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APERIODIC TILING USING P SYSTEM

S. Jebasingh[1], T. Robinson[2] and Atulya K. Nagar[3]
[1]*Department of Mathematics, Karunya University, India*
E-mail: jebasinghs@karunya.edu
[2]*Department of Mathematics, Madras Christian College, India*
E-mail: robin.mcc@gmail.com
[3]*Department of Computer Science, Liverpool Hope University, United Kingdom*
E-mail: nagara@hope.ac.uk

Abstract

Tile Pasting P System is a computational model, based on the P System model, to generate two-dimensional tiling patterns using pasting rules at the edges of the regular polygons. Computational mechanism plays an important role in understanding the various complexities involved in the formation of complex patterns. In this paper we study the construction of non-periodic tiling patterns using corner tiles and aperiodic Wang tiles using the computational model Tile Pasting P System. We show that the Tile Pasting P System requires a minimum of four membranes to generate the non-periodic Wang tiling.

Keywords:

P System, Pasting Rules, Periodic Tiling, Aperiodic Wang Tiles

1. INTRODUCTION

Membrane Computing (or P System) [1] is an area of computer science aiming to abstract computing models from the structure and functioning of living cells. A P System consists of a membrane structure, multisets of objects placed inside the membranes, evolution rules governing the modification of these objects in time, and transfer of objects from one membrane to another membrane (inter-cellular communication). The computational power and efficiency of P Systems is of great interest and the same has been studied by measuring the amount of space, time and number of communication rules required for solving computationally difficult problems.

P System with different multiset representations (strings, numbers and vectors) are found in the literature. Recently, a new computational model, namely, Tile Pasting P System, based on the P System model, was introduced and studied in [2]-[4] for generating two dimensional tiling and tessellation patterns, that are formed by gluing (pasting rule) regular polygon tiles thereby covering the infinite Euclidean plane without gaps or overlaps. A Tile Pasting P System consists of a hierarchical membrane structure, labelled regular tile objects placed inside the membranes and pasting rules with target indications associated with the membranes for the construction of tiling patterns. In this mechanism the tiling of a plane or constructing a finite region of pattern is initiated from a membrane containing the axiom tile, subsequently new tiles are added to the axiom tile by pasting rules in a maximally parallel and error free manner. The resultant pattern is communicated to the outer/inner membrane or retained in the same membrane. In each step the computed pattern is exposed to a new set of pasting rules defined in the membrane and the pattern is collected in the output membrane.

Tile Pasting P System is found to generate periodic and non-periodic patterns. In this paper we study the system by considering the generation of non-periodic tiling patterns using corner tiles and aperiodic Wang tiles. In section 2 we define the Tile Pasting P System with an example to tile the plane non-periodically using the corner tile set. In section 3 we discuss the generation of non-periodic tiling of the Euclidean plane using the aperiodic Wang tile set. We show that the system requires a minimum of four membranes to construct the non-periodic Wang tiling pattern.

2. TILE PASTING P SYSTEM

A tile is a two-dimensional topological region (disk) with distinguished boundary points as vertices determining the boundary edges. A unit-square tile with color-coded corners is called a corner tile. Also corner tiles whose edges labelled are called edge labelled corner tiles. For the generation of tiling patterns we consider only regular polygons (squares, triangles,...) as tiles. A tiling of a plane is filling all of the Euclidean plane or part of it using regular polygons called tiles. A tiling in the Euclidean plane is a countable family of tiles that cover the plane without gaps or overlaps. Hence the intersection of any finite set of tiles in a plane tiling has zero area. Such an intersection will consist of a set of points (vertices) and lines (edges). Two adjacent tiles have an edge in common. A tile, its vertices and edges may be labelled distinctively. A tiling set which can tile the plane is said to have a valid tiling and the tiling set is called a valid tile set. Tiling problem introduced by Wang enquires whether a given set of tiles can tile the whole plane. This problem was found to be undecidable. A valid tiling is periodic with the period $(h, v) \in z^2$ if the tile at the position (i, j) is the same as the tile position $(i + h, j + v)$ for all $(i, j) \in z^2$. In the following we define pasting rules to glue the tiles and a computational model called Tile pasting P System to generate tiling patterns.

Definition 2.1: A pasting rule $(A, B)_X$, is an unordered pair of edge labels of tiles M and N with target indication $X \in \{in, out, here\}$. If A is the label of the right edge of M and B is the label of the left edge of N, then an application of the rule $(A, B)_X$ in a membrane region pastes side by side (or joins edge to edge) the two tiles M and N and communicates the resultant tiling pattern to another membrane region or retain it in the same region according to the specified target X.

Tiling patterns are thus made up of tiles, 'glued' (or pasted edge to edge) together. The pasting rule defined above between two tiles is extended for pasting a tile with a tiling pattern. A tile may be pasted with a tiling pattern along the boundary edges when there are pasting rules abutting the tile with the tiling pattern. Rotation of tile or tiling pattern is not allowed while pasting two tiles / patterns.

