

A SIMPLE BUT EFFICIENT SCHEME FOR COLOUR IMAGE RETRIEVAL BASED ON STATISTICAL TESTS OF HYPOTHESIS

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Abstract

This paper proposes a simple but efficient scheme for colour image retrieval, based on statistical tests of hypothesis, namely test for equality of variance, test for equality of mean. The test for equality of variance is performed to test the similarity of the query and target images. If the images pass the test, then the test for equality of mean is performed on the same images to examine whether the two images have the same attributes / characteristics. If the query and target images pass the tests then it is inferred that the two images belong to the same class i.e. both the images are same; otherwise, it is assumed that the images belong to different classes i.e. both the images are different. The obtained test statistic values are indexed in ascending order and the image corresponding to the least value is identified as same / similar images. The proposed system is invariant for translation, scaling, and rotation, since the proposed system adjusts itself and treats either the query image or the target image as sample of other. The proposed scheme provides cent percent accuracy if the query and target images are same, whereas there is a slight variation for similar, transformed.

Keywords:

Variance, Mean, Query Image, Target Image, Tests of Hypothesis

1. INTRODUCTION

Handling images, that is, manipulation, storing, analyzing, indexing, matching, retrieval, display etc., are very complicated when compared to that of text manipulation. So, it needs proper image database systems, which can support the aforesaid image manipulations. It is observed from the literature that remarkable progress has been made in both theoretical research and system development. However, still it is a challenging problem for the researchers in the area of visual data mining to design an automatic retrieval system, because real-world images usually contain complicated objects and colour information.

The content-based image retrieval (CBIR) system has attracted many researchers in recent years. In this system, the researchers concentrated on developing low-level global visual features namely colour properties, shape, texture, and spatial relationship etc., that are used as a query for the retrieval process [1]-[4]. These features are used to represent and index the images. Jing *et al.* [5] suggested that a single signature computed for the entire image cannot sufficiently capture the important features of individual objects and there is a gap between the visual features and semantic concepts of images. To overcome this problem, the region-based system is developed [6]-[10], which represents the focus of the user's perceptions of the image contents. In this system, the entire image is classified / segmented [11]-[16] into various regions according to the objects / structures present in the image, then the region-to-region comparison is made to measure the similarity between

two images [3], [12], [13]. This system requires one or several regions from the query image to start a query session. Automatic and precise extraction of image objects is still beyond the ability of the retrieval system available with the computer vision [17]. Therefore, the above system tends to partitioning of one object into several regions; none of them is representative for the semantic object. Edge oriented segmentation algorithms are available for both region-based and content-based retrieval systems. This type of techniques use features of local edges, and the edges are classified based on two factors such as orientation and correlation between neighboring edges [18]. The main drawback of this system is to determine which region to be used as feedback query image example for retrieval [5]. In this type of systems, when either the target image or the query image is rotated with a particular angle, the corresponding regions of both the images may not match exactly. Hence, it is not robust for the same image with different angles.

Su *et al.* [19] suggested that the content-based retrieval system yields results with low accuracy and slow response time, because there is a big gap between semantic concepts and low-level image features. A concept, relevance feedback is used to bridge the gap [20]-[22]. In [19], a new relevance feedback approach is proposed, which uses Bayesian classifier and treats positive and negative feedback images with different strategies. In this technique, the user has to provide positive and negative feedback images to improve the system's performance. Again the problem arises as discussed in [5], how to incorporate the negative and positive examples to refine the query and how to adjust the similarity measures according to the feedback [19]. The degree of relevance (to the query) of each of the n positive feedback images is determined by the user at each feedback iteration. Since, the positive feedback images are chosen by the user with the use of iterative technique, this technique demands computational complexity. Minka and Picard [23] proposed the FourEyes system, which also has two disadvantages: (i) it uses the region-to-region similarity measure; (ii) the re-clustering of all the features when a new image is added. Thus it is not very scalable [5]. Jing *et al.* [5] proposed a system, which requires high computational efforts because, it computes probabilistic interpretation and it has been used in region matching; region codebook is used; the SVM based classifier and clustering techniques are employed. Above all these things, it requires the positive and negative query image examples. Again the problem arises how to select the negative and positive image examples to refine the query. To avoid this problem, many researchers [9], [17], [24]-[27] have taken efforts to integrate the information from all the regions in an image and then the image-to-image similarity measure (integrated information) is used to compare the images. For instance, SIMPLicity system [17] uses an

integrated region matching as the similarity measure. In summary, the review highlights that the region-based technique again tends to the concept of the content-based retrieval by using the integrated information. The region-based technique also requires high computational efforts.

