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QUANTUM COMPUTING: ITS APPLICATIONS IN MACHINE LEARNING & OTHER AREAS

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Abstract

Classical computers have been present for a long time and they have played a significant role in scientific advancements. Quantum computing has shown good results in solving complex problems. Quantum computers use phenomena of quantum superposition and quantum entanglement to form states that scale exponentially with the number of qubits or quantum bits [1]. Classical computers use individual bits, 0 and 1 to store information as binary data & quantum computers use the probability of state before it is measured [2]. Therefore, it gives them a potential to process exponentially more data as compared to classical computers. Unlike classical computers that use binary bits, quantum computers use qubits that are produced by quantum state of object to perform operations. Since, these qubits are quantum, they follow phenomena like quantum superposition and entanglement. Superposition is ability of a quantum system to be in multiple states at the same time. Entanglement is the strong correlation among quantum particles. These phenomena help quantum computers work with 0, 1 and superposition of 0 and 1, giving it advantage of doing complex calculations that classical systems cannot do or take a significant amount of time to get desired results [3]. Quantum computing is used because of its potential for changing time and space complexity of many algorithms we have been using as a solution to linear system of equations [4]. Quantum simulation is one of the most prominent areas of quantum computers, it has the potential to solve the complexities of molecular and chemical interactions which can lead to the discovery of new medicines and materials. Various applications of quantum computing in several significant areas of computer science, such as cryptography, machine learning, deep learning and quantum simulations. They also use various real-life scenarios such as risk analysis, logistics and satellite communication [6].

Keywords:

Quantum Computing, Qubits, Quantum Computers, Cryptography, Machine Learning, Deep Learning, Quantum Annealing, Quantum Neural Networks, Markov Models, Natural Language Processing

1. INTRODUCTION

Quantum computing represents a paradigm shift in computational science, harnessing the peculiarities of quantum mechanics to potentially solve problems that are intractable for classical computers. Unlike classical bits, which are binary and can only exist in a state of 0 or 1 at any given time, quantum bits or qubits can exist in superpositions of 0 and 1 simultaneously [5]. This superposition allows quantum computers to perform many calculations in parallel, vastly increasing their computational power for certain types of tasks. One of the most promising applications of quantum computing is in optimization problems, where finding the best solution among a vast number of possibilities is essential but computationally intensive. Quantum algorithms like Grover's algorithm can provide a quadratic speedup over classical algorithms in searching unsorted databases, offering significant efficiency gains. Similarly, Shor's algorithm demonstrates potential exponential speedup for factoring large numbers, a crucial capability for breaking many classical encryption schemes. Moreover, quantum computers excel in simulating quantum systems, which are notoriously complex and difficult to model accurately with classical computers [9]-[11]. This capability has profound implications for fields such as chemistry, materials science, and drug discovery, where understanding and predicting the behavior of molecules and materials at a quantum level could lead to revolutionary advancements. However, the path to practical quantum computing is fraught with challenges. Maintaining the delicate quantum states of qubits against environmental interference (decoherence) is a major hurdle. Error correction in quantum systems is also complex and requires sophisticated techniques to ensure the accuracy and reliability of computations. Building scalable quantum computers that can outperform classical systems consistently across a wide range of applications remains a formidable goal. In summary, while quantum computing holds immense promise for transforming fields ranging from cryptography to scientific research, realizing its potential requires overcoming significant technical barriers. As research and development progress, quantum computing stands poised to revolutionize computational capabilities and unlock new frontiers in science and technology in the decades to come.

2. APPLICATIONS

2.1 CRYPTOGRAPHY

The first applications of quantum computing, i.e. use of Shor's algorithm [7], can break most widely used public-key cryptosystems, such as RSA that use complex mathematical problems such as integer factorisation as basis for security. Given an integer $N= p\times q$ for some prime numbers p and q, Shor was able to determine p and q in time O [log (N^3)]. This is exponentially faster as compared to any existing classical algorithms. Shor's algorithm is analogous to the hidden subgroup problem (HSP) for finite Abelian groups [12]. The HSP is described by a group G, in the case of Shor's algorithm $G=\mathbb{Z}$.

