PERFORMANCE EVALUATION OF MASSIVE MIMO SYSTEMS IN RICIAN FADING CHANNEL

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Abstract
Massive MIMO is the most significant technology in 5G that improves spectral efficiency by employing massive number of antennas at the base station. This paper presents the variation of uplink achievable rates of massive MIMO systems in Rician fading channels using maximum ratio combining and zero forcing receivers for Perfect and Imperfect CSI. . The theoretical and approximated values of MRC and ZF receiver in Rician fading for Perfect and Imperfect CSI is formulated. With the results of simulations, it is shown that massive MIMO system can deliver high sum rate with the increase of antennas and also with the high Rician factor The impact of large-scale fading parameter in Rician fading using ZF receiver is analysed in this paper and compared with MRC receiver.

Keywords:
Massive MIMO (MM), Rician Factor (RK), Path Loss Factor (PLF), Channel State Information (CSI), Non-Line of Sight (NLOS)

1. INTRODUCTION

. Recently, Massive MIMO has gained much attention of the researchers as it serves multiple users at a time by using massive no. of antennas at the base station (BS) in the same frequency-time resource [1]. Several efforts have been made for improvement of SE in Massive MIMO systems. Using Law of Power Scaling, the transmit power can be minimized at the base station. For example, if the system with single antenna is compared with Massive MIMO, the power required by the latter is only 1% of the former to keep the same quality of service [2]. Since the mathematical calculations are simple, the Rayleigh fading is assumed in majority of the works. But it lacks LOS component between the transmitter and receiver. Also, this Rayleigh fading is not accurate because multipath propagation has both independent and identically distributed scatters and line of sight components. So, need to consider more generalized fading models. Rician fading is considered in this work. Rician factor (R_k) is used to find the strength of the LOS component. It can be calculated by taking the ratio of LOS and NLOS components. Many of the works assumed Rician factor is same for all the users [3]. But in practical scenario, the Rican factor between the users and the BS is different which is considered in this work. The upper bounds of uplink sum rate in multicell MM system using ZF receiver is derived for imperfect CSI case [4]. The asymptotic analysis of multicell MM system over Rician fading is discussed over downlink using Maximum Ratio Transmission and Regularized ZF [5]. An accurate scheme for channel estimation is derived for 5G MM systems [6]. A novel iterative approach is designed for uplink MM systems for Imperfect CSI [7]. The optimum power allocation and the optimal number of users to be used in a cell is found [8]. The variation of spectral efficiency with no. of antennas at the BS at different values of Rician Factor is investigated [9]. The effect of large-scale fading parameter in Rayleigh fading using MRT precoding for downlink case is studied [10]. The effect of large-scale fading parameter in Rayleigh fading using MRC receiver for uplink case is shown [11] whereas the effect of large-scale fading parameter in Rician fading using ZF receiver for uplink is analyzed and compared with the MRC receiver and achieved better results.

2. SYSTEM MODEL

A Massive MIMO system with one N antenna BS and K one antenna users distributed randomly in circular shaped single cell operating in TDD mode is considered. The $N\times 1$ vector y received at the BS is

$$y=\sqrt{P_u}Gx+w \tag{1}$$

where P_u gives power of single user, x is $K\times 1$ vector consists of transmitted signal from all users, G is channel matrix of order $N\times K$ and w is AWGN vector of order N by 1. $g_{nk} = [G]_{nk}$ represents the channel coefficient between the k^{th} user and antenna 'n' of the BS. g_{nk} models fast fading, shadow fading and geometric attenuation and is given by:

$$g_{nk}=h_{nk}\sqrt{\alpha_k}\ ,\ n=1,2,\ldots,N \tag{2}$$

where h_{nk} is fast-fading element and $\sqrt{\alpha_k}$ is shadow fading coefficient. So, G can be represented as

$$G = HD^{0.5} \tag{3}$$

where H represents $N\times K$ small scale fading matrix and $D^{0.5}$ represents $K\times K$ diagonal matrix. The small-scale fading matrix H consists of LOS component $\bar{H}$, and NLOS component H_r. H can be expressed as:

$$H=\bar{H}\left(\frac{F}{F+I_K}\right)^{0.5}+H_r\left(\frac{1}{F+I_K}\right)^{0.5} \tag{4}$$

where $\bar{H}$ is a $N\times K$ matrix in which $\left[\bar{H}\right]_{nk}=e^{-j(n-1)\frac{2\pi d}{\lambda}\sin(\theta_k)}$ where d is the spacing between antennas, λ is the wavelength, θ_k is the arriving angle of the k^{th} user signal. F is a diagonal matrix of order $K\times K$ with $[F]_{kk}=R_k$, The ratio of the deterministic component power to the scattered components power is denoted by the Rician Factor, R_k. H_r denotes the NLOS component the entries of which are assumed to be independent and identically distributed Gaussian random variables with zero-mean and unit variance. From Eq.(4), Eq.(3) can be expressed as will be:

$$G=\bar{G}\left(\frac{F}{F+I_K}\right)^{0.5}+G_r\left(\frac{1}{F+I_K}\right)^{0.5} \tag{5}$$

where $\bar{G}$ represents the deterministic component of G and G_r denotes the random component of G.

3. UPLINK ACHIEVABLE RATE

3.1 PERFECT CSI

Considering the BS with Perfect CSI. Let D be the detector matrix of dimension $N \times K$ which depends on G. By multiplying with the conjugate transpose of the linear detector matrix, the BS processes its received signal vector as:

$$r = D^H y = \left(D^H \sqrt{P_u} Gx + D^H w\right) \tag{6}$$

The k^{th} element of r is given by

$$r_k = \sqrt{P_u}\hat{d}_k^H g_k x_k + \sqrt{P_u}\sum_{i=1, i\neq k}^{K} \hat{d}_k^H g_i x_i + \hat{d}_k^H w \tag{7}$$

where g_k is the k^{th} column of the matrix, x_k is the k^{th} user signal and d_k is k^{th} column of matrix D.

The achievable rate of any user 'k' is expressed by:

$$R = \log_2\left(1 + \frac{P_u\left|\hat{d}_k^H g_k\right|^2}{P_u \sum_{i=1, i\neq k}^{K}\left|\hat{d}_k^H g_i\right|^2 + \left\|\hat{d}_k\right\|^2}\right) \tag{8}$$

The sum rate per cell is given by

$$SR = \sum_{k=1}^{K} R(k) \text{ (bits/s/Hz)} \tag{9}$$

3.1.1 *Maximum Ratio Combining (MRC):*

The achievable rate for Rician channel in MRC receiver can be obtained by replacing $\hat{d}_k = \hat{g}_k$ in the Eq.(8) and is given by

$$R_{MRC-PCSI} = \log_2\left(1 + \frac{P_u\left|g_k\right|^2}{P_u \sum_{i=1, i\neq k}^{K}\left|g_k^H g_i\right|^2 + \left\|g_k\right\|^2}\right) \tag{10}$$

The Eq.(10) can be approximated as

$$R_{MRC-PCSI-Approx} = \log_2\left(1 + \frac{P_u \alpha_k\left[2NR_k + N + N^2\left(R_k+1\right)^2\right]}{P_u\left(k_n+1\right)\sum_{i=1}^{K}\alpha_i \Delta_1 + N\left(R_k+1\right)^2}\right) \tag{11}$$

where

$$\Delta_1 = \frac{\left[R_k R_i \varphi_{ni}^2 + N\left(R_k + R_i\right) + N\right]}{\left(R_i+1\right)} \text{ and}$$

$$\varphi_{ni}^2 = \frac{\sin\left(\frac{N\pi}{2}\left[\sin\left(\theta_k\right) - \sin\left(\theta_i\right)\right]\right)}{\sin\left(\frac{\pi}{2}\left[\sin\left(\theta_k\right) - \sin\left(\theta_i\right)\right]\right)}$$

3.1.2 *Zero-Forcing Receiver (ZF):*

The estimate of the channel using ZF receiver is given as

$$D = G(G^H G)^{-1} \tag{12}$$

With ZF, $D^H = (G^H G)^{-1} G^H$ or $D^H G = I_K$. The achievable rate of ZF receiver in Rician channel can be calculated by substituting the k^{th} column of $A^H G = I_K$ i.e. $d_k^H g_i = \delta_i$ in Eq.(8) and is given by:

$$R_{ZF-PCSI} = E\left\{\log_2\left(1 + \frac{P_u}{\left[\left(G^H G\right)^{-1}\right]_{kk}}\right)\right\} \tag{13}$$

The achievable rate of any user k for ZF receiver can be approximated as

$$R_{ZF-PCSI-Approx} = \log_2\left(1 + \frac{P_u \alpha_k\left(M-K\right)}{\left[\hat{\Sigma}^{-1}\right]_{kk}}\right) \tag{14}$$

where

$$\hat{\Sigma} = \left(F + I_N\right)^{-1} + \frac{1}{M}\left[F\left(F+I_N\right)^{-1}\right]^{0.5} \bar{H}^H \bar{H}\left[F\left(F+I_N\right)^{-1}\right]^{0.5}$$

3.2 IMPERFECT CSI

The CSI is not known and it is to be estimated in real scenario. Considering Imperfect CSI for Rician model, it is assumed that $\bar{G}$ and F are known at the BS and at the user to make the analysis simple. Then, the estimate of G can be given by

$$G = \bar{G}\left(\frac{F}{F+I_K}\right)^{0.5} + \hat{G}_r\left(\frac{1}{F+I_K}\right)^{0.5} \tag{15}$$

where $\hat{G}_r$ is the random component estimate. The estimation of channel can be done with the assistance of uplink pilots. Let the number of symbols used for uplink training be τ per coherence time interval T [12]. All the users send the pilot sequences of same length τ at a time and it can be represented by $\tau \times K$ matrix $\sqrt{P_P}\varphi$ that satisfies $T^H T = I_K$ where $T = \varphi[(F+I_K)^{-1}]^{0.5}$ and $P_P = \tau P_u$ is the pilot power. The noisy pilot matrix received at the BS is given by:

$$Y_P = \sqrt{P_P}\varphi^T + W \tag{16}$$

where W is an $N \times \tau$, AWGN matrix with independent and identically distributed CN(0,1). since the LOS components is assumed to be known and it can be removed and substituting the remaining terms into Eq.(16) then the received pilot matrix becomes:

$$Y_{P,r} = \sqrt{P_P} G_r\left(\frac{1}{F+I_N}\right)^{0.5}\varphi^T + W \tag{17}$$

This can be simplified as

$$Y_{P,r} = \sqrt{P_P} G_r Q^T + W \tag{18}$$

The estimation of the Channel is done to estimate G_r from $Y_{p,r}$. The MMSE estimate of Rician fading channel $\hat{G}_r$ from $Y_{p,r}$ is obtained as:

$$\hat{G}_r = \frac{1}{\sqrt{P_p}} \cdot Y_{P,r} Q * \tilde{B} = \left(G_r + \frac{1}{\sqrt{P_p}} U\right)\tilde{B} \tag{19}$$

where $\tilde{B} = \left(\frac{1}{P_p} D^{-1} + I_K\right)$ and $U \triangleq WF^*$

Let $E = \hat{G} - G$ be the error in this estimation and $\hat{D}$ denote the estimate of receiver that depends on $\hat{G}$. By multiplying the received signal with the conjugate- transpose of the *D*:

$$\hat{r} = \hat{D}^H y \tag{20}$$

$$\hat{r} = \hat{D}^H \left(\sqrt{P_u} \bar{G} x - \sqrt{P_u} E x + w \right) \tag{21}$$

The k^{th} user estimated signal is given by

$$\hat{r}_k = \sqrt{P_u} \hat{a}_k^H \hat{g}_k x_k + \sqrt{P_u} \sum_{i \neq k}^{K} \hat{a}_k^H \hat{g}_1 x_i - \sqrt{P_u} \sum_{i=1}^{K} \hat{a}_k^H \varepsilon_i x_i + \hat{a}_k^H w \tag{22}$$

In the Eq.(22), $\hat{g}_i$ and ε_i represents the i^{th} columns of $\hat{G}$ and *E*. The desired user signal is given by the first term and the remaining three terms represents unwanted user interference. The variance of the channel estimation error is given by

$$E\left\{ \left| [\varepsilon]_{nk} - E[\varepsilon]_{nk} \right|^2 \right\} = \frac{P_u \alpha_k}{(1 + P_P \alpha_k)(R_k + 1)} \tag{23}$$

where *n*=1 to *N* and *k*=1 to *K*. Then the achievable rate for user ‘k’ is represented as

$$R = \log_2 \left(1 + \frac{P_u \left| \hat{d}_k^H \hat{g}_k \right|^2}{P_u \sum_{i=1, i \neq k}^{K} \left| \hat{d}_k^H \hat{g}_i \right|^2 + P_u \left\| \hat{d}_k \right\| \sum_{i=1}^{K} \frac{\alpha_i}{(\tau P_u \alpha_i + 1)(R_k + 1)} + \left\| \hat{d}_k \right\|^2} \right) \tag{24}$$

