDD using model MD2

### *5.1.2 PSO-SRVM Models:*

The gaussian, linear, laplacian and polynomial kernel functions have developed SRVM models and optimized by PSO technique for predicting the OMC and MDD of soil. The performance of the PSO-SRVM model in predicting the OMC of soil has been calculated, as given in Table.6.

Table.6. Performance of PSO-SRVM models in predicting OMC

| Model ID | Kernel | Phase | RMSE | R | MAE |
|---|---|---|---|---|---|
| MC5 | Gaussian | Train | 0.4136 | 0.9930 | 0.2886 |
| | | Test | 1.9353 | 0.9685 | 0.8982 |
| MC6 | Linear | Train | 0.4646 | 0.9912 | 0.3845 |
| | | Test | 1.9858 | 0.9686 | 0.9538 |
| **MC7** | **Laplacian** | Train | 0.0057 | 1.0000 | 0.0035 |
| | | **Test** | **1.2461** | **0.9847** | **0.8060** |
| MC8 | Polynomial | Train | 0.4431 | 0.9920 | 0.3603 |
| | | Test | 1.8334 | 0.9707 | 0.9340 |

*Bold values correspond to the better-performing model

The Table.6 shows that model MC7 has achieved over 98% (R=0.9847) accuracy in the testing phase, comparatively higher than models MC5, MC6, and MC8. Model MC7 has computed the OMC of soil with the RMSE of 1.2461% and MAE of 0.8060%, comparatively better than other PSO-SRVM models. A relationship has been drawn between actual and predicted OMC using model MC7, as depicted in Fig.6.

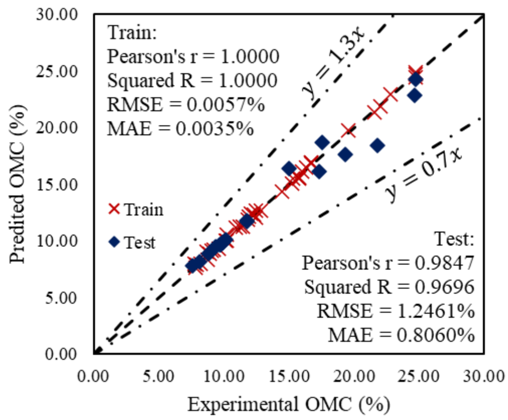



Fig.6. Actual vs predicted plot for OMC using model MC7

The Table.7 represents that the laplacian kernel-based PSO-optimized SRVM model has achieved 100% (R=1) and 95.45% (R=0.9545) accuracies in the training and testing phase, respectively. Model MD7 has computed maximum dry density with the RMSE of 0.1327g/cc and MAE of 0.0019g/cc, comparatively less than other PSO-optimized SRVM models. The Table.7 demonstrates that the polynomial kernel function-based model MD8 has attained the least performance in predicting the MDD of eighteen soil samples.

In addition, a statistical relationship has been plotted between actual and predicted MDD of soil using model MD7, as graphically presented in Fig.7. Also, the fitness curve has been plotted for model MD7 in the training phase, as shown in the appendix.

Table.7. Performance of PSO-SRVM models in predicting MDD

| Model ID | Kernel | Phase | RMSE | R | MAE |
|---|---|---|---|---|---|
| MD5 | Gaussian | Train | 0.0123 | 0.9921 | 0.0096 |
| | | Test | 0.1703 | 0.9254 | 0.0034 |
| MD6 | Linear | Train | 0.0233 | 0.9715 | 0.0185 |
| | | Test | 0.1549 | 0.9470 | 0.0024 |
| **MD7** | **Laplacian** | Train | 0.0009 | 1.0000 | 0.0007 |
| | | **Test** | **0.1327** | **0.9545** | **0.0019** |
| MD8 | Polynomial | Train | 0.1139 | 0.2791 | 0.0912 |
| | | Test | 0.3403 | 0.4215 | 0.0176 |