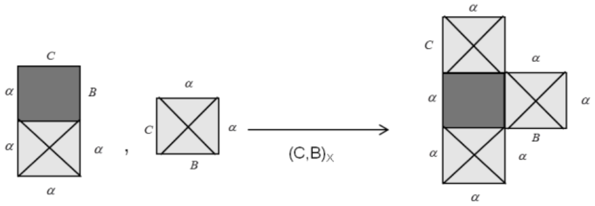


Fig.1. Pasting of tiles with a tiling pattern: Here the pasting rule $(C, B)_X$ glues the tile along the vertical edge B as well as on the horizontal edge C

Definition 2.2: A Tile Pasting P system (TPPS) Π is a construct with $(\mu, \Sigma_1,\Sigma_2,...\Sigma_n, , \gamma, R_1, \ldots, R_n, \delta, m_k)$ where μ is a membrane structure with n membranes $m_1, m_2,...m_n$; $\Sigma_1,\Sigma_2,...\Sigma_n$ are the finite set of tiles associated with the membranes $m_1, m_2,..m_n$ respectively; γ is the seed tile present in the membrane m_i; $R_1, \ldots, R_n$ are the finite set of pasting rules associated with the n membranes of the system, δ is the transition rule defined in each membrane of the system and m_k is the output membrane.

Definition 2.3: The transition rule of the system is defined as δ: $(R_i \times X \times \tau)_i \rightarrow (\tau')_j$, where $X \in \{in, out, here\}$. In each time unit the application of transition rule implies that a set of pasting rules with a specified target X, present in the set R_i associated with a membrane m_i, is applied to all possible edges of the pattern τ and the resultant pattern τ' is communicated to the membrane m_j (either outside the membrane region or inside the membrane region or retained in the membrane itself) according to the specified target X. The application of transition rule starts from the membrane m_i, containing the axiom tile γ. If a membrane contains transition rules with two or three different target indication, then any one of the transition rule is chosen non-deterministically for application.

If a transition rule with a specified target is selected then the pasting rules in that target are applied in a maximally parallel manner to all possible edges of the tiling pattern present in the membrane. Also in each time unit the computation takes place in only one membrane of the system. Such a computation is called a sequential computation. The successive application of transition rule results in communication of tiling pattern from one membrane to another membrane. The system is said to have computed a tiling pattern successfully only if it is collected in the output membrane and the computation halts in the output membrane.

Definition 2.4: The tiling patterns collected in the output membrane m_k forms the language of tiling patterns generated by the TPPS Π and is denoted by $L(\Pi) = \{\tau / \tau \in m_k\}$. The TPPS is said to generate a language of patterns when every tiling pattern obtained in the output membrane is a member of the desired language of patterns and the non-deterministic computation of the system does not produce patterns which are not members of the desired language of patterns in the output membrane. The class of languages generated by a TPPS with at most m membranes is denoted by PL_m (TPPS).

In the following we give an example of non-periodic tiling of the plane using corner tiles [5]. They tile the plane by placing them without gaps or overlaps such that neighbouring tiles have matching corner colours. If we allow n colours for the corners, then there can be at most n^4 distinct tiles because the tiles are not allowed to be rotated or reflected. The complete tile set of corner tiles with two colours is shown below. Horizontal edges and vertical edges with same corner colors are labelled uniformly.

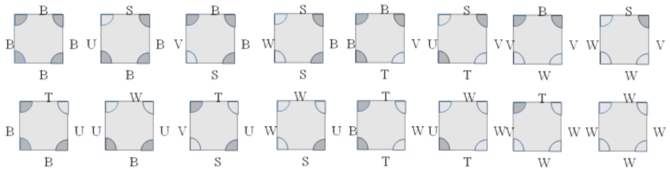


Fig.2. Corner tile set with two colours

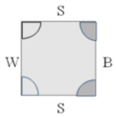
Example 2.1: Consider a TPPS with two membranes to generate the non-periodic tiling pattern using the corner tiles with two colors.

$$\Pi = (\mu, \Sigma_1, \Sigma_2, \gamma, R_1, R_2, \delta, m_2)$$

where,

$\mu = [_1[_2]_2]_1$ with two membranes m_1, m_2

Σ_1 is the edge labelled corner tile set, shown in Fig.2, associated with the membrane m_1, $\Sigma_2 = \phi$, no tiles are associated with the membrane m_2

$\gamma =$ 

is the axiom tile present in the membrane m_1

$R_1 = \{(B, B)_{here}, (W, W)_{here}, (S, S)_{here}, (T, T)_{here}, (B, B)_{in}, (W, W)_{in}, (S, S)_{in}, (T, T)_{in}\}$, $R_2 = \{\varphi\}$

δ:$(R_1 \times here \times \gamma)_1 \rightarrow (\tau)_1$, $(R_1 \times in \times \gamma)_1 \rightarrow (\tau)_2$, $(R_1 \times here \times \tau)_1 \rightarrow (\tau')_1$, $(R_1 \times in \times \tau)_1 \rightarrow (\tau')_2$ where γ is the axiom tile and τ, τ' are arbitrary patterns constructed from the axiom tile.

m_2 is the output membrane.