3.2.1 Maximum Ratio Combining:

By replacing $\hat{a}_k = \hat{g}_k$ for MRC in the Eq.(24) then achievable rate of MRC receiver for Rician channel is

$$R_{MRC-PCSI} = \log_2 \left(1 + \frac{P_u \left| \hat{g}_k \right|^4}{P_u \sum_{i=1, i \neq k}^{K} \left| \hat{g}_k^H \hat{g}_1 \right|^2 + P_u \left\| \hat{g}_k \right\|^2 \sum_{i=1}^{K} \frac{\alpha_i}{(\tau P_u \alpha_i + 1)(R_k + 1)}} \right) \tag{25}$$

The achievable rate for MRC receiver can be approximated as

$$R_{MRC-PCSI-Approx} = \log_2 \left(1 + \frac{P_u \alpha_k \left[2M^2 R_k^2 + \left(2MR_k + 2M^2 R_k\right)\eta_k + \left(M + M^2\right)\eta_k^2 \right]}{P_u (k_n + 1) \sum_{i=1}^{K} \alpha_i \Delta_2 + M P_u \alpha_k \frac{(R_k + \eta_k)}{(1 + P_p \alpha_k)} + M (R_k + 1)(R_k + \eta_k)} \right) \tag{26}$$

where

$$\Delta_2 = \frac{\left[R_k R_i \varphi_{ni}^2 + N \eta_k (R_i + 1) + N R_k \right]}{(R_i + 1)}$$

3.2.2 ZF receiver:

The interference can be reduced to zero using Zero-forcing receiver by making $\hat{d}_k^H \hat{g}_l = \delta_{ki}$. Substituting this in Eq.(24), the SINR is

$$SINR = \frac{P_u}{\sum_{i=1}^{K} \frac{P_u \alpha_i}{\tau P_u \alpha_i + 1} \left[\left(\hat{G}^H \hat{G} \right)^{-1} \right]_{kk}} \tag{27}$$

The achievable rate for Rician channel using ZF receiver is

$$R_{ZF} = \frac{P_u}{\sum_{i=1}^{K} \left(\frac{P_u \alpha_i}{(\tau P_u \alpha_i + 1)(k_i + 1)} + 1 \right) \left[\left(\hat{G}^H \hat{G} \right)^{-1} \right]_{kk}} \tag{28}$$

The achievable rate for ZF receiver can be approximated as

$$R_{ZF-Approx} = \log_2 \frac{P_u \alpha_k (N - k)}{\sum_{i=1}^{N} \left(\frac{P_u \alpha_i}{(\tau P_u \alpha_i + 1)(k_i + 1)} + 1 \right) \left[\hat{\Sigma}^{-1} \right]_{kk}} \tag{29}$$

where

$$\hat{\Sigma} = \Lambda + (F + I_N)^{-1} + \frac{1}{M} \left[F (F + I_N)^{-1} \right]^{0.5} \bar{H}^H \bar{H} \left[F (F + I_N)^{-1} \right]^{0.5}$$

and Λ is a diagonal matrix consisting of

$$\left[\frac{\eta_1}{R_1 + 1}, \ldots, \frac{\eta_2}{R_2 + 1}, \ldots, \frac{\eta_K}{R_K + 1} \right]$$

The sum rate can be calculated as $SR = \sum_{k=1}^{K} R(k)$ where *R* is the achievable rate of user *k* for any receiver.

4. SIMULATION AND ANALYSIS

Computer simulations are performed in MATLAB R 2018a for verifying the results. A circular region with a radius of 1000m around the base station is considered. The number of users is taken as 10. The distance of each user from BS is taken randomly between 100m to 1000m and is represented as r_k. The large-scale fading is given by $\alpha_k = \frac{z_k}{\left(\frac{r_k}{r_h} \right)^v}$. In this, z_k represents log-normal shadowing that has standard deviation σ of 8dB and path loss factor ν of 3.8. The Monte Carlo trials used for averaging the plots is 5000. The simulated and approximated values of achievable rate vs. *N* with Perfect CSI and Imperfect CSI for MRC receiver and ZF receiver in Rician channel is shown in Fig.1 and Fig.2.

The approximated values and simulated results are closer for different configurations of BS antennas for two CSI cases. It can be observed that SE can be improved by using a more no. of antennas at the BS. It also shows that ZF performs better than MRC in both cases. The variation of achievable rate with respect to Rician factor is shown in Fig.3. For analysis, the approximation Eq.(11), Eq.(26), Eq.(14) and Eq.(29) are used because the approximated values are closer to simulated values. The assumed values for simulation are *K* = 10, P_u = 10dB, and *N* = 100. The sum rates for both CSI cases tending to the same value as R_k increases.

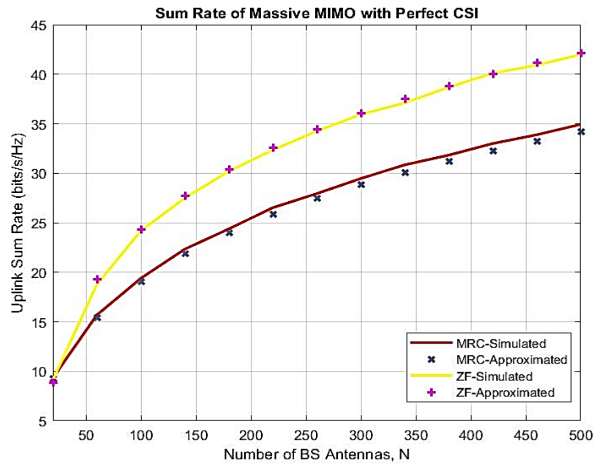

Fig.1. Sum Rate versus N antennas at BS with K = 10 users and transmit power P_u = 10dB for Perfect CSI

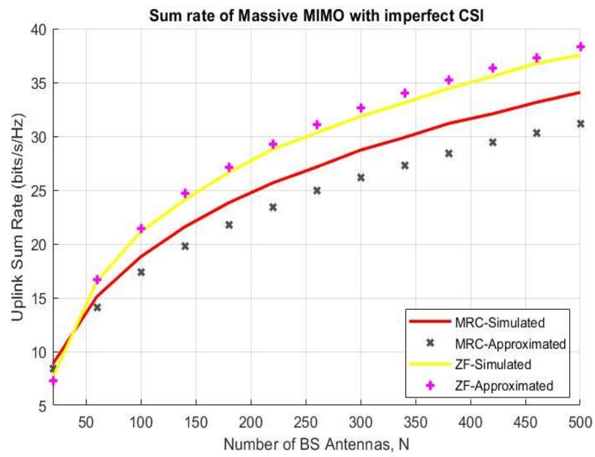

Fig.2. Sum Rate vs. N antennas at BS with K = 10 users and transmit power P_u = 10dB for Imperfect CSI

Also, the sum rates increases with rise in R_k, except for ZF perfect CSI case because the performance of ZF receivers can be badly limited if the channel matrix is ill-conditioned [13]. The effect of inter-user interference and error in estimation is very less for this case. As R_k increases, the channel matrix will become same as that of $\bar{H}$, where the singular values are having a larger spread. Then, the condition number of $\bar{H}$ attains very high values leads to ill-conditioned. But the estimation error is the main limiting factor for the case of ZF with imperfect CSI. As the value of R_k improving, the channel estimation becomes more robust since the random quantities become deterministic in nature. The two cases of MRC receivers also follow the same. The same holds true for both cases of MRC receivers, where a higher Rician K-factor reduces the effects of inter-user interference.

The Uplink Sum rate versus the number of users in a cell is shown in Fig.4. The Sum rate rises with the increase in no. of users as the channel vector between users become orthogonal and results in elimination of interference between them. The point at which the sum rate is maximum is taken as the optimal number of users in ZF receivers. The Sum rate increases up to the optimal number of users because of spatial diversity and multiplexing gain. Beyond the optimal number of users, the sum rate reduces because of the dominant inter user interference. So, to achieve the maximum sum rate, the value of K i.e. number of users should be carefully designed.

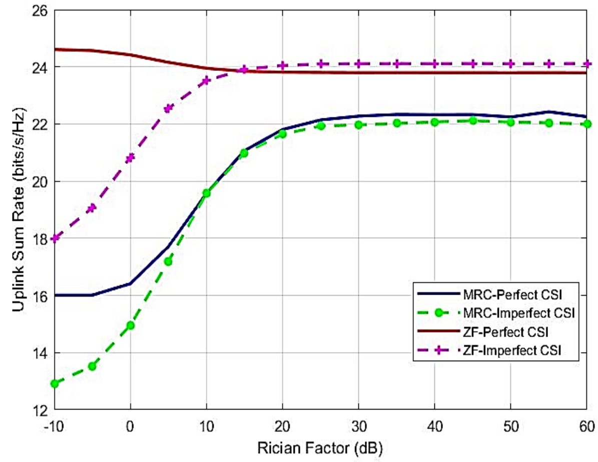

Fig.3. Sum Rate versus Rician Factor (R_k) for MRC and ZF receivers

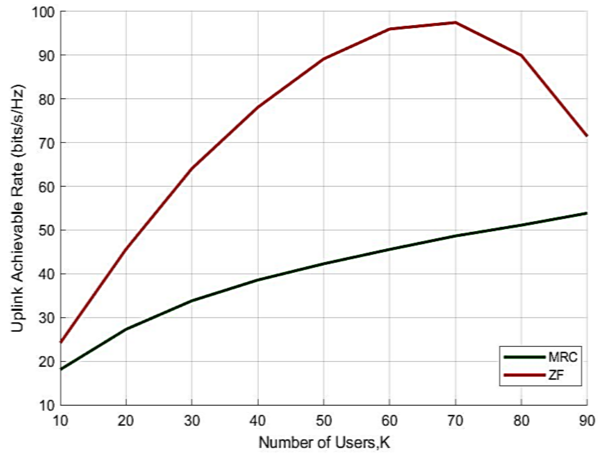

Fig.4. Sum Rate versus Number of Users K for MRC and ZF receivers

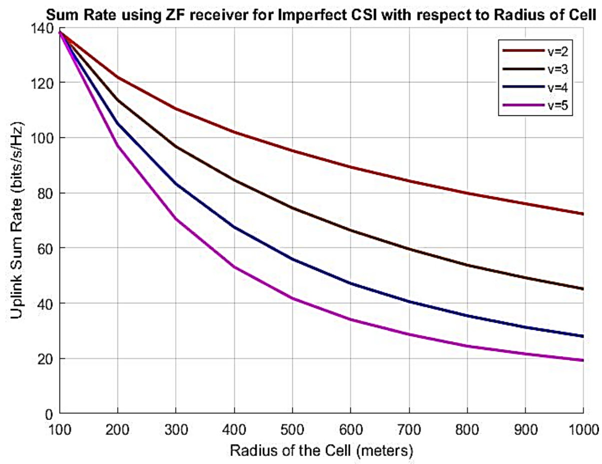

Fig.5. Sum Rate versus Radius of the Cell for ZF receivers

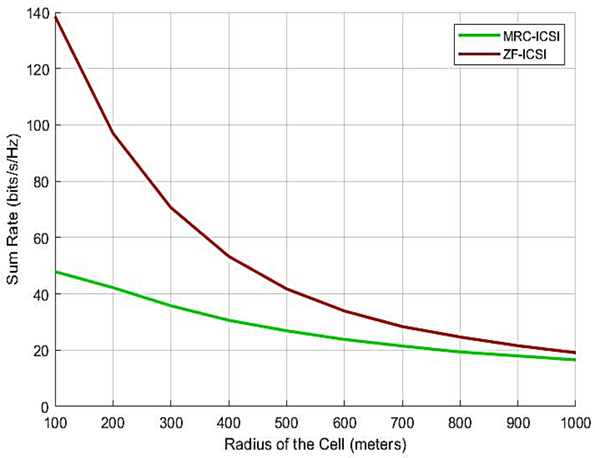

Fig.6. Sum Rate versus Radius of the Cell for MRC and ZF receivers

The influence of large-scale fading parameter on SE with respect to radius of the cell for different values of path loss factor for ZF receiver is shown in Fig 5. The chosen values of PLF are 2 for free space, 3 for urban cell radio, 4 for urban shadowed cell radio and 5 for obstructed in building. Irrespective of PLF, the SR is maximum when the user is at 100m and it reduces as the user is moving away from the base station. The reduction in SR is also higher for higher values of PLF indicating that the effect of PLF is more as the user is away from the BS. The values of SR using ZF receiver are more than the values of SR using MRC receiver in and that comparison is shown in Fig 6.

5. CONCLUSION

The Massive MIMO performance over Rician fading channel is analyzed in this paper. The expressions of uplink achievable rates and its approximations for MRC and ZF receiver are derived in both the Perfect and Imperfect CSI cases. From the simulations, it is observed that the sum rate increases when both the Rician factor and the number of BS antennas become large. This proves that Massive MIMO systems are more appropriate for using in a Rician fading environment, particularly when there is strong LOS component. The Sum Rate of these receivers' increments as the number of BS antennas N increments. The effect of Path Loss Factor is more when the position of the users far from the BS irrespective of MRC or ZF receiver. The performance of ZF receiver is better compared to MRC receiver.

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