*Bold values correspond to the better-performing model

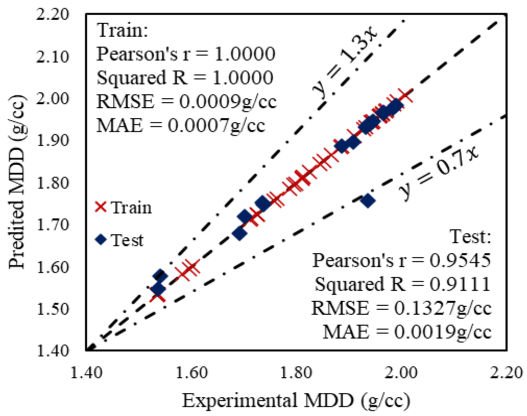



Fig.7. Actual vs predicted plot for MDD using model MD

GA-HRVM Models

In predicting the compaction parameters of soil, models MC1 (OMC model) and MD2 (MDD model) have been recognized as better-performing models because models have attained higher performance than other models SRVM models. Models MC1 and MD2 have been developed by gaussian and linear kernel functions. Therefore, models MC9, MC10, and MC11 have been constructed with dual kernel functions. The performance of models MC9, MC10, and MC11 is given in Table.8.

Table.8. Performance of GA-DRVM models in predicting OMC

| Model ID | Kernel | Phase | RMSE | R | MAE |
|---|---|---|---|---|---|
| MC9 | Gaussian + Linear | Train | 0.4231 | 0.9927 | 0.3625 |
| | | Test | 1.7753 | 0.9715 | 0.9361 |

| | | | | | |
|---|---|---|---|---|---|
| **MC10** | **Gaussian + Laplacian** | Train | 0.0008 | 1.0000 | 0.0004 |
| | | **Test** | **0.8194** | **0.9956** | **0.7920** |
| MC11 | Gaussian + Polynomial | Train | 0.6151 | 0.9845 | 0.5051 |
| | | Test | 1.6234 | 0.9747 | 0.9514 |

*Bold values correspond to the better-performing model

Table.8 illustrates that model MC10 has predicted the OMC of eighteen soil samples with the RMSE of 0.8194%, MAE of 0.7920, and R of 0.9956, comparatively less than models MC9 and MC11. It can be stated that the combination of gaussian and laplacian kernel functions predicts OMC better than other combinations for dual or parallel relevance vector machines. The regression plot between actual and predicted OMC using model MC10 has been drawn, as shown in Fig.8.

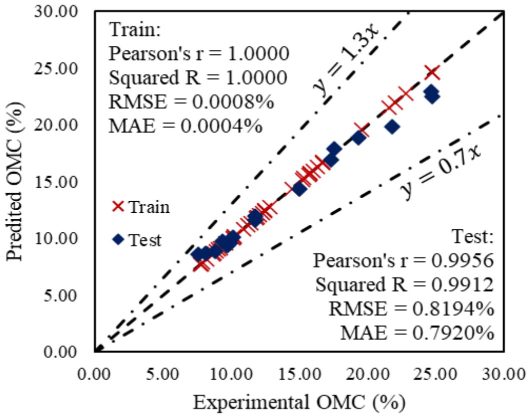



Fig.8. Actual vs predicted plot for OMC using model MC10

Similarly, the linear kernel function has been selected as the primary kernel function, and models MD9, MD10, and MD11 have been developed using different combinations of kernel functions. The performance of models MD9, MD10, and MD11 are shown in Table.9.

Table.9. Performance of GA-DRVM models in predicting MDD

| Model ID | Kernel | Phase | RMSE | R | MAE |
|---|---|---|---|---|---|
| **MD9** | **Linear + Gaussian** | Train | 0.0233 | 0.9715 | 0.0185 |
| | | **Test** | **0.1437** | **0.9675** | **0.0014** |
| MD10 | Linear + Laplacian | Train | 0.0233 | 0.9715 | 0.0185 |
| | | Test | 0.1549 | 0.9470 | 0.0024 |
| MD11 | Linear + Polynomial | Train | 0.0247 | 0.9680 | 0.0206 |
| | | Test | 0.1604 | 0.9551 | 0.0019 |

*Bold values correspond to the better-performing model

The Table.9 shows that model MD9 has predicted the maximum dry density of eighteen soil with higher performance in the testing phase. Model MD9 has attained over 96% (R=0.9675) accuracy in the testing phase, comparatively higher than other GA-optimized DRVM models. It can be observed that model MD9 has predicted MDD of soil with the least prediction error, i.e., RMSE = 0.1437g/cc and MAE = 0.0014g/cc. The Fig.9 depicts the relationship between actual and predicted MDD of soil using model MD9. It can be stated that the combination of linear and gaussian kernel functions predicts the MDD better than other combinations. The fitness curve of models MC11 and MD9 is plotted, as shown in the appendix.