2.2 UNSTRUCTURED SEARCH

Unstructured data gives rise to a significant portion of total data generated. It consists of text, dates and values that result in data not organised in any pre-defined manner. In this search within a list of k elements, assuming n=2k for the index to become an 'n' bit string, function f is given such that $f:\{0, 1\}n\rightarrow\{0, 1\}$ to tell us whether that specific unique element is present or not. Grover's algorithm, based on quantum computing, was devised in 1997 for searching in an unstructured data set [8]. Grover's algorithm does not use any internal structure of the given function f, even if it has one. This algorithm requires a time complexity of O(√N) , which is an improvement by a quadratic factor over the classic computational models. Fig. below shows the complete working of Grover's search algorithm for 3 qubits. The amplitude of the marked state becomes negative through the oracle, and then that state is amplified. After an appropriate number of iterations, the amplitude of the desired state is maximised [13]. Sysoev proposed an improvised algorithm based on Grover's algorithm to solve NP-tasks, which would be exponentially faster than the speed achieved by Grover's algorithm. However, this requires the use of two quantum systems at the same time to alternate the roles between each iteration, and this kind of quantum computational model is yet to be developed [14].

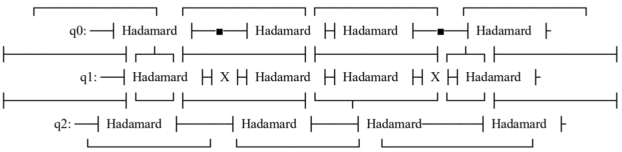

Fig.1. A 3-Qubit Grover's search algorithm on a quantum computer.

2.3 AMPLITUDE AMPLIFICATION AND ESTIMATION

Let's assume that the probability of finding an element x0 in the list of elements X [x1, x2, …, xn] is p. Each time we execute this search algorithm, the probability of finding the element would increase by p, making the probability 2p, 3p, …, and so on. Applying the same logic in quantum computations, we get the concept of amplitude amplification. We can consider a Boolean function f(x), x ∈ X, wherein it's value is true if x0 = x otherwise, it's false. In amplitude amplification, instead of increasing the probability after each iteration, we would be increasing the amplitude of being in one amongst the two possible states (true/false) residing in a complex separable Hilbert space. The quantum algorithm which was proposed by Brassard [15] is a generalisation of Grover's algorithm [8] where there has to be a unique solution only. This algorithm can find the element in O(1/√p) time, which is a quadratic speedup over classical algorithms. Amplitude estimation uses the ideas behind Grover's[8] and Shor's algorithms[7] to obtain the approximate number of times a 'True value" is obtained in the simulation.

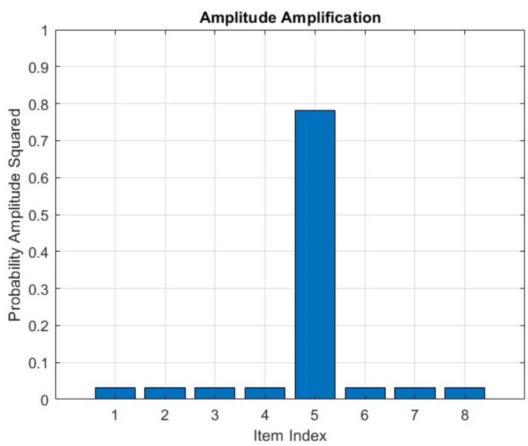

Fig.2. Amplitude Amplification

2.4 APPLICATIONS OF SEARCH

Grover's search and amplitude amplification can be used as subroutines for more complicated quantum algorithms. A quantum algorithm by Durr and Hoyer [16] can be used to find the minimum of an unsorted list of N integers with O(√N) evaluations. More generally, it finds the minimum of an unknown function f :{0, 1}^{n} ⟶ ℤ. Their algorithm applies Grover's algorithm to a function g:{0, 1}n ⟶ {0, 1} defined by g(x) = 1 if and only if f(x) < T, where T is some threshold initially set randomly. The threshold is then updated as inputs x are found such that f(x) < T. A classical computer requires time O(N^2) in the worst case, where N is the number of vertices in the graph. It give a quantum algorithm that runs in time O(N^3/2), up to logarithmic factors [17]. Efficient algorithms for other graph problems, such as strong connectivity, minimum spanning tree and shortest path were also proposed. A fundamental problem in text processing and bioinformatics is pattern matching. There is an algorithm that can find a given pattern p of length m within a text t of given length n. The required time is of the order O(√n+√m) up to logarithmic factors. The best possible classical complexity is O(n+m) [18]-[22].