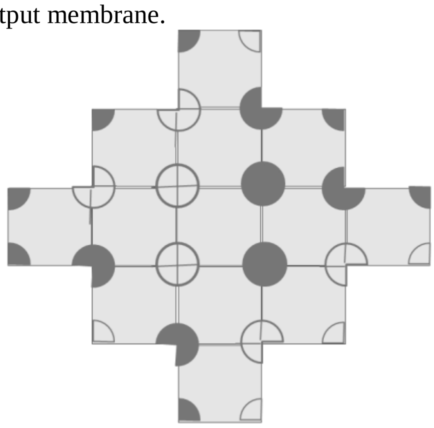


Fig.3. Non-periodic tiling using corner tiles

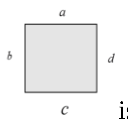
The membrane $m_1$contains the axiom tile and the tiles of the set Σ_1. It is assumed that the tiles defined in the set Σ_1 are available in unlimited numbers during the computation of the pattern. The construction of the pattern can begin from membrane m_1 either by applying the rule $(R_1 \times here \times \gamma)_1 \rightarrow (\tau)_1$ or $(R_1 \times in \times \gamma)_1 \rightarrow (\tau)_2$.

Case (i): If the system applies the transition rule $(R_1 \times in \times \gamma)_1 \to (\tau)_2$ then pasting rules from the set R_1 with the target indication in will be used to paste the tiles on the four edges of the axiom tile and the resultant pattern will be communicated to m_2. The computation of the system stops in m_2 as it has no transition rule to be applied in m_2.

Case (ii): If the system non-deterministically applies the transition rule $(R_1 \times here \times \gamma)_1 \to (\tau)_1$ then pasting rules from the set R_1 with the target indication here will be used to paste on the four edges of the axiom tile γ and the resultant pattern τ will be retained in the membrane m_1. This process will be repeated if the system non-deterministically selects the transition rule $(R_1 \times here \times \tau)_1 \to (\tau')_1$ resulting in the construction of the non-periodic tiling pattern. The computation of the system stops when the system selects the rule $(R_1 \times in \times \tau)_1 \to (\tau')_2$ and the resulting pattern is collected in the output membrane m_2.

3. APERIODIC WANG TILING USING TPPS

Tiling the Euclidean plane non-periodically is a significant example of aperiodicity in two dimensional planes. A finite set of tiles is aperiodic if and only if it has at least one valid tiling, but does not have valid periodic tiling. Such tile sets have been shown to exist with Wang tiles. The aperiodic Wang tile set constructed by Karel Culik [6] is the smallest set of tiles known to tile the plane aperiodically. The construction of aperiodic tile set by Culik uses dynamical system and Beatty sequences to label the sides of the Wang tiles. Stanley et al [7] expounded the aperiodic Wang tile set and showed the existence of a generalized aperiodic Wang tile set and the non-periodic tiling of the plane. In the following we briefly outline the method of construction of the non-periodic tiling of the plane using the aperiodic Wang tile set. For a detailed discussion on the construction of the tile set and results, the reader may refer [7].

A tile 

is a multiplier tile with the multiplier $q > 0$ if $q.a + b\text{-}d = c$. The aperiodic Kari-Culik tile sets consist of two multiplier tile sets, namely tile sets ⅓ and tile set 2.

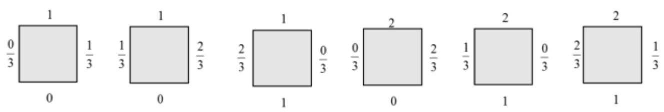


Fig.4. Tile set ⅓

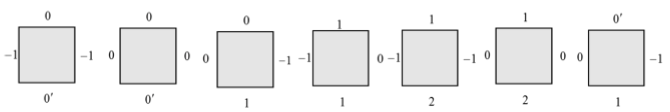


Fig.5. Tile set 2

The 13 tiles of tile set ⅓ and 2 are aperiodic tile set as it has a valid tiling of plane but has no periodic tiling [7]. In a valid non-periodic tiling τ using the aperiodic tile sets ⅓ and 2, each horizontal row consists of tiles either exclusively from the tile set ⅓ or from the tile set 2 [7] and there must be rows with tiles exclusively from the tile set ⅓. Also in a valid tiling τ if a row consist exclusively of tiles from the tile set ⅓ then the row immediately below and above it consist exclusively of tiles from the tile set 2 [7]. Hence there cannot be two consecutive rows from the same tile set.

The Wang tile set equivalent to the tile set ⅓ and 2 is obtained by tweaking the side colours and top-bottom colours of the tiles. The purpose of labelling the edges 0 as $0/3$ is to ensure that each row corresponds to a single multiplicand and different colours are assigned for different 0's. This ensures that the rows of the non-periodic tiling pattern are constructed exclusively from that tile set. In a similar manner the top-bottom colours of the tiles are introduced in such fashion that the tile set is aperiodic. The Wang tile set equivalent to the aperiodic tile set ⅓ and 2 are shown below with edge labels indicating the colours on the edges. Also one can observe that the Wang tile set equivalent for the tile sets ⅓ and 2 are having tiles with same edge labels (colours) in the upper horizontal edge and lower horizontal edge.

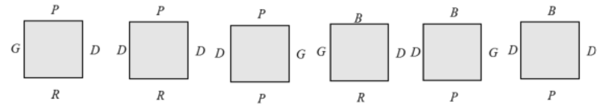


Fig.6(a). Aperiodic Wang tile set ⅓

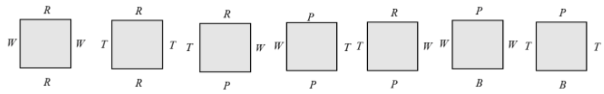


Fig.6(b). Aperiodic Wang tile set 2

Consider a piecewise multiplicative function
$$F(x)=\begin{cases} q_1 x, x_0 \le x < x_1 \\ q_2 x, x_1 \le x < x_2 \\ \quad . \\ \quad . \\ q_k x, x_{k-1} \le x < x_k \end{cases}$$
defined on a finite interval $[x_0, x_k)$ where $\{x_0, x_1, \ldots x_k\} \in N$, $\{q_1, q_2, \ldots q_k\}$ are positive rational numbers chosen so that F(x) is a invertible bijection of $[x_0, x_k)$ onto itself. The generalized aperiodic Wang tile set is derived from the basic tile construction which gives values of the edges of a basic tile $B(x, q_i, n)$, for $x \in (x_i \le x < x_{i+1})$, a fixed rational number q_i and $n \in N$. For a fixed rational number q_i, x in a bounded interval there are only a finite number of tiles derived using the basic tile construction.

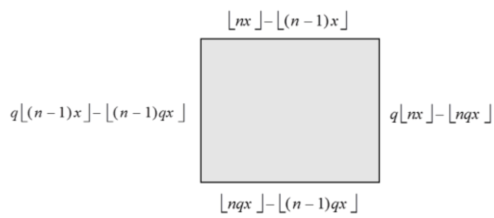


Fig.7. Basic tile construction

The values on the upper horizontal edges form the Beatty difference sequence for x and the bottom horizontal edges form the Beatty difference sequence for qx. The row of tiles from $B(q_1x, q_2, n)$ fit together naturally with the row of tiles from $B(x, q_1, n)$ as the bottom horizontal edge values of the tile set $B(x,q_1,$

n) are the Beatty difference sequence for q_1x, which are also the top horizontal edge values of the tile set $B(q_1x, q_2, n)$. Thus the tile sets $\{B(x, q_i, n)\}$, $i = 1, 2,..., k$, fit together in a natural order. Hence for the dynamical function $F(x)$ with k pieces, we construct a finite set of finite number of tiles $\{B(x, q_i, n)\}$, for 1, 2,..., k.

The Wang tile set derived from the basic tile construction by incorporating both side and top-bottom colour changes is an aperiodic tile set [7] and the side colours of each tile set are disjoint. It has been proved in [7] that the Wang tiling constructed by natural way of fixing the rows together using the aperiodic tile set is a non-periodic tiling. Also in a valid Wang tiling using the aperiodic tile set, there cannot be two consecutive rows from the same tile set and every row of tiling must be exclusively constructed from the same tile set [7].

Lemma 3.1

Let $F(x) = \begin{cases} q_1x, x_0 \le x < x_1 \\ q_2, x, x_1 \le x < x_2 \\ . \\ . \\ q_k x, x_{k-1} \le x < x_k \end{cases}$ be a piecewise multiplicative function defined on a finite interval $[x_0, x_k)$, $\{x_0, x_1,...x_k\} \in N$, $\{q_1, q_2,...q_k\}$ are positive rational numbers chosen so that F(x) is a invertible bijection of $[x_0, x_k)$ onto itself. The Wang tile set equivalent for the finite set of tiles $B(x, q_i, n)$ where $x \in (x_i \le x < x_{i+1})$, a fixed rational number q_i and $n \in N$, will have tiles with same edge labels (colours) in the upper and lower horizontal edges.