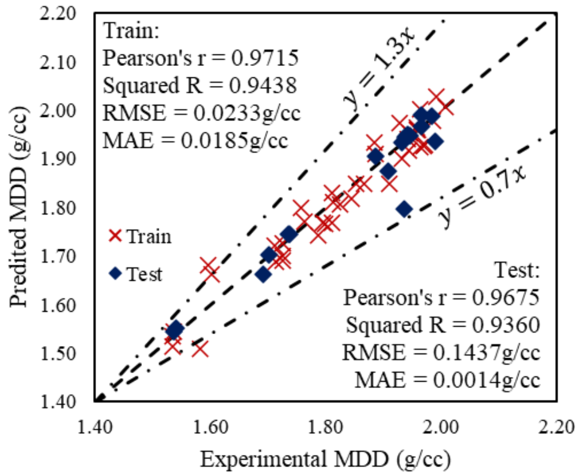



Fig.9. Actual vs predicted plot for MDD using model MD9

### *5.1.3 PSO-SRVM Models:*

In the prediction of compaction parameters of soil, laplacian kernel function-based models MC7 and MD7 have been identified as the better-performing models. Therefore, the laplacian kernel function has been used as the primary kernel function for developing dual kernel function-based DRVM models. Three models, MC12, MC13, and MC14, have been developed to predict the OMC of soil. The performance of the models is mentioned in Table.10.

Table.10. Performance of PSO-DRVM models in predicting OMC

| Model ID | Kernel | Phase | RMSE | R | MAE |
|---|---|---|---|---|---|
| **MC12** | **Laplacian + Gaussian** | Train | 0.0042 | 1.0000 | 0.0025 |
| | | Test | 2.0602 | 0.9877 | 0.8815 |
| MC13 | Laplacian + Linear | Train | 0.4616 | 0.9913 | 0.3801 |
| | | Test | 2.0112 | 0.9680 | 0.9616 |
| MC14 | Laplacian + Polynomial | Train | 0.5389 | 0.9881 | 0.4322 |
| | | Test | 1.5046 | 0.9759 | 0.9145 |

*Bold values correspond to the better-performing model

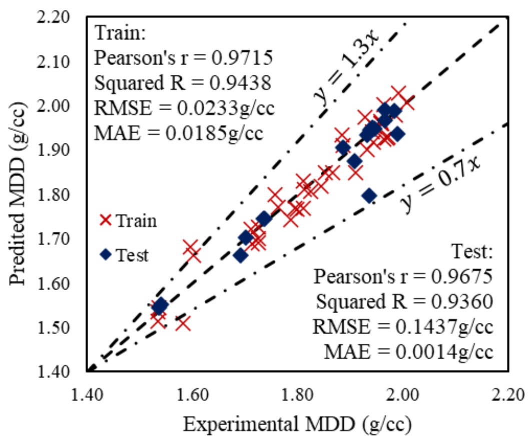



Fig.10. Actual vs predicted plot for OMC using model MC12

The Table.10 demonstrates that model MC12 has attained over 98% (R=0.9877) accuracy in the testing phase, comparatively better than models MC13 and MC14. Model MC12 has predicted the OMC of eighteen soil samples with the RMSE of 2.0602% and MAE of 0.8815% and has been recognized as a better-performing model in predicting the OMC of soil. The relationship between actual and predicted OMC using model MC12 has been plotted, as shown in Fig.10.

Similarly, the laplacian kernel function has been used as a primary kernel function for developing DRVM models for predicting MDD of soil. Three models, MD12, MD13, and MD14, have been constructed to predict soil MD. The performance of models MD12, MD13, and MD14 is given in Table.11.