2.5 ADIABATIC QUANTUM COMPUTATION (OR QUANTUM ANNEALING):

Adiabatic quantum computation (AQC) or quantum annealing is a unique way to solve optimisation problems. It is used to find the global minimum value from the dataset with the help of a function. The ground state (lowest energy state) of a complicated Hamiltonian describes the solution to the problem. Initially, we take a simple Hamiltonian in its ground state to solve the problem. Thereafter, a complicated Hamiltonian evolves adiabatically from the simple Hamiltonian. The Fig.3 provides a graphical representation for the quantum annealing process [23]. According to adiabatic theorem, the system will always remain in the ground state. The processor D-Wave '2X' from the D-wave company, developed recently, can outperform classical processors implementing quantum Monte Carlo and simulated annealing [24]. Like the Shor's algorithm, quantum annealing can be used to factor integers into primes. This makes it very important from the perspectives of cryptography. Burges in [25] did fundamental research in this direction. The author used factoring of biprimes as a framework for solving combinatorically hard problems using optimisation algorithms. His work was further improved by the authors[26].

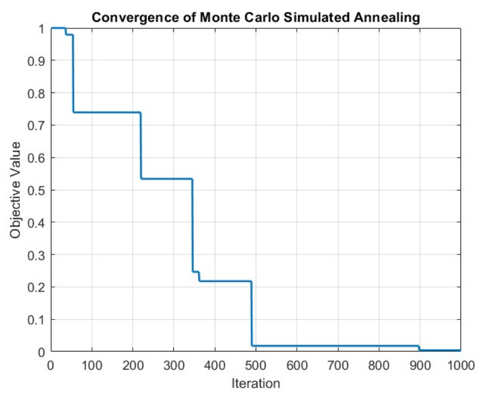

Fig.3. Graphical representation of Quantum Annealing

3. QUANTUM COMPUTING IN MACHINE LAERNING

In machine learning, we develop algorithms which can learn from the inputs and give desired outputs from which we expect the algorithms to predict values for future unknown inputs. The most common use of quantum computing in machine learning is using computational speed-ups achieved by quantum algorithms, for classical machine learning algorithms. While deep learning algorithms use hardware such as quantum annealers (quantum computers based on AQC) to enhance performance.

3.1 NEAREST NEIGHBOUR CLASSIFICATION AND K-MEAN CLUSTERING

A standard algorithm in machine learning, K-nearest neighbour (KNN) algorithm takes all the previous data under consideration while evaluating a new data item that we need to classify based on how similar it is and how it's neighbours are classified. The closer a vector is to another vector, the more similar they are. Standard methods for evaluating closeness or distance are the inner product, the Hamming distance, or the Euclidean. In [27], the authors use a technique of overlap or fidelity ⟨a b⟩ of two quantum states ⟨a⟩ and ⟨b⟩ to measure the similarity between vectors. The overlap is acquired through a subroutine known as a swap test. Based on [27], the authors in [28] proposed a quantum algorithm that takes time O(log MN). This introduced an exponential speed-up [29]. The authors of [30] have presented algorithms for measuring the distance between feature vectors. The approach is based on the swap test that provides methods for calculating Euclidean distance both directly and using the inner product. It is coupled with the use of amplitude amplification applied together with Grover's search. However, the representation of classical information through qubits is different. In the worst case, the algorithm leads to polynomial reductions when compared to Monte-Carlo algorithms.

