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WAVELET BASED METHOD FOR REMOVING GAUSSIAN NOISE BY REPRESENTING IMAGES IN HEXAGONAL LATTICE

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Abstract
Human Visual System (HVS) is for acquiring, processing, analyzing, and understanding the images and this is a natural process. The image acquisition, processing, analysing and understanding the images are electronically duplicated in Computer vision. At present the hardware available for acquiring, processing and displaying is based on square pixel. But, retina of the human eye closely resembles a hexagonal grid space. So if we could able to represent the image in hexagonal domain, the computer vision will be as close to human vision. Keeping this in mind, we proposed a wavelet based image de-noising on hexagonally re-sampled images. In addition to the biological inspiration the hexagonal representation has many other advantages. By considering these advantages of hexagonal representation, in this work we proposed a wavelet based image denoising scheme in hexagonal grid space. For analysing and comparing result obtained in hexagonal grid with square grid image, Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) is used.

Keywords:
Gabor Filter, Wavelets, Interpolation, Hexagonal Grid

1. INTRODUCTION

One of the major challenges in the field of image processing and computer vision is image de-noising. The goal of the image de-noising is to estimate the original image by suppressing noise from a noisy image. Usually the noise is introduced in the images during image acquisition (digitization) or during image transmission. Sensor temperature and light levels are major factors that affecting the amount of noise in the images, while acquiring images with CCD camera. Images are also corrupted during transmission of images. The principal reason of noise is due to interfering in the channel used for image transmission. A noisy image can be modeled as shown in Eq.(1)

$$A(x, y) = I(x, y) + N(x, y) \tag{1}$$

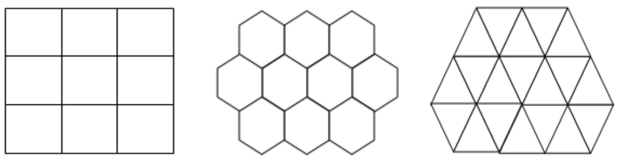
where, $I(x,y)$ is the original image, $N(x,y)$ is the noise and $A(x,y)$ is the resulting noisy image. Typical noises usually present in an image are uniform or quantization noise, salt & pepper noise, Gaussian noise, Gamma noise and Rayleigh distribution noise [25]. In this work we concentrated on eliminating Gaussian noise present in an image.

Gaussian noise is an example of statistical noise. The probability density function (PDF) of this type of noise is same as that of the normal distribution (Gaussian distribution), hence it is known as Gaussian noise. That is in Gaussian noise the values taken by the noise are Gaussian-distributed [28]. If the values are identically distributed and statistically independent, then it is a special case and it is known as white Gaussian noise. The probability density function 'P' of a Gaussian random variable 'z' is given by [28].

$$P(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \tag{2}$$

In this equation the parameter μ represents the mean value and σ represents the standard deviation of the distribution. In the case of images z represents the grey level. A lot of algorithms have been proposed to improve the performance of image de-noising scheme.All these works are based on the images on rectangular Grid. The work proposed in this paper is based on hexagonal sampling grid to remove the Gaussian noise in an image as it has many advantageous.

According to [10] there exist only three possible regular tessellation schemes to tile a plane without overlapping among the samples and gaps between them. They are the tessellation with squares, with hexagons, and with regular triangles (Fig.1).



(a) (b) (c)

Fig.1. Three Scheme of Regular Tessellation (a) Squares (b) Hexagons (c) Triangles

The Fig.1(a) is the square, which is familiar and usual because it is aligned with the standard Cartesian axes, which helps to make operations simple. The Fig.1(c) illustrates the triangular case, which yields a denser packing than the square case. This means that more information is contained in the same area of the image. The tessellation in Fig.1(b) is the hexagonal case. It is believed to be the most efficient tessellation scheme among them because of its advantages like smaller quantization error [11, 12, 13, 14], Consistent Connectivity [15], pixel is equidistantly adjacent to their six neighbours along the six sides of the pixels and Grater Angular Resolution[1]. In addition to these advantages the primary motivation behind using a hexagonally re-sampled image is that retina of the eye closely resembles a hexagonal grid space [16] (Fig.2). So we can obtain natural behaviour to realize the computer vision by using the hexagonally sampled images.

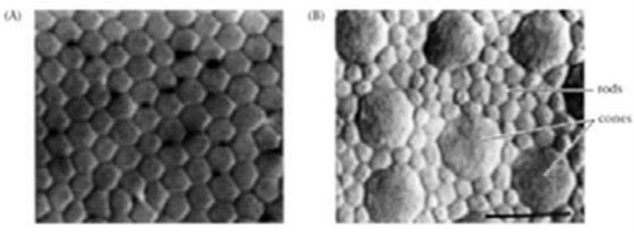


Fig.2. Arrangement of Rods and Cones in Retina

The main limitation in the use of hexagonal image structure is due to lack of hardware for capturing and displaying hexagonal-based images. So conversion has to be done from square image to hexagonal image before the hexagonal-based image processing. Resampling is the process of transforming a discrete image which is defined at one set of coordinate locations to a new set of coordinate points i.e., converting from rectangular to hexagonal grid. There have been several ways to simulate a hexagonal grid on a regular rectangular grid [27]. The common simulations methods are Mimic Hexagonal Pixels Using Square Pixels, Pseudo Hexagonal Pixel [17], Mimic Hexagonal Structure [18, 19] and Virtual Hexagonal Structure [19].

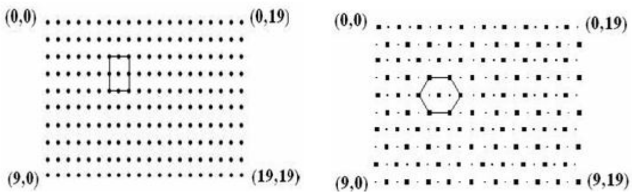
A method of simulating hexagonal grid on rectangular grid by alternatively suppressing rows and columns of the existing rectangular grid was proposed by E.G. Rajan et al. [20]. According to [20], “all the pixels of the rectangular grid that do not have any correspondence to those in their hexagonal counterparts are suppressed. By suppressing it means to assign the value of these pixels to be zero. While processing this sub sampled image the suppressed pixels are not considered in computation”. The sub sampled hexagonal grid is shown in Fig.3.

Another method of simulating a hexagonal grid is half pixel shift method. This method was proposed by Senthil Periasamy [21]. In half-pixel shift method, for each odd line, find the midpoint between two adjacent pixels by simple linear interpolation (i.e., mid = (left + right)/2). Discard the left and right, keeping only the mid values. This gives us a hexagonal mapping from a regular square or rectangular [21, 29]. This gives us a hexagonal mapping from a regular square or rectangular grid as shown in Fig.4.

In this paper we proposed a new method for de-noising by combining Wavelet and Hexagonally Re-sampled Image. By combining wavelet and processing of image in hexagonal grid will give better performance, because hexagonal wavelet includes the advantage of hexagonal grid along with the wavelet.

2. PROPOSED METHODOLOGY

The method proposed in this work for image de-noising is performed in the noisy image which is represented in hexagonal lattice. Since there is no hardware for acquiring and displaying the images in hexagonal grid, conversion has to be done from square to hexagonal lattice before hexagonal based image processing. In this work we used alternate pixel suppressal method for hexagonal representation. There may be loss in image quality during resampling of image from square to hexagonal lattice. So we used Gabor filter based interpolation method for maintaining the image quality. The block diagram for performing the proposed method is shown in Fig.5.



(a) (b)

Fig.3. (a) Rectangular Grid and (b) Simulated Hexagonal Grid obtained using alternate Pixel Suppression

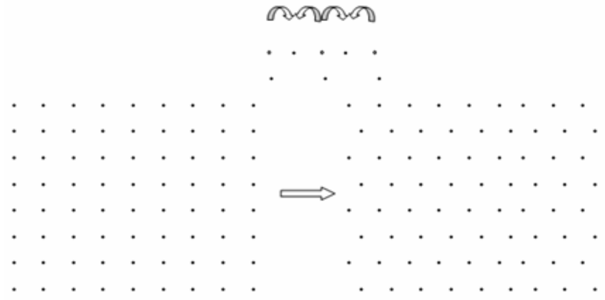


Fig.4. (a) Rectangular Lattice (b) Hexagonal Lattice using Half-Pixel Shift method

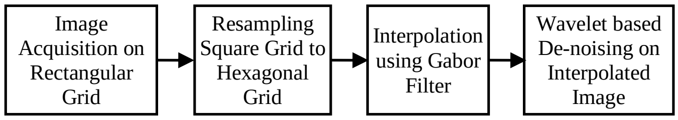
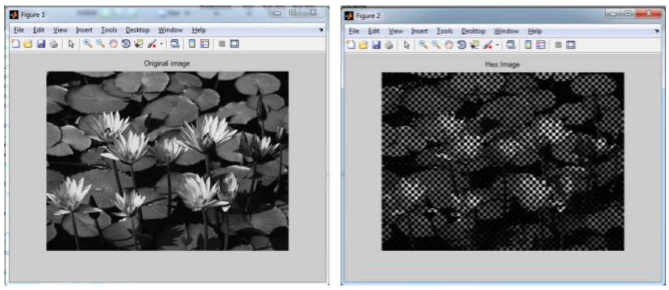


Fig.5. Block Diagram for the Proposed Method

Alternate pixel suppressal method, which is explained in the introduction part, is used in this work for hexagonal resampling to obtain image based on hexagonal lattice. The Fig.6 shows the original image and its hexagonal lattice representation using alternate pixel suppressal method.

During the resampling process of image from rectangular lattice to the hexagonal lattice, it was observed that there is a considerable loss in image quality. So, in order to maintain the quality we need to use the interpolation techniques. Image reconstruction through interpolation is a routine task in image processing during all transformation that is made on an image. In this work Gabor filter is used for image interpolation.



(a) (b)

Fig.6. (a) Original Image (b) Hexagonal Image Representation using Alternate Pixel Suppressal method

Gabor filter [22] is the only filter with orientation selectivity that can be expressed as a sum of two separable filters. In the spatial domain, a 2D Gabor filter is a Gaussian kernel function modulated by a sinusoidal plane wave. The following equations represent the 2D-Gabor function which was proposed by Daugman [23]. The Eq.(3) and Eq.(4) represent the real and imaginary part of the function respectively [23],

$$g\lambda\theta\sigma_{\gamma\varphi}(x,y)=\exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda}+\varphi\right) \qquad (3)$$

$$g\lambda\theta\sigma_{\gamma\varphi}(x,y)=\exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\sin\left(2\pi\frac{x'}{\lambda}+\varphi\right) \qquad (4)$$

where, $x' = x\cos\theta + y\sin\theta$ and $xy' = -x\sin\theta + y\cos\theta$.

The parameters used in these equations are as follows:

- 'x' and 'y' - position of x and y coordinate of the image
- sigma(σ) - The standard deviation (σ) of the Gaussian factor determines the size of the receptive field
- gamma(γ) - aspect ratio specifies the ellipticity of the Gaussian factor. The value of γ vary in a limited range of $0.23 < \gamma < 0.92$.
- lambda(λ) - The wavelength. $1/\lambda$ the spatial frequency of the cosine factor.

The interpolation is done by using Gabor filter using following manner. Hexagonal sampled grid has 3directional symmetry in 0^0, 60^0 and 120^0 orientations. Due to these three axes of symmetry of hexagonal grid, we select three different orientation of Gabor filter along in 0^0, 60^0, and 120^0 and the filtering is done in these three orientations. The three filtered image is then superimposed to get the interpolated image [25]. The Fig.7 shows the image after Interpolating using Gabor Filter. The interpolated image is then used for wavelet based de-noising.

Fig.7. Interpolated Image Using Gabor Filter

2.1 PROPOSED SCHEME FOR IMAGE DE-NOISING

Different De-noising schemes exist nowadays. In existing methods, de-noising is performed on normal images which are based on rectangular grid. In this work, instead of using normal rectangular grid, we used hexagonally resample image, which is interpolated by using Gabor filter. So by combining wavelet and processing of image in hexagonal grid we achieved a better result than conventional method, because hexagonal wavelet includes the advantage of hexagonal grid along with the wavelet. The procedure used for de-noising is stated below.

1. Noisy image is first resampled using alternate pixel suppressal method to get image in hexagonal lattice.
2. Perform interpolation using Gabor filter to improve the quality of image represented in hexagonal lattice.
    a. Hexagonal sampled grid has directional symmetry in 0^0, 60^0 and 120^0 orientations
    b. Select three different orientation of Gabor filter along 0^0, 60^0, and 120^0
    c. Perform filtering in these three orientations
    d. The three filtered image is then superimposed to get the interpolated image.
3. Perform wavelet decomposition of image obtained after interpolation. (We get LL, LH, HL & HH components).
4. Perform de-noising using the MATLAB function 'wdencmp'.

3. RESULTS AND DISCUSSIONS

Wavelet based image de-noising is performed on rectangular as well as hexagonally resampled images. In order to maintain the quality of the image represented in hexagonal lattice obtained by using alternate pixel suppressal method, we used interpolation technique based on Gabor filter. Since hexagonal sampled grid has 3 directional symmetry in 0^0, 60^0 and 120^0 orientations, we select three different orientation of Gabor filter along in 0^0, 60^0, and 120^0 and the filtering is done in these three orientations. The three filtered image is then superimposed to get the interpolated image. The Fig.8 shows the superimposed Gabor Kernel in 0^0, 60^0, and 120^0 orientation. Finally de-noising based on wavelet is applied to the interpolated image. Application of wavelets on images results in the decomposition of image to various frequency components. The Fig.9 shows the wavelet decomposition on hexagonally sample image. We performed this method on several images and the result is found better in hexagonal images.

For the performance analysis, Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR), two commonly used measures for quantifying the error between images, are used. MSE indicate the average difference of the pixel throughout the image. If MSE is higher the difference of the pixel between the original and the processed image is also higher. The MSE between two images P and Q is defined by,

$$MSE = \frac{1}{N}\sum_{i}\sum_{j}\left(Q_{ij} + P_{ij}\right)^2 \tag{5}$$

where, the sum over i and j denotes the sum over all pixels in the images, and N is the number of pixels in each image. The PSNR between two images is given by,

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \tag{6}$$

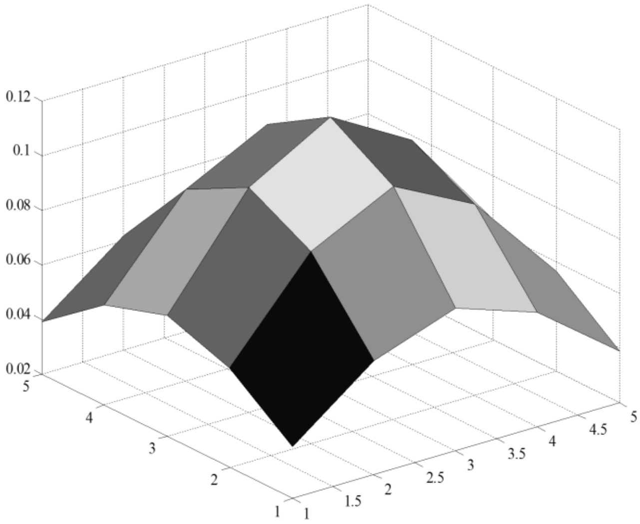


Fig.8. Superimposed Gabor Kernel in 0^0, 60^0 and 120^0 orientation

Performance measures in terms of Peak Signal-to-Noise ratio (PSNR) and Mean Squared Error were analyzed and listed in Table.1, which shows marginal improvement in PSNR and MSE for hexagonally sampled Image. Also we can see that as threshold value increases, hexagonal sampling gives better results. The Fig.10 and Fig.11 shows the noisy and de-noised conditions of two different images on rectangular as well as hexagonal domain.

The number of pixels used for representing same amount of information is less in the case of hexagonal representation. So, when we convert a noisy image from rectangular domain to hexagonal domain, some amount of noise is reduced. After wavelet based de-noising, the amount of noise reduction is much better than that of rectangular domain. The wavelet transform used is 'Daubechies' and we tested more than 50 images with different noise levels (σ). The results of two different images are summarized in the Table.1.

(a) (b) (c)

Fig.10. (a) Noisy image (water.jpg) (b) De-noised image on Rectangular Domain (c) De-noised image on Hexagonal Domain

(a) (b) (c)

Fig.10. (a) Noisy image (girl.jpg) (b) De-noised image on Rectangular Domain (c) De-noised image on Hexagonal Domain

4. CONCLUSION

We simulated the proposed method and it has been found that the wavelet based de-noising on hexagonal domain is effective than conventional images. MSE and PSNR are found to be better in hexagonal domain. Combining wavelet and processing of image in hexagonal grid we achieved a better result than conventional method, because hexagonal wavelet includes the advantage of hexagonal grid along with the wavelet. As a future work, different algorithms for conventional images can be modified for hexagonal images and if it gives better results we can replace hexagonal structure for computer vision. Also it will give the missing corner stone of artificial human visual system.

Table.1. PSNR of de-noised images in Rectangular and Hexagonal domain for different noise level

Test Image	**Noise Level**	**PSNR**	
		Image in Rectangular Domain	**Image in Hexagonal Domain**
water.jpg	10	34.0000	38.7395
	20	31.3158	37.6269
	30	29.8968	36.2039
	40	29.1576	34.8422
	50	28.7073	33.5059
girl.jpg	10	35.1109	42.4189
	20	31.5584	41.4518
	30	30.0138	39.8926
	40	29.1672	37.9447
	50	28.7332	36.4542

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