Table.11. Performance of PSO-DRVM models in predicting MDD

| Model ID | Kernel | Phase | RMSE | R | MAE |
|---|---|---|---|---|---|
| **MD12** | **Laplacian + Gaussian** | Train | 0.0007 | 1.0000 | 0.0005 |
| | | Test | 0.1310 | 0.9941 | 0.0008 |
| MD13 | Laplacian + Linear | Train | 0.0058 | 0.9982 | 0.0042 |
| | | Test | 0.1387 | 0.9499 | 0.0022 |
| MD14 | Laplacian + Polynomial | Train | 0.0079 | 0.9968 | 0.0065 |
| | | Test | 0.1377 | 0.9383 | 0.0029 |

*Bold values correspond to the better-performing model

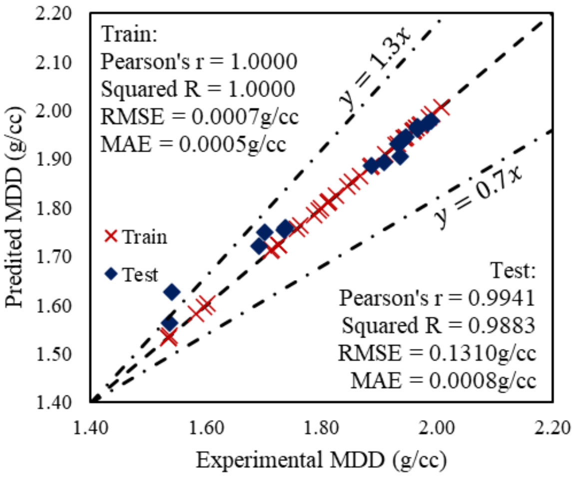



Fig.11. Actual vs predicted plot for MDD using model MD12

The Table.11 shows that model MD12 has predicted MDD of soil with RMSE of 0.1310g/cc, MAE of 0.0008g/cc, and R of 0.9941. The performance comparison illustrates that model MD12 has attained higher performance in predicting the MDD of eighteen soils. The regression relationship between actual and predicted MDD using model MD12 has been mapped, as shown in Fig.11.

Based on the performance, model MD12 has been identified as the better-performing model in predicting MDD of soil. The fitting curve of models MC12 and MD12 has been drawn, as graphically presented in the appendix.

## 5.2 SCORE ANALYSIS

The score analysis is a method for comparing the performance of soft computing models to identify the best architecture model. In this analysis, a score of 'k' (in this work, k =4), i.e., the better-performing soft computing models) is assigned to the models which have achieved the higher value for each performance parameter. The higher value shows the best model and the lower value, i.e., shows the worse model. In the subsequent steps, the value of performance parameters is summed up for each training and testing phase. In the next step, the overall score is calculated by adding the score of each training and testing phase. Tables 12 and 13 show the details of the score analysis for the better-performing models of OMC and MDD, respectively.

Table.12. Score analysis for better-performing models of OMC

| Model ID | Phase | RMSE | R | MAE | Total | Overall Total |
|---|---|---|---|---|---|---|
| MC1 | Train | 1 | 1 | 1 | 3 | 10 |
| | Test | 2 | 3 | 2 | 7 | |
| MC7 | Train | 2 | 2 | 2 | 6 | 13 |
| | Test | 3 | 1 | 3 | 7 | |
| **MC10** | **Train** | **4** | **2** | **4** | **10** | **22** |
| | **Test** | **4** | **4** | **4** | **12** | |
| MC12 | Train | 3 | 2 | 3 | 8 | 12 |
| | Test | 1 | 2 | 1 | 4 | |

*Bold values correspond to the best architectural model

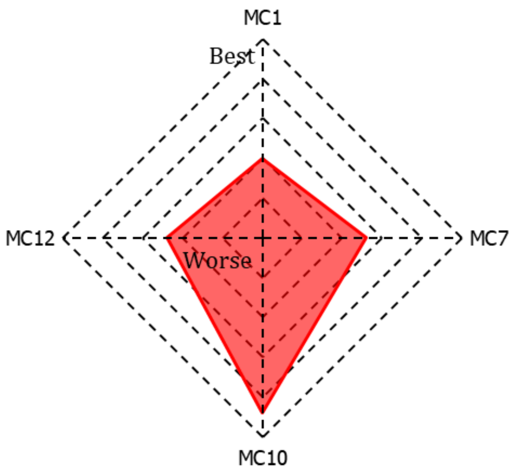



Fig.12 Score analysis for better-performing models of OMC

Table.12 shows that model MC10 has attained a higher score in predicting the OMC of soil. Model MC10 has 10 and 12 scores in the training and testing phase, respectively. Finally, model MC10 has attained the highest overall score, i.e., 22. Based on the score analysis, MC10 has been recognized as the best architecture model. The comparison of the overall score of the better-performing models of OMC has been graphically presented in Fig.12.