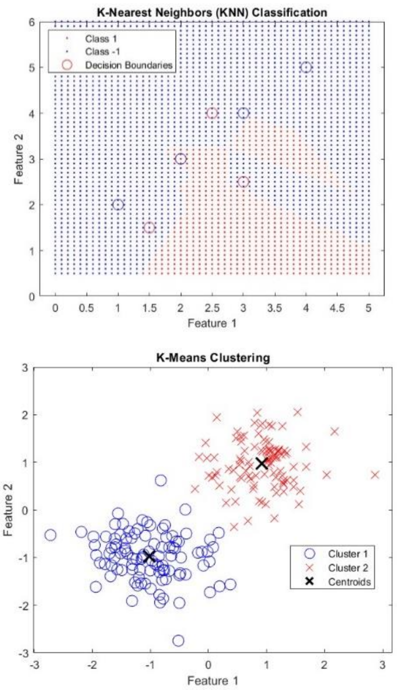

Fig.4. Nearest Neighbour classification and K-mean clustering

3.2 SUPPORT VECTOR MACHINES (SVM)

SVMs are the supervised machine learning algorithms for classification of data models. They are used for classification analysis and also for regression. It uses a test sample for training the data model and assigning each value to one of the categories available. The task in such problems is to find an optimal hyperplane that separates two-class regions very clearly and acts as a decision boundary for future inputs. In the early 2000s, the authors of [30]-[31] proposed the first version of the quantum SVM, which used a variant of Grover's search. More powerful methods have been developed recently. The data input can come from, sources such as qRAM accessing classical data, or it can be a quantum subroutine preparing quantum states. Specifically, quantum phase estimation and matrix invasion are used to create the optimal hyperplane and test the input vector, which in principle requires time poly log(N). 'N' is the dimension of the matrix that is required to produce a quantum version of the hyperplane vector. The methods described in [32]-[34] can be used to analyse data using the HHL algorithm.

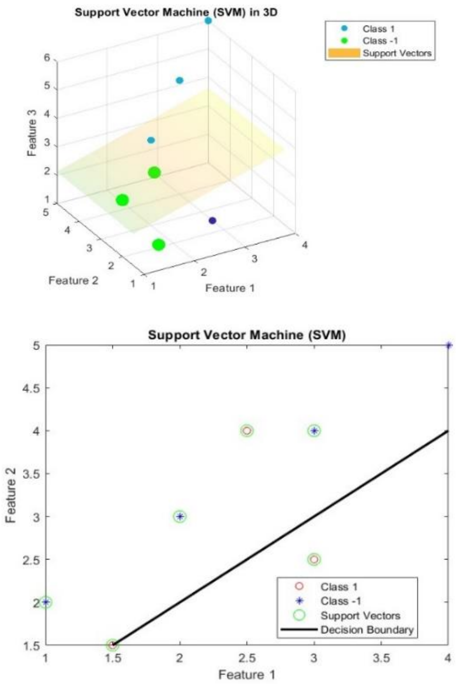

Fig.5. Support vector machines (SVM)

3.3 QUANTUM NEAURAL NETWORKS (QNN) AND DEEP LEARNING

QNNs are computational neural networks working on principles governed by quantum mechanics. Artificial neural networks are researched because of their help in pattern recognition and big data applications. It is believed that concepts such as entanglement, parallelism and interference can help. An increasing number of advancements have explored the idea of quantum artificial networks[35]-[37]. Current work in the field uses the concept of replacing the classical binary bit with a qubit, thus creating a neural unit that is in a superposition of the activated and not activated states. Quantum annealers are easily scalable and commercially available and well suited for constructing deep quantum learning networks [38]. A deep learning network that is the Boltzmann machine is the easiest to approximate[39]. The quantum Boltzmann machine outputs quantum data which is in qubits. Schuld[40] have concluded in their survey that there are no proposals that truly harness the power of quantum computers. The reason why this is just theoretical till now is that quantum states need to be normalised. Moreover, classical neural networks have non-linear dynamics, whereas QNN has linear dynamics.

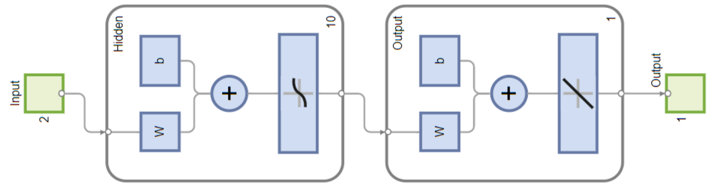

Fig.6. Neural Network

3.4 HIDDEN QUANTUM MARKOV MODELS

A Markov model is a stochastic model that models temporal or sequential data which helps in predicting future value based on the current information. A hidden Markov model (HMM) is a Markov model where states of the model are hidden and can be observed only when it is given as output by the state. HMM is particularly used to model sequential data in fields such as NLP (Natural Language Processing). In 2010, authors of [41] first introduced hidden quantum Markov models (HQMMs). HQMMs have an edge that they are a generalisation when compared to the classical HMM. In [42], the authors have proposed open quantum systems with instantaneous feedback to implement the HQMM. They also note that HQMMs can find application as simulators of stochastic processes. Recently an iterative maximum likelihood algorithm has been proposed [43]. The algorithm could successfully learn HQMM and better model certain sequential data.