At low-level image analysis, colour plays important role because it is the most widely used visual features and is invariant to image size and orientation [28], [29]. In the proposed system, a colour image is segregated into red, green and blue (RGB) colour model that are not only represent the level of combination of colours in the pixel, but also represent the energy level and other features like textures, shape etc. For instance, the wall street bull 1 and wall street bull 2 images presented in Fig. 1 are seem to be almost same in terms of visual perception, but the individual RGB colour based graphical analysis clearly exposes that the two images slightly differ, which is demonstrated in Figs. 2 and 3. This is the main motivation of the proposed work to employ statistical tests of hypothesis for colour image retrieval.

In this paper, a simple but efficient technique is proposed for automatic retrieval of colour images based on the statistical tests of hypotheses such as test for equality of means and homogeneity of variances. In the proposed technique, mean and variance are used as representatives of both query and target images. The test for variance examines the strength (closeness) of relationship between the query and target images and it measures the linear dependency of the pixels within the image. The test for mean tests the equality of spectrum of energy of the query and target images.

The rest of the paper is organized as follows: Section 2 discusses the test for equality of mean and variance of the query and target images. Section 3 deals with the image indexing and retrieval. The experiments and results are demonstrated in Section 4. Finally Section 5 concludes with conclusion.

2. TEST STATISTIC FOR IMAGE MATCHING

In the proposed technique, first, the similarity of the query and target images are tested, if they passes the test then it can be proceeded to test the mean values of the two images; otherwise, the test is dropped. To test the similarity of the two images, the Bartlett's test is performed, and the test for equality of two means is employed to test whether the means of two images are equal or not. Here, either the query image or the target image is treated as sample, and other is treated as population. By testing the query and target images, it is inferred that whether there is variation and interaction among the pixels between the two images and within the image. If the means and variances pass the tests, then it can be concluded that the two images are same; otherwise, it is assumed that they differ.

2.1 TEST FOR SIMILARITY OF IMAGES

Generally, when capturing an image through camera (digital or analog) or scanning through scanner, there are many possibilities to include noise in the image. The inclusion of noise is a random process, which is independent and identically distributed Gaussian random variable. Let X be a random variable that represents the intensity value with additive noise of a pixel at location $(k, 1)$ in a colour image. Hence, an image is

assumed to be Gaussian Markov Random field [30], [31]. The pixel $X(k, 1) \in \mathfrak{R}^3$ is a linear combination of three colours, namely, red, green, and blue, i.e., $X(k, 1) = \{r(k, 1), g(k, 1), b(k, 1)\}$. The mean intensity value of each colour is represented by μ , and the variation is denoted by σ^2 . The normal density function of $X(k, 1)$ for each colour is given by

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right), -\infty < x, \mu < \infty, \sigma > 0 \quad (1)$$

The density function in Eq. (1) can be denoted as $N(x/\mu, \sigma^2)$ and the distribution law as $N(\mu, \sigma^2)$.

2.1.1. Test Statistic for Similarity of Images

Either the query or target image is treated as samples while the other is treated as population. To test whether the two images are same or not, first, the variation among the intensity values of the two images are tested i.e., $H_0: \sigma_q = \sigma_t$, where σ_q and σ_t represent the variation among the intensity values of the query and target images. To achieve this, the test for homogeneity of variances is employed i.e. Bartlett's test.

Hypotheses:

$$\begin{aligned} H_0: \sigma_q &= \sigma_t \quad (\text{Similarity}) \\ H_a: \sigma_q &\neq \sigma_t \quad (\text{Non-similarity}) \end{aligned}$$

Test Statistic: The Bartlett's test statistic (B) is defined in Eq.(2), which is applied to test whether the images are similar or not.

$$B = \frac{(N - k) \ln(S_p^2) - \sum_{i=1}^k (N_i - 1) \ln(S_i^2)}{1 + (1/(3(k-1))) \left(\left(\sum_{i=1}^k 1/(N_i - 1) \right) - 1/(N - k) \right)} \quad (2)$$

where, s_i^2 is the variance of the i^{th} group, N is the total sample size, N_i is the sample size of the i^{th} group, k is the number of groups and s_p^2 is the pooled variance. The pooled variance is the weighted average of the group variances and is defined in Eq.(3).

$$S_p^2 = \sum_{i=1}^k (N_i - 1) S_i^2 / (N - k) \quad (3)$$

The variances are judged to be same, if $B > \chi_{[\alpha, k-1]}^2$, where, $\chi_{[\alpha, k-1]}^2$ is the upper critical value of the Chi-square distribution with $(k-1)$ degrees of freedom at a significance level α .

2.1.2 Test for equality of Means

As discussed in the previous section, if the variations among the intensity values of the query and target images passed the test for homogeneity of variances, then it is proceeded to test the equality of means of the two images with the basic assumption that there is no variation among the intensity values between the query and target images. After observing the outcome of the two tests, it can be concluded that the two images are same or not. To achieve this, the test for equality of means i.e. t-test is performed on the images. The tests of hypotheses are assumed to be as follows.

Hypotheses:

$$\begin{aligned} H_0: \mu_q &= \mu_t \quad (\text{Similarity}) \\ H_a: \mu_q &\neq \mu_t \quad (\text{Non-similarity}) \end{aligned}$$

The test statistic (T) is defined in Eq.(4),

$$T = \frac{\bar{X}_q - \bar{X}_t}{S_p \sqrt{\frac{1}{N_q} + \frac{1}{N_t}}} \quad (4)$$

where $S_p^2 = \frac{(N_q - 1)s_q^2 + (N_t - 1)s_t^2}{N_q + N_t - 2}$ is pooled variance; N_q

and N_t are the number of pixels in the query and target images;

\bar{X}_q and \bar{X}_t are the mean values of the query and target images;

s_q^2 and s_t^2 are the variation among the intensity values of the query and target images.

Critical region: It is concluded that the query and target images are same, if $T < -t_{(\alpha/2, \nu)}$ or $T > t_{(\alpha/2, \nu)}$, where $t_{(\alpha/2, \nu)}$ is the critical value of the t-distribution with $\nu = (N_q + N_t - 2)$ degrees of freedom at level of significance $\alpha/2$; otherwise it is concluded that the images are different.

3. IMAGE INDEXING AND RETRIEVAL

The query image is matched with the images in the image database by adopting the test statistic discussed in the previous section. The target images are identified and marked in the image database based on the output obtained from the tests. According to the users' requirements of the number of same / similar images, they can fix the level of significance of the tests.

The proposed system matches and retrieves the same image (target and query images are same) from the image database for 0% level of significance (100% accuracy-same images); almost same images for 1% to 2% (98% accuracy) level of significance; similar images for the level of significance from 3% to 5%; related images for the level of significance from 6% to 8%. The selected images are indexed (ranked) from lowest to highest i.e. in ascending order based on the test statistic values obtained. The image which corresponds to the first value in the indexed list is marked as target image and is retrieved from the image database.

4. EXPERIMENTS AND RESULTS

Different types of images have been considered, that is, textured images, which have been taken from Brodatz Album, some images considered from Corel image database. Based on these image collection; an image database has been constructed with more than 1050 images. For sample, some of them have been presented here. First, similar types of images with size 128 pixels width and 96 pixels height have been considered for the

experiment, which are presented in Fig. 1. Using Eq. (2), B is computed, and T is computed based on Eq. (4). The query and target images are considered in various combinations, and the computed B and T values are indexed in ascending order. The image, which corresponds to the least values (topmost values in the indexed list) of B and T, is identified as required target image, and then it is displayed. The obtained retrieval results of the various combinations of the query and target images are presented in Table.1 for Wall Street bull images. From the results obtained it is observed that the proposed system produce zero for both B and T, if the query and target images are same while it produces other than zero for similar or different images. While the wall street bull 1 image is given as input query image to the system and the level of significance is fixed at 0%, the proposed system produces the retrieval result with wall street bull 1 image only; for the level of significance at 1% and 2%, the system gives the retrieval results are wall street bull 1 and wall street bull 2 images; for the level of significance from 3% to 5%, the system yields the retrieval results wall street bull 1, wall street bull 2 and wall street bull 3 images.