Similarly, the score analysis has been performed for the better-performing models of MDD. The results of the score analysis is given in Table.13.

Table.13. Score analysis for better-performing models of MDD

| Model ID | Phase | RMSE | R | MAE | Total | G.Total |
|---|---|---|---|---|---|---|
| MD2 | Train | 1 | 1 | 1 | 3 | 6 |
| | Test | 1 | 1 | 1 | 3 | |
| MD7 | Train | 3 | 3 | 3 | 9 | 16 |
| | Test | 3 | 2 | 2 | 7 | |
| MD9 | Train | 1 | 1 | 1 | 3 | 11 |
| | Test | 2 | 3 | 3 | 8 | |
| **MD12** | **Train** | **4** | **3** | **4** | **11** | **23** |
| | **Test** | **4** | **4** | **4** | **12** | |

*Bold values correspond to the best architectural model

The Table.13 demonstrates that model MD12 has obtained 11 and 12 scores in the training and testing phase, respectively. Model MD12 has achieved a 23 overall score in predicting the MDD of soil. Therefore, model MD12 has been recognized as the best architecture model for predicting the maximum dry density of soil. The overall score analysis of models MD2, MD7, MD9, and MD12 has been graphically presented in Fig.13.

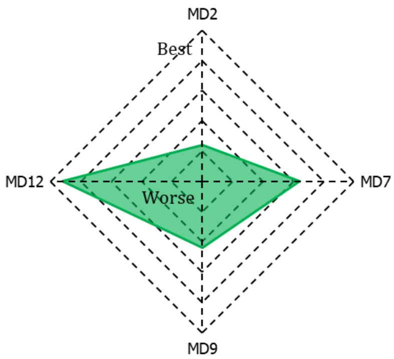



Fig.13. Score analysis for better-performing models of MDD

## 5.3 DISCUSSION OF RESULTS

This research introduces a robust relevance vector machine model to predict the compaction parameters of soil. For this purpose, 59 soil samples have been collected from the literature. Four SRVM models have been developed for predicting each OMC and MDD of soil, which each GA and PSO technique have optimized. Select the better-performing algorithm (from the model), the dual (parallel) kernel functions have been implemented, and the developed models have predicted the compaction parameters of soil. The RMSE, MAE, R, and R2 statistical methods have measured the performance and accuracy of the developed models.

The performance comparison demonstrates that the gaussian kernel-based GA-optimized SRVM model has predicted the OMC of soil with an accuracy of 0.9883 and is recognized as a better-performing model. On the other side, the laplacian kernel-based PSO-optimized SRVM model has predicted OMC with an accuracy of 0.9847 and is identified as the better-performing model. The GA-optimized SRVM model attains higher accuracy than the PSO-optimized SRVM model in predicting the OMC of soil. In the case of dual kernel-based RVM models, the combination of the gaussian and laplacian kernels-based DRVM model has gained an accuracy of 0.9956 in predicting the OMC of soil, which has found more than the GA-optimized SRVM model. Similarly, in the dual kernel-based DRVM model, the laplacian and gaussian kernel-based DRVM model has gained 0.9877 accuracy, which is comparatively higher than the PSO-optimized SRVM model.

In the case of MDD prediction, the GA-optimized gaussian kernel-based SRVM model has attained an accuracy of 0.9470. Still, the PSO-optimized laplacian kernel-based SRVM model has achieved an accuracy of 0.9545. The performance of both SRVM models (GA and PSO-optimized) has been increased by implementing the second kernel function in predicting the MDD of soil. The GA-optimized DRVM (developed by gaussian + laplacian kernel) and PSO-optimized DRVM (created by laplacian + gaussian kernel) models have gained 0.9675 and 0.9941 accuracies, respectively.

Finally, the GA-optimized DRVM model MC10 and PSO-optimized DRVM model MD12 have been recognized as the best architecture model in predicting OMC and MDD of soil, respectively.

## 6. VALIDATION WITH LITERATURE

A comparison between the best architecture models (MC10 and MD12) and available models in the literature survey has been mapped to validate the performance and accuracy of the best architecture models in the testing phase. The comparison of test performance is given in Table.14.