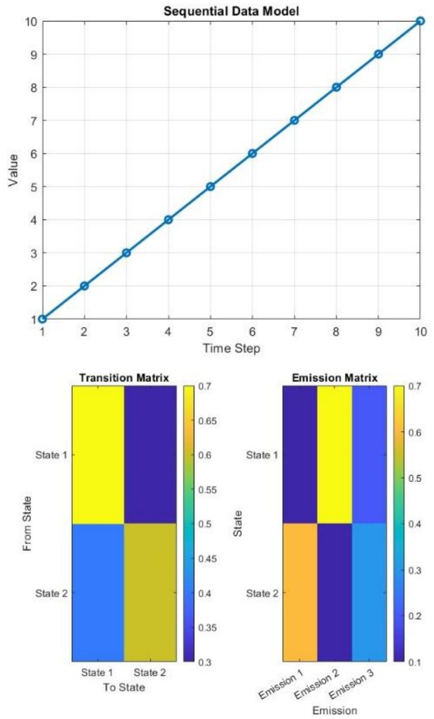

Fig.7. Sequential data and 2-States matrices

4. QUANTUM COMPUTING IN OTHER AREAS

4.1 FINANCIAL RISK ANALYSIS

Suppose we have a portfolio of financial products and the value of these products profits and losses depends on future prices. These future prices are uncertain, and we do not know how they will develop with time. This uncertainty raises many questions, such as whether a particular investment will yield profit or loss. To get these future estimations, many algorithms have been developed over time. Value at Risk (VaR) and Conditional VaR (CVaR) are the two units for calculating the risk. VaR is used to determine the loss distribution, while the CVaR is used to determine the expected loss for losses greater than the VaR. CVaR is more sensitive to extreme events in the loss distribution. Monte Carlo simulations are the most widely used methods to find these predictions on classical computers. Monte Carlo simulation is the process of generation of random objects or processes that can be achieved on a computer. It follows a stochastic model to sample the future prices. Also, calculations performed on a classical computer takes very long time for big datasets. Quantum methods can achieve this very quickly. Quantum techniques not only help machine learning to solve financial problems, but can also optimise risk returns for the financial assets and portfolios.

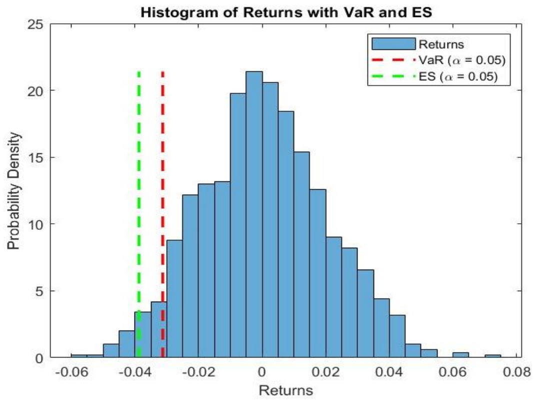

Fig.8. Financial risk analysis (VaR, ES and Returns)

4.2 QUANTUM RANDOM NUMBERS

Random numbers serve as the fundamental element in several applications. Statistical methods, such as bootstrap method, require random numbers to work. Random numbers play a very significant role in cryptography. It is used in generation of crypto codes, which serves as the base for many modern cryptographic algorithms. Not just cryptography, it is also used in many other programming aspects and well-known algorithms, such as Monte Carlo simulation. The deterministic system of present computers does not generate truly random numbers. Computers follow complex algorithms to generate pseudo-random numbers. These pseudo-random numbers serve as a base for cryptography which is very critical for privacy. To solve this problem, a random number generