PARAMETRIC EXTRACTION AND EQUIVALENT CIRCUIT MODELLING OF SINGLE BAND ANTENNAS

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Abstract

A simple and powerful equivalent circuit modeling suitable and novel method for extracting distributed parameters of a single band antenna which can be generalized is presented. From these two techniques, an equation for distributed components in terms of dimensional parameters and effective dielectric constant is developed. Validity of the developed equation is analyzed using simulation software and the results are in close matching. In future, this method can be elaborated to develop generalized modeling for multi band and wide band antennas.

Keywords:

Antenna, Equivalent Circuit Modelling, RLC Circuit, Series Resonance, Q Factor, Bandwidth

1. INTRODUCTION

Antenna is a radiating structure which is capable of emitting/receiving electromagnetic radiation at a certain frequency. Antenna is designed to be operated in a band of frequencies and that is termed as antenna bandwidth. It is often very difficult for antenna engineers to design an antenna with an accurate resonant frequency with a particular bandwidth, because it is depended on various physical, electrical and magnetic parameters. There comes the use of equivalent circuit modelling. Equivalent circuit modelling is the process of constructing a lumped element circuit which possess the resonant and bandwidth characteristics of antenna. From this model, a technical person is easily capable of calculating various parameters of the antenna. If the developed equivalent circuit model can be generalized, it can be used for accurate designing antennas and can be used for easily converting one antenna with particular characteristics into another.

Two general power sources based complicated equivalent circuit model in introduced in [1]. Wang et al. [2] presents a new equivalent circuit for antenna which is transformed from Sdomain parameters. Floquet Modal Expansion of Surface Current Distributions based equivalent circuit modelling is presented in [3]. Circuit modelling for general PCB meta-rings antenna is discussed by the authors in [4]. An analytical model for proximity coupled microstrip antenna is introduced in [5]. Equivalent circuit model suitable for antennas with lumped components like PIN diodes is explained in [6]. Circuit modelling of frequency re configurable Ku band antennas is discussed by Karmakar et al. [7]. A Schottky diode based large signal equivalent model of microwave circuits is presented in [8]. Hossain et al. [9] introduces a novel circuit modelling for wideband absorbers, which is suitable for low frequencies in microwave range. A circuit approach for anisotropic frequency-selective surfaces and meta-surfaces is presented in [10]. Naseri et al. [11] presents a modelling which is suitable for linear and circular polarized

microwave surfaces. Anyhow, in all the above discussed design or modelling, the equivalent circuit formation and parameter extraction is rather complex and time consuming. Various signal processing strategies are also incorporated in some models. Another problem with these is lack of generalization. Presented modelling is suitable for the discussed antennas/surfaces only.

In this article, we are presenting a simple method for parametric extraction of distributed components and generalized equivalent circuit modelling which is very much suitable for single band antennas. Parametric extraction is done with the help of well-known series resonance equations. The extracted parameters are analysed for different dimensional factors and a generalized equation for calculating distributed components is arrived. Validity of the developed equations are also analysed and presented.

2. THEORY OF SERIES RESONANCE

Antenna is considered as a self-resonating structure which having its on resonant frequency caused due to various structural parameters. According to [12], a single band antenna is equivalent to a series RLC circuit having a resonant frequency and selectivity. A series RLC circuit with AC expiation is shown in Fig.1.



Fig.1. Series RLC Circuit with AC Excitation

The magnitude of impedance of the circuit is given by

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2} \tag{1}$$

At DC condition, the capacitor acts as a blocking capacitor and the current through the circuit is zero. As frequency of excitation increases, a complex current (current with a phase shift with voltage) will flow through the circuit. The magnitude and phase shift of the current waveform will be a function of frequency and circuit parameters. According to well-known characteristics of inductance and capacitance, it is clear that the inductive reactance increases while the capacitive reactance decreases with frequency. At a magic point called resonance, the capacitive and inductive reactance will have same magnitude but opposite phase, and thus they will cancel each other resulting in an impedance which is equal to resistance in the circuit. Equation for finding out resonant frequency is as follows:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{2}$$

At resonance

$$Z=R$$
; At resonance (3)

As we deviate the frequency towards left of resonant frequency, the capacitive reactance will be larger and which results in a magnitude of impedance greater than R. The same phenomena will happen when we move towards the right to the resonant frequency too with a difference that, here the inductive reactance is larger. Thus, impedance will be minimum at resonance which implies that the current will be maximum at resonance. Current will decrease from maximum value, as we move from resonant frequencies to either side. This will create a band pass characteristic to current curve which indicates that the resonant circuit has a bandwidth which is defined as the range of frequencies for which current is greater than or equal to 0.707 times the maximum current.

Selectivity or Q-factor or Quality factor is the ability of the circuit to select a particular frequency from a range of frequencies. Q-factor and bandwidth of the circuit are inversely proportional [13] and related to the resonant frequency and bandwidth through the well-known equation.

$$Q = \frac{f_r}{BW} = \frac{2\pi f_r L}{R} = \frac{1}{2\pi f_r CR}$$
(4)

Thus from Eq.(2), it is clear that the resonant frequency is a function of L and C only. Eq.(4) clearly states that the Q is a function of all the three-circuit component viz. R, L and C.

3. THEORETICAL ANTENNA MODELLING

Antenna is also a resonating structure with a resonant frequency and bandwidth. Its reflection co efficient will gives both these parameters. By equating the series RLC circuit discussed above with a single band antenna, the equivalent circuit parameters of the antennas can be extracted using simple mathematical methods.

Algorithm for extracting distributed *R*, *L* and *C* parameters of the antenna is described as follows.

- **Step 1:** Obtain the Resonant Frequency f_r , S_{11} value and -10dB Bandwidth (BW) from reflection coefficient
- **Step 2:** Calculate the value of *R* by solving the Eq.(5).

$$10^{\frac{S_{11}}{20}} = \frac{R - Z_0}{R + Z_0} \tag{5}$$

where Z_0 is the characteristic impedance of the transmission line used. In our case, it is 50 Ω .

- **Step 3:** Calculate Q using first expression of Eq.4.
- Step 4: Calculate Inductance by the formula

$$L = \frac{QR}{2\pi f_r} \tag{6}$$

This is obtained by rearranging 2^{nd} part of Eq.(4).

Step 5: Calculate Capacitance by the formula

$$C = \frac{1}{2\pi Q R f_r} \tag{7}$$

This is obtained by rearranging 3^{rd} part of Eq.(4).

After extracting the circuit parameters from reflection co efficient, we have to relate these values to the dimensional parameters of the antenna. For this another algorithm is used and the stepwise explanation of this is as follows.

- **Step 1:** Perform parametric analysis of the antenna; extract the distributed parameters for all the variations.
- **Step 2:** Obtain the curve between each dimensional parameter and extracted component. Using curve fitting method of statistics, we can obtain a relation between dimensional and distributed parameter.
- **Step 3:** By combining all the individual relation of onedimensional parameter, obtain the offset values (constant) if any.
- **Step 4:** Generalize the relation obtained in step 3 by replacing dimension with fractional guided wavelength or by incorporating effective dielectric constant

As an example, extraction of circuit parameters of single band dipole antenna fed by a coplanar strip discussed in [14] is discussed here. Antenna has four different dimensional parameters which are crucially affects the resonance. Structure of slot line fed antenna under study is shown in Fig.2.



Fig.2. Structure of Slot-line fed single band Dipole

The reflection co efficient curves obtained using dimensional variations in L_1 , W_1 , L_2 and W_2 are depicted in Fig.3.



Fig.3. Parametric Analysis

Since antenna offers perfect matching at resonance, the resistance value obtained using Eq.(5) in each variation is nearly 50Ω . The capacitance and inductance of the antenna varies considerably with all these parameters and the curve showing variation of distributed parameters with dimensional parameters are shown in Fig.4.



Fig.3. L and C with dimensional parameters

On curve fitting the graphs obtained in Fig.4, the relations of various dimensional parameters on C and L are obtained are given in Table.1.

 Table.1. Expression for L and C with different dimensional parameters

Parameter	Relation					
L_1	$L = L_1(0.0116L_1 - 0.218) + 1.6931$ $C = L_1(0.0667L_1 - 1.24) + 11.5833$					
L_2	$C = \frac{L = L_2(-0.0151L_2 + 0.0834) + 0.6037}{L_2} - \frac{1}{L_2} \left(\frac{31.4243}{L_2^2} - \frac{10.92}{L_2} - 0.199\right) + 6.7251$					
W_1	$L = W_1(-0.0097W_1 + 0.3304) - 2.021$ $C = W_1(0.2063W_1 - 5.85) + 47.5829$					
W_2	$L = W_2(-0.0104W_2 + 0.1304) + 0.431$ $C = W_2(0.0955W_2 - 0.7655) + 7.58$					

By combining all the above relations, we can obtain the expression for capacitance as:

$$C = 54.64 + L_1 \left(0.0667L_1 - 1.24 \right) + W_1 \left(0.2063W_1 - 5.85 \right) + W_2 \left(0.0955W_2 - 0.7655 \right) + \frac{1}{L_2} \left(\frac{31.4243}{L_2^2} - \frac{10.92}{L_2} - 0.199 \right)^{(8)}$$

The inductance can be combined and written as

$$L = -1.3036 + L_1(0.0116L_1 - 0.218) + L_2(-0.0151L_1 + 0.0834) + W_1(-0.0097W_1 + 0.3304) + W_2(-0.0104W_2 + 0.1304)$$
(9)

As the next step, we have to generalize the equation for all dielectric constants by incorporating effective dielectric constant in Eq.(8) and Eq.(9). For this convert all length by equivalent fraction of guided wavelength. Then the generalized equations are:

$$\begin{split} C &= 20.27 + 0.379 \sqrt{\varepsilon_{ef}} \left[L_1 \sqrt{\varepsilon_{ef}} \left(0.0667 L_1 \sqrt{\varepsilon_{ef}} - 1.24 \right) \right] \\ &+ \frac{1}{L_2 \sqrt{\varepsilon_{ef}}} \left(\frac{31.4243}{\varepsilon_{ef} L_2^2} - \frac{10.92}{L_2 \sqrt{\varepsilon_{ef}}} - 0.199 \right) + W_1 \sqrt{\varepsilon_{ef}} \end{split}$$
(10)
$$\begin{split} &\left(0.0955 W_2 \sqrt{\varepsilon_{ef}} - 5.85 \right) + W_2 \sqrt{\varepsilon_{ef}} \left(0.0955 W_2 \sqrt{\varepsilon_{ef}} - 0.7655 \right) \\ &\text{and} \\ L &= -1.3036 + 0.364 \sqrt{\varepsilon_{ef}} \left[L_1 \sqrt{\varepsilon_{ef}} \left(0.012 L_1 \sqrt{\varepsilon_{ef}} - 0.22 \right) \right] \\ &+ L_2 \sqrt{\varepsilon_{ef}} \left(-0.0151 L_2 \sqrt{\varepsilon_{ef}} + 0.0834 \right) \\ &+ W_1 \sqrt{\varepsilon_{ef}} \left(-0.0097 W_1 \sqrt{\varepsilon_{ef}} - 0.3304 \right) \\ &+ W_2 \sqrt{\varepsilon_{ef}} \left(-0.0104 W_2 \sqrt{\varepsilon_{ef}} - 0.1304 \right) \end{split}$$

4. RESULTS AND DISCUSSIONS

Relations obtained in Eq.(10) and Eq.(11) are validated for different commercially available substrates using the high frequency simulation tool Ansoft HFSS. All the antennas are designed to operate in 2.4GHz ISM bands and the structural parameters of the antenna used for simulations are depicted in Table.2.

Table.2. Antenna parameters

Name	Dielectric constant	L_1	L_2	W_1	W_2
Antenna 1	10.2	7.95	2.43	8.15	2.1
Antenna 2	6.15	10	3.04	10.21	2.5
Antenna 3	4.4	11.5	3.5	11.75	3
Antenna 4	2.2	14.9	4.55	15.25	3.9

Obtained inductor and capacitor values using Eq.10 and 11 and a comparison of resonating frequency and bandwidth are depicted in Table.3.

Table.3. Validation of Equations

Nome	L C		BW (GHz)		Fr (GHz)	
Iname	(nH)	(pF)	HFSS	Calculated	HFSS	Calculated
Antenna 1	0.7275	6.0926	0.457	0.435	2.40	2.39
Antenna 2	0.7296	6.1300	0.460	0.430	2.38	2.38
Antenna 3	0.7313	6.1501	0.455	0.431	2.38	2.37
Antenna 4	0.7236	6.0758	0.442	0.436	2.42	2.40

The simulated and calculated values are close enough and which indicated the universal validity of the equations for antenna structures given in Fig.2.

5. CONCLUSION

In this paper, a simple, powerful and generalized equivalent circuit modelling suitable for single band antenna is presented. A novel method for extracting distributed parameters of the antenna is also discussed. From these two techniques, a generalized equation for distributed components in terms of dimensional parameters is developed. Validity of the developed equation is analysed using simulation software and the results are in close matching. In future, this method can be elaborated to develop generalized modelling for multi band and wide band antennas.

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