Table.14. Comparison of the best architecture models and published models available in the literature

| Reference | Approach | R Test | |
|---|---|---|---|
| | | OMC | MDD |
| V Hohn et al. (2022) | Empirical models | 0.8724 | 0.8735 |
| Yousif et al. (2022) | Empirical models | 0.9281 | 0.9850 |
| Othman (2021) | ANN | 0.9503 | 0.9633 |
| Jalal et al. (2021) | GEP | 0.8580 | 0.7770 |
| Wang & Yin (2021) | MEP | 0.9607 | 0.9263 |
| Özbeyaz et al. (2020) | KB_SVR | 0.9300 | 0.9300 |
| Kurnaz et al. (2020) | ANN | 0.9191 | 0.9219 |
| **Present Study** | **RVM** | **0.9956** | **0.9941** |

The Table.14 demonstrates that the developed models based on the RVM approach have outperformed the models available in the literature study while predicting the compaction parameters of soil. Hence, the RVM approach can be used to predict the compaction parameters of soil because it predicts the compaction parameters with high accuracy and the least prediction error.

## 7. CONCLUSIONS AND FUTURE SCOPE

In this research, twenty-eight RVM models have been developed, trained, tested, and analyzed in predicting the compaction parameters of soil. The following conclusions are mapped:

- GA-optimized models (SRVM and DRVM) predict the optimum moisture content of soil better than PSO-optimized models (SRVM and DRVM)
- PSO-optimized models (SRVM and DRVM) predict the maximum dry density of soil better than the GA-optimized model (SRVM and DRVM).
- The performance of the relevance vector machine can be enhanced by implementing two kernel functions together. Also, the dual kernel based RVM model gives the most optimistic prediction of the compaction parameters of soil.
- The sensitivity analysis presents that the specific gravity, liquid limit, and plasticity index of soil are the most influencing parameters in predicting the compaction parameters of soil. Also, the sand content strongly influences the prediction of the maximum dry density of soil.

The performance of RVM models demonstrates the prediction ability. It appears that the RVM approach can be used to solve other geotechnical problems. Also, the present work may be extended by implementing the various metaheuristic algorithms to enhance the performance of the RVM model and map a comparative study between them.