In the proposed system, the query and target images are assumed to be Gaussian distributions with mean μ and variance σ^2 . The μ and σ^2 are global representations of the two images. The mean represents measure of central tendency whereas the variance represents the spatial interaction / correlation among the colour pixels in the image. The mean vector represents the first moment of the image data and the variance represents the second moment. As discussed in Section 1, these two moments are sufficient to represent the features in the images. Since the global features are extracted from the distributional differences of query and target images, and the sizes of those images are same with different angles (i.e. the two sample points are same with different angles).

4.1 INVARIANT FOR SCALING AND ROTATION

Since either the query image or target image is treated as sample of other, the proposed retrieval system is invariant for scaling. For example, let us consider the images (a) and (d) in Fig. 4. Image (d) is proportionately scaled down from image (a), it means that only the size (number of pixels) is reduced proportionately; otherwise there is no loss or change in the properties of the images (virtually the sample points are same).

The proposed system adjusts itself whereas the sizes of the query and target images differ. To prove this, several query and target images with different sizes have been experimented, which are given in Figs. 1, 4 and 5. Eq. (2) is used to calculate B, and Eq. (4) is employed to compute T. The calculated B and T values for the images in Fig. 4 are indexed in ascending order and they are given in Table.2. The image, which corresponds to the least values, (topmost value in the indexed list) of B and T is identified as required target image, and then it is retrieved.



Fig.1. Wall Street Bull - row 1: actual image with size 75×100 ; row 2: actual image with size 96×128 ; row 3: images in row 2 are rotated clockwise with 90 degrees; row 4: images in row 2 are rotated clockwise with 180 degrees

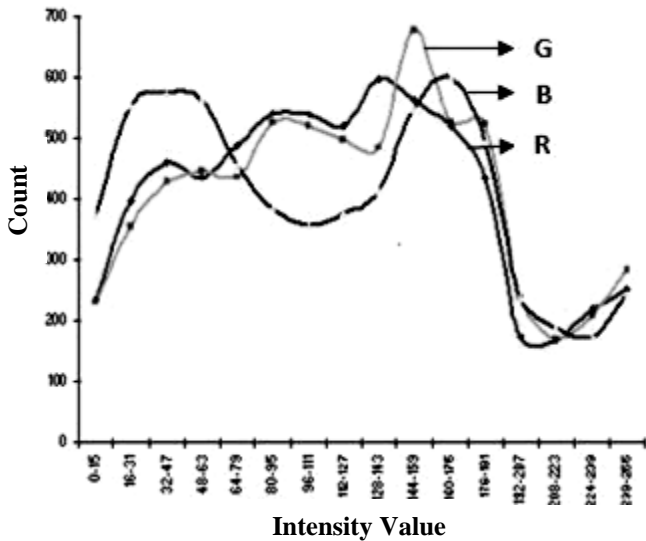


Fig.2. Line Graph for RGB colours of the Wall Street Bull Image - 1

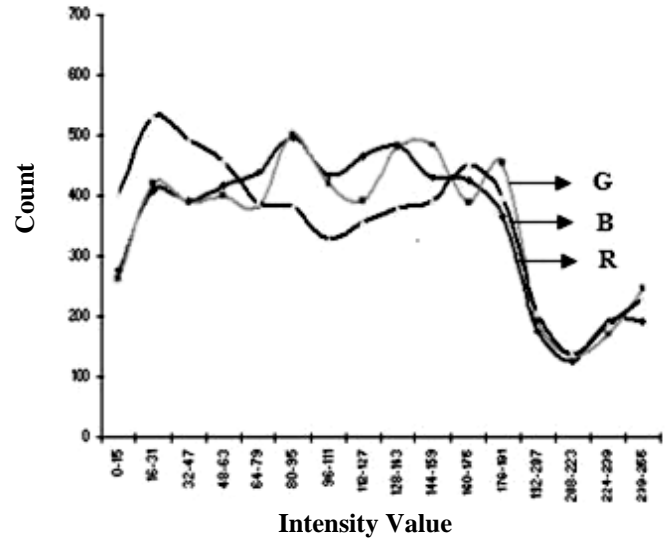


Fig.3. Line Graph for RGB colours of the Wall Street Bull Image - 2

Table.1. Wall Street Bull – Query Image vs. Target images

Vs.	Bull-1		Bull-2		Bull-3		Bull-4		Bull-5	
	B	T	B	T	B	T	B	T	B	T
Bull-1	0.0000	0.0000	0.8021	0.0431	0.9953	0.0613	1.2572	0.3060	1.6053	0.2594
Bull-2	0.8021	0.0431	0.0000	0.0000	0.8661	0.1423	0.9041	0.1938	1.1498	0.2450
Bull-3	0.9953	0.0613	0.8661	0.1423	0.0000	0.0000	0.8831	0.1708	1.0956	0.2564
Bull-4	1.2572	0.3060	0.9041	0.1938	0.8831	0.1708	0.0000	0.0000	0.9867	0.1978
Bull-5	1.6053	0.2594	1.1498	0.2450	1.0956	0.2564	0.9867	0.1978	0.0000	0.0000

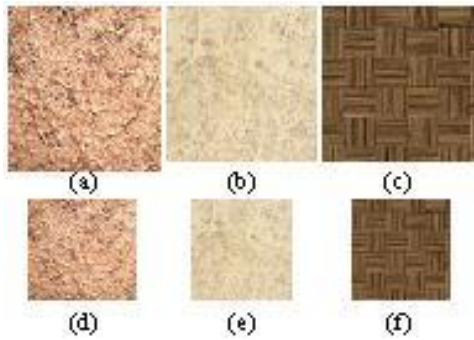


Fig.4. Texture images – row 1: actual images; row 2: scaled down images

In the proposed system, the query and target images are assumed to be Gaussian distributions with mean μ and variance σ^2 . The μ and σ^2 are global representations of the two images. The mean represents measure of central tendency whereas the variance represents the spatial interaction / correlation among the colour pixels in the image. The mean vector represents the first moment of the image data and the variance represents the second moment. As discussed in section 1, these two moments are sufficient to represent the features in the images. Since the global features are extracted from the distributional differences of query and target images, and the sizes of those images are same with different angles (i.e. the two sample points are same with different angles), the proposed system is invariant for rotation.

To prove the proposed system is sufficiently efficient for rotation invariant, several images considered for the experimentation and that are rotated clockwise with different angles, viz. 90° , 180° . The procedures used in sub-Sections 2.1.1 and 2.1.2 are adopted here to compute B and T values for the images presented in Fig. 5. The proposed retrieval system gives zero for T for all the images shown in first row whereas it gives different values for B, but they are not significant. The results obtained are presented in Table.3. This proves the robustness of the proposed system is invariant for rotation.

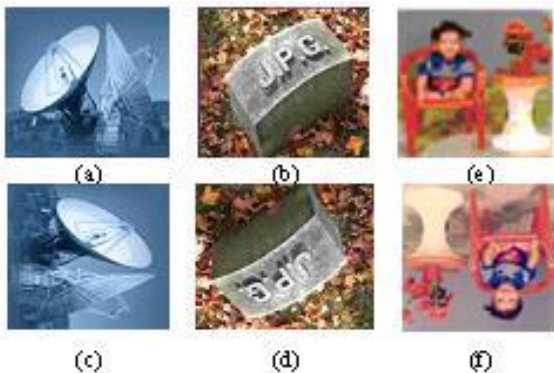


Fig.5. Actual images vs. images rotated with different angles

Table.2. Actual image vs. scaled-down image

Images	B	T
(a) vs. (c)	0.0010	0.0000
(b) vs. (d)	0.0024	0.0000
(e) vs. (f)	0.0059	0.0000

Table.3. Actual image vs. image rotated with different angles

Images	B	T
(a) vs. (c)	0.0012	0.0000
(b) vs. (d)	0.0034	0.0000
(e) vs. (f)	0.0011	0.0000

5. CONCLUSION

In this paper, a simple but efficient scheme for image retrieval based on statistical test of hypothesis viz. test for homogeneity of variance, test for equality of mean. The proposed system is invariant for transformation (translation, rotation, and scaling), since, the query image is treated as a sample / population of the target image and vice-versa. First, the variation among the intensity values of the query and target images are tested. If the two images pass the test (variation among the intensity values of the query and target images are same) then the mean values of the two images are tested. If the images pass the tests, it is concluded that the two images are identical; otherwise, it is assumed that the images are different. The proposed system provides cent percent accuracy and precision (B and T statistic values are zero, which are shown in Table.1) for the same images, and even if the target and query images are with a slight variations such as translated, rotated, scaled. The computed B and T values are indexed in ascending order. The image, which corresponds to the lowest values (first) in the list is chosen as the required image and is retrieved.

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