Proof: A finite number of tiles are constructed for $x \in (x_i \le x < x_{i+1})$, a fixed rational number q_i and $n \in N$, denoted by $B(x, q_i, n)$. From the basic tile construction we observe that the top horizontal edges and bottom horizontal edges of $B(x, q_i, n)$ form the Beatty difference sequence $B_n(x)$ and $B_n(q_ix)$ respectively. For $x \in (x_i \le x < x_{i+1})$, where $\{x_i, x_{i+1}\} \in N$, the Beatty difference sequence $B_n(x) \in \prod_{-\infty}^{\infty}\{x_i, x_{i+1}\}$. Since F is an invertible function, $f_i(x) = q_ix \in (x_{i+1} \le x < x_{i+1})$, and the Beatty difference sequence $B_n(q_ix) \in \prod_{-\infty}^{\infty}\{x_{i+1}, x_{i+2}\}$. Thus the Beatty difference sequence $B_n(x)$ and $B_n(q_ix)$ are not disjoint sequences. Therefore the finite set of tiles $B(x, q_i, n)$ constructed using the basic tile construction for $x \in (x_i \le x < x_{i+1})$ will have tiles with same non-zero numerical values in the upper horizontal edge and lower horizontal edge.

The Wang tile set equivalent to the finite set of tiles $B(x, q_i, n)$ is obtained by tweaking the side colors and top- bottom colours for the different 0's to ensure that each tile set correspond to a single multiplicand [7] and also for the fixing of different rows non-periodically. The edges having non-zero numerical values were assigned uniform colours [7]. Hence the Wang tile set equivalent for the finite set of tiles $B(x, q_i, n)$, will have tiles with same edge labels (colours) in the upper horizontal edge and lower horizontal edge.

Lemma 3.2

The non-periodic tiling of the plane using the generalized aperiodic Wang tile set cannot be constructed by TPPS if two rows of tiling are constructed (where each row of the pattern is constructed from a specific set of tiles) using a single membrane.

Proof: Let $F(x)$ be a piecewise multiplicative function with k pieces and $\{T_i\}$, $i = 1, 2,...k$ denote a finite set of finite number of tiles constructed for each piece of $F(x)$ using the basic tile construction. The tile set $\{T_i\}$, $i = 1, 2,...k$ is an aperiodic tile set [7] such that the vertical edge labels of each tile set are disjoint [7]. The rows of the tiling fit together naturally since the bottom edge labels (Beatty difference sequence of qx) of the tile set T_n matches with the top edge labels (Beatty difference sequence of x) of the tile set T_{n-1}. The tile set $\{T_i\}$, $i = 1, 2,...k$ constructs only non-periodic Wang tiling when the tiling is constructed by fixing the tiles of successive pieces of the dynamical function one over the other in the natural order [7]. Consider a Tile Pasting P System $(\mu, \Sigma_1, \Sigma_2,..\Sigma_n, \gamma, R_1, \dots, R_n, \delta, m_f)$ to construct the non-periodic Wang tiling of a plane using the aperiodic Wang tile set.

Case (i): Let $T_u = \{B(x, q_i, n)\}$ and $T_V = \{B(q_ix, q_j, n)\}$ denote two successive set of tiles constructed for $x \in (x_i \le x < x_{i+1})$, $q_ix \in (x_{i+1} \le x < x_{i+2})$, a fixed rational number q_i and q_j respectively and $n \in N$ and assume that the membrane m_k contains the tile set $\Sigma_k = T_u \cup T_v$ to construct two successive rows of tiling. Let Γ be the tiling pattern computed from the axiom tile and communicated to the membrane m_k for the natural way of fixing of the rows from the tile set T_u and T_V one above the other. Suppose that the membrane m_k can produce two successive rows of tiling $\Gamma_{(i, j)}$ and $\Gamma_{(i+1, j)}$ such that the rows are constructed from the tile set T_u and T_V respectively one over the other.

Let $\delta:(R_k \times X \times \tau)_k \rightarrow (\tau')_l$ be the transition rule with a target indication $X \in \{in, out, here\}$ defined in the membrane m_k such that the rows $\Gamma_{(i, j)}$ and $\Gamma_{(i+1, j)}$ from the tile set T_u and T_V are constructed successively. The transition rule $\delta:(R_k \times X \times \tau)_k \rightarrow (\tau')_l$ will apply the pasting rules on the pattern Γ , construct a row of tiling and communicate the pattern inside the region or outside the region or retain it in the same membrane as per the target indication $X \in \{in, out, here\}$. Since two consecutive rows of tiling has to be computed in the membrane m_k, after the completion of the first row the pattern must be retained in m_k itself for the construction of the second row over the first row. If target indication is in or out then after the completion of the first row the pattern will be communicated inside or outside the region without constructing the second row. Also if the target indication is here then the resultant pattern will never be communicated to other membranes of the system. Therefore the membrane m_k with the transition rule $\delta:(R_k \times X \times \tau)_k \rightarrow (\tau')_l$ with pasting rules in one target indication $X \in \{in, out, here\}$ cannot construct two consecutive rows of tiling, one over other and communicate it to the next membrane.

Therefore the membrane m_k must have two transition rules $\delta:(R_k \times here \times \tau)_k \rightarrow (\tau')_l$ and $\delta:(R_k \times X \times \tau)_k \rightarrow (\tau')_l$, where $X \in \{in, out\}$ to construct two consecutive rows of tiling $\Gamma_{(i, j)}$ and $\Gamma_{(i+1, j)}$. Let $(R_k \times here \times \tau)_k \rightarrow (\tau')_l$ be the transition rule to retain the pattern after the construction of first row, and $(R_k \times X \times \tau)_k \rightarrow (\tau')_l$, $X \in \{in, out\}$ be the rule to construct the second row and communicate the pattern to the next membrane m_l.

Assume that the system non-deterministically first selects the transition rule $(R_k \times X \times \tau)_k \rightarrow (\tau')_l$, $X \in \{in, out\}$ for the construction of the row using the tile set T_V. Since the natural way of fixing the row of tiles over the pattern Γ is by the tile set T_u, the transition rule may or may not construct the row of tiles from the tile set T_V. Pasting of tiles from the set T_V, instead of tiles from the set T_u, is possible as the upper and lower horizontal edge labels of the tile set T_u and T_v are not disjoint sets by lemma 3.1. When a complete or incomplete row of tile is constructed from the set T_v the resultant pattern will be communicated to the next membrane m_l without constructing the row of tiles from the other set. If the transition rule $(R_k \times X \times \tau)_k \rightarrow (\tau')_l$, $X \in \{in, out\}$ is applicable over Γ then the system will select the transition rule $\delta:(R_k \times here \times \tau)_k \rightarrow (\tau')_l$ for the construction of the row over Γ. Thus the system constructs invalid Wang tiling patterns when the transition rule $(R_k \times X \times \tau)_k \rightarrow (\tau')_l$, $X \in \{in, out\}$, is selected first non-deterministically. The invalid pattern thus constructed will reach the output membrane with incomplete rows as the pasting rules in the successive membranes are not disjoint. We now discuss the case of the system selecting the set of pasting rules with the target here below.

Suppose that the system non-deterministically selects the transition rule $\delta:(R_k \times here \times \tau)_k \rightarrow (\tau')_l$ to construct the row of tiling $\Gamma_{(i,\ j)}$ and the newly constructed pattern is retained in the membrane m_k. After constructing the row $\Gamma_{(i,\ j)}$, the system has to select the transition rule $(R_k \times X \times \tau)_k \rightarrow (\tau')_l$, $X \in \{in, out\}$, to construct the row $\Gamma_{(i+1,\ j)}$ and communicate it to the next membrane. As the computation is non-deterministic the system may again select the rule $\delta:(R_k \times here \times \tau)_k \rightarrow (\tau')_l$ and the row of tiles from T_u will be repeated wherever pasting is possible. The pasting of tiles from the same tile set T_u over the row $\Gamma_{(i,j)}$ using the set of pasting rules with the target here is possible as the upper and lower horizontal edges of the tiles of the set T_u are not disjoint by lemma 3.1. In a valid Wang tiling using the aperiodic tile set $\{T_i\}$, $i = 1,2,..k$, formation of two consecutive rows from the same tile set T_u is forbidden, as it will lead to incomplete tiling of the plane. Thus the system constructs invalid Wang tiling patterns when the transition rule $\delta:(R_k \times here \times \tau)_k \rightarrow (\tau')_l$ is selected first non-deterministically. The invalid pattern thus constructed will reach the output membrane with incomplete rows as the pasting rules in the successive membrane are not disjoint. The two rows of valid Wang tiling is constructed in a single membrane only when the transition rule $\delta:(R_k \times here \times \tau)_k \rightarrow (\tau')_l$ is first applied, followed by the application of the rule$(R_k \times X \times \tau)_k \rightarrow (\tau')_l$, $X \in \{in, out\}$. The non-deterministic computation of the system has no preference in selecting the transition rule with a particular target first. Therefore the membrane m_k having the transition rule with two target indications constructs invalid Wang tiling patterns which are not members of the desired language of patterns.

Case (ii): Let T_l and T_{l+r} denote the two set of tiles constructed for l^{th} and $(l+r)^{th}$ piece of the dynamical function $F(x)$. Let Γ be the tiling pattern computed from the axiom tile and communicated to the membrane m_k for the natural way of fixing of the row from the tile set T_l. Suppose that the membrane m_k can produce two (non-consecutive) rows of tiling $\Gamma_{(i,\ j)}$ and $\Gamma_{(i+r,\ j)}$ such that the rows are constructed from the tile set T_l and T_{l+r} respectively. The membrane m_k contains the tile sets $\Sigma_k = T_l \bigcup T_{l+r}$ to construct two successive rows of tiling and should have the transition rules $\delta:(R_k \times in \times \tau)_k \rightarrow (\tau')_l$ and $\delta:(R_k \times out \times \tau)_k \rightarrow (\tau')_l$ to produce the rows from the tile set T_l and T_{l+r} respectively. Since the natural way of fixing the row over the tiling pattern Γ is by the tile set T_l, the top edge labels of the pattern Γ and bottom edge labels of the tile set T_l should have the same Beatty difference sequence according to the construction of the Wang tile set using the basic tile construction method for the dynamical function $F(x)$. Suppose that the system non-deterministically selects the transition rule $\delta:(R_k \times out \times \tau)_k \rightarrow (\tau')_l$ to construct a row of tiles instead of the rule $\delta:(R_k \times in \times \tau)_k \rightarrow (\tau')_l$. This will lead to the construction of a row of tile from the set T_{l+r} with gaps as the top edge labels of the pattern Γ does not naturally fix with the bottom edge labels of the tile set T_{l+r} Thus the system constructs invalid Wang tiling patterns when the rule $\delta:(R_k \times out \times \tau)_k \rightarrow (\tau')_l$ is selected first non-deterministically. The invalid pattern thus constructed will reach the output membrane with incomplete rows as the pasting rules in the successive membranes are not disjoint. Therefore the construction of two non-consecutive rows in a membrane results in the deviation of natural way of fixing the rows and a tiling pattern with gaps.

The construction of row of tiling from the tile set T_l in the natural way of fixing of tiles is done only when the transition rule $\delta:(R_k \times in \times \tau)_k \rightarrow (\tau')_l$. is selected non-deterministically. However the non-deterministic computation of the system has no preference in selecting the transition rule $\delta:(R_k \times in \times \tau)_k \rightarrow (\tau')_l$. than the transition rule $\delta:(R_k \times out \times \tau)_k \rightarrow (\tau')_l$. Hence the construction of two non-consecutive rows in a membrane will result in computation of unsuccessful computations or invalid tiling patterns in the output membrane. Thus the non-periodic tiling of the plane using the aperiodic tile set $\{T_i\}$, $i = 1,2,..k$ cannot be constructed non-deterministically if two rows of tiling are computed in a single membrane.

Theorem 3.3: The non-periodic Wang tiling using two sets of aperiodic Wang tiles can be constructed using a TPPS with four membranes and not less than that $PL_m(TPPS) \subset PL_4(TPPS), m = 3,2,1$.

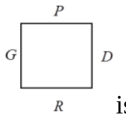
Proof: Consider a TPPS with four membranes to generate the non-periodic Wang tiling using the aperiodic KC tile set.

$$\Pi = (\mu, \Sigma_1,\ldots,\Sigma_4, \gamma, R_1,\ldots, R_4,\delta, m_4)$$

where,

$\mu = [_1[_2[_3[_4]_4]_3]_2]_1$

Σ_1 is the aperiodic tile set ⅓ as shown in Fig.6(a), Σ_2 is the aperiodic tile set 2 as shown in Fig.6(b), Σ_3 is the aperiodic tile set ⅓ as shown in Fig.6(a), $\Sigma_4 = \phi$



$\gamma =$ is the axiom tile present in m_1

R_1 = {(G, G) here, (D, D) here,(G, G) in, (D, D)in }

R_2 = {(R, R) in, (P, P) in, (B, B) in, (W, W) in, (T, T) in},

R_3 = {(R, R) out, (P, P) out, (B, B) out, (G, G) out, (D, D) out, (R, R) in,(P, P) in, (B, B) in, (G, G) in, (D, D) in },

$R_4 = \{\varphi\}$

δ: $(R_1\times here \times \gamma)_1\rightarrow(\tau)_1$ $(R_1\times here \times \tau)_1\rightarrow(\tau')_1$, $(R_1\times in \times \gamma)_1\rightarrow(\tau)_2$ $(R_1\times in \times \tau)_1\rightarrow(\tau')_2$, $(R_2\times in \times \tau)_2\rightarrow(\tau')_3$ $(R_3\times out \times \tau)_3\rightarrow(\tau')_2$, $(R_3\times in \times \tau)_3\rightarrow(\tau')_4$

m_4 is the output membrane.

The membrane m_1 contains the axiom tile and the tiles of the set Σ_1. The computation starts at the membrane m_1, which is having the axiom tile from the tile set ⅓. A row of tiling is constructed exclusively from the tile set ⅓ when the system applies the transition rule δ: $(R_1 \times here \times \gamma)_1\rightarrow(\tau)_1$ to the axiom tile and the repeated application of the rule $(R_1 \times here \times \tau)_1\rightarrow(\tau')_1$ to the successive pattern. This process stops when the system non-deterministically applies the rule $(R_1 \times in \times \tau)_1\rightarrow(\tau')_2$ and communicates the pattern to the membrane m_2.

The membrane m_2 contains the tiles of the set Σ_2. In m_2 the system applies the rules $(R_2 \times in \times \tau)_2\rightarrow(\tau')_3$ to construct a row of tiling exclusively from the tile set 2, above and below the row constructed in m_1 and communicates the pattern to the membrane m_3. The membrane m_3 contains the tiles of the set Σ_3. In the membrane m_3 the system either selects the rule $(R_3 \times out \times \tau)_3\rightarrow(\tau')_2$ or $(R_3 \times in \times \tau)_3\rightarrow(\tau')_4$ If the system non-deterministically selects the rule $(R_3 \times out \times \tau)_3\rightarrow(\tau')_2$ then it constructs a row of tiling exclusively from the tile set ⅓ and communicates the pattern to the membrane m_2 and the process repeats. If the system non-deterministically selects the rule $(R_3 \times in \times \tau)_3\rightarrow(\tau')_4$ then it constructs a row of tiling exclusively from the tile set ⅓ and communicates the pattern to the membrane m_4 and the computation stops.

Every membrane in the system is necessary, as the non-periodic tiling cannot be computed if we combine the computation of two membranes in a single membrane. Suppose there is a TPPS Π with three membranes to generate the non-periodic Wang tiling such that the system is obtained by combining the rules of any two membranes to perform the same computation. Consider the case where the system is obtained by combining the membranes $m_1$and m_2. The membrane m_1 now has to construct a row of tiling exclusively from the tile set ⅓, followed by the construction of row of tiles (above and below) from the tile set 2 and communicate the pattern to membrane m_2. Hence the membrane m_1 must contain the aperiodic tile set ⅓ and 2 (as shown in Fig.6(a) and Fig.6(b)). By lemma 3.2 such a system cannot construct the non-periodic tiling of the plane. Therefore m_1 and m_2 cannot be merged together. Similarly the membranes m_2 and m_3 cannot be merged together to construct the rows of tiling exclusively from the tile set 2, followed by the construction of row of tiles from ⅓. Also the membrane m_3 and m_4 cannot be merged together otherwise the computation of the system will never halt. Therefore there is no TPPS with three membranes or less than that to generate the non-periodic Wang tiling using the aperiodic tile set.

Theorem 3.4: The non-periodic Wang tiling using $n(n \geq 2)$ sets of aperiodic Wang tiles, can be constructed using a TPPS with n+2 membranes and not less than that $PL_m(TPPS) \subset PL_{n+2}(TPPS)$, $m = n+1, n,..1$.

Proof: The proof of the theorem is by mathematical induction. The statement is true for $n = 2$ by theorem 3.3. Assume that the statement is true for $n = k$, that is $PL_m(TPPS) \subset PL_{k+2}(TPPS)$, $m = k+1, k,...1$. The non-periodic tiling using k sets of aperiodic Wang tiles is constructed using a TPPS with k+2 membranes and not less than that. The graph of the membrane structure is shown below.

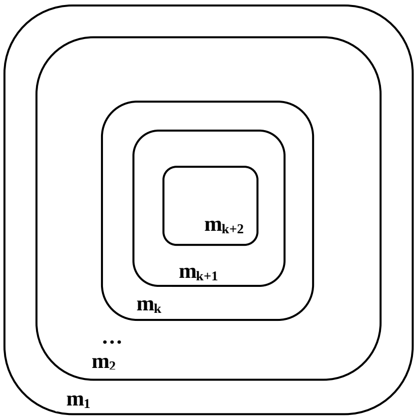


Fig.8. Membrane structure of the system

Consider a non-periodic tiling using k+1 sets of aperiodic Wang tiles. By assumption to construct a valid Wang tiling using k sets of aperiodic tile set we require k+2 membranes. By the natural way of fixing the rows, the $(k+1)^{th}$ aperiodic tile set should be fixed above the row constructed using the k^{th} aperiodic tile set. Therefore in the graph shown above, place a membrane after the $(k+1)^{th}$ membrane to construct the row of tiling using the $(k+1)^{th}$ aperiodic tile set. The computation of adjacent membranes cannot be combined together to construct two rows of tiling in a single membrane, as the system cannot generate the valid Wang tiling non-deterministically by lemma 3.2. Hence each membrane is necessary for the construction of the valid Wang tiling. Therefore $(k+3)$ membranes are required to generate the non-periodic tiling using $(k+1)$ aperiodic Wang tile set and not less than that.

4. CONCLUSION

We have presented here a computational model where non-periodic tiling patterns are generated using pasting rules in a parallel manner, into a hierarchical membrane structure. The minimum number of membranes required in the construction of non-periodic Wang tiling has been studied for the generalized aperiodic Wang tile set. The model can be used in the generation of various architectural tiling and wall papers which are periodic and non-periodic patterns in nature. The implementation of the model consist of parallel application of pasting operation on regular polygons, membranes as regions of computation, communication between the region and various other routines for developing application programs. It will be interesting to develop a bio-inspired graphic tool for tessellation and tiling patterns with the distinct features of membrane computing techniques.

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