## APPENDIX

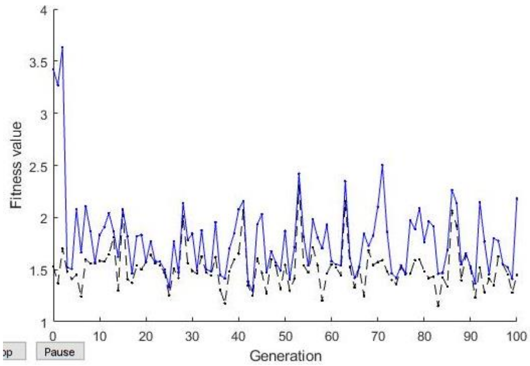



Fig.14. Fitness curve of model MC1

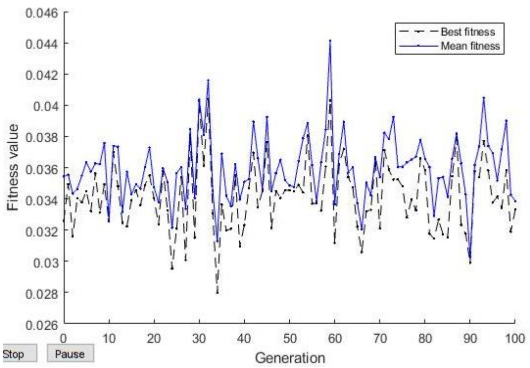



Fig.15. Fitness curve of model MD2

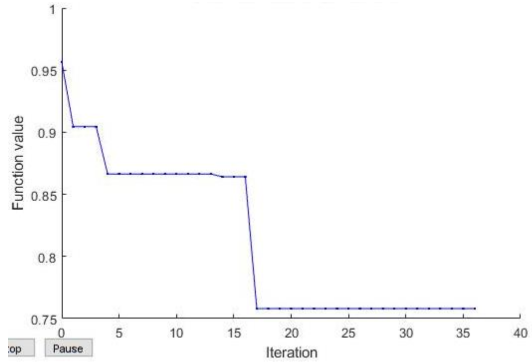



Fig.16. Fitness curve of model MC7

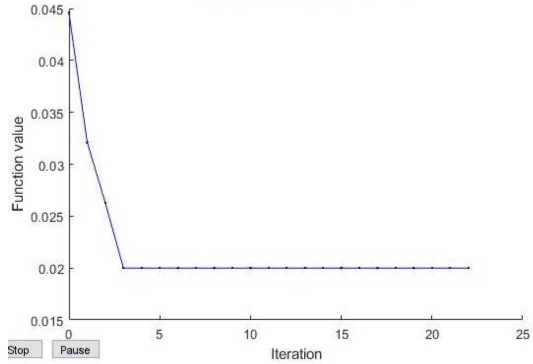



Fig.17. Fitness curve of model MD7

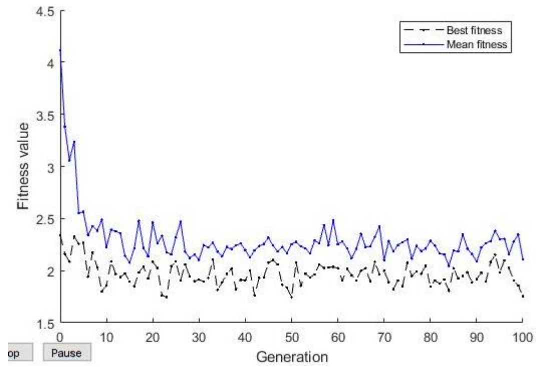



Fig.18. Fitness curve of model MC10

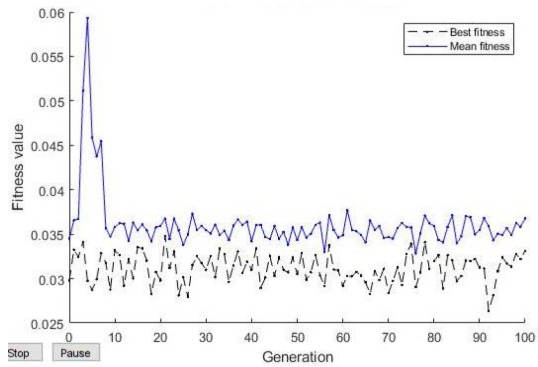



Fig.19. Fitness curve of model MD9

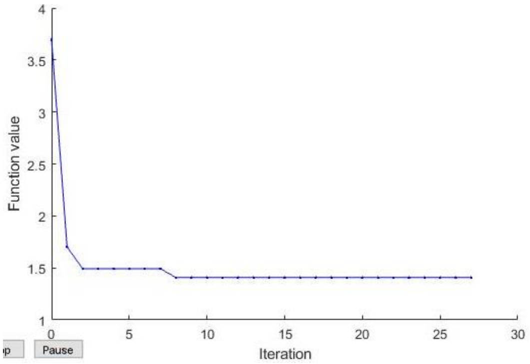



Fig.20. Fitness curve of model MC12

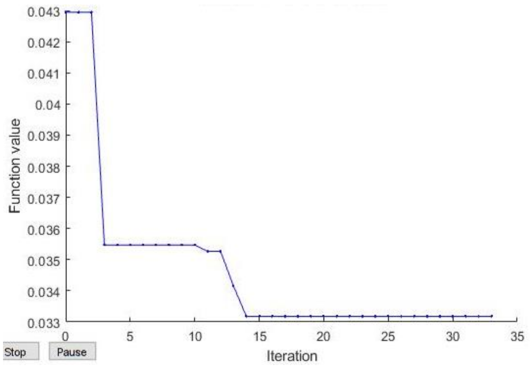



Fig.21. Fitness curve of model MD12

## ABBREVIATION AND NOTATIONS

$\gamma_s$ – Specific unit weight

BRNN – Bayesian regularization neural networks

C – Clay content

CE – Compaction energy

CL – Confidence level

COD – Coefficient of determination

DT – Decision tree

ELM – Extreme learning machine

FC – Fine content

G – Gravel content

GEP – Gene expression programming

GMDH – Group method for data handling

HL – Hidden layers

KB_SVR – Kernel-based support vector regression

LL – Liquid limit

LSSVM – Least square support vector machine

MARS – Multivariate adaptive regression splines

MEP – Multivariate expression programming

MLP_NN – Multilayer perceptron neural network

MLR – Multilinear regression

MLS_SVR – Multivariate support vector regression

PI – Plasticity index

PL – Plastic limit

S – Sand content

SG – Specific gravity

StDev – Standard deviation

SVM – Support vector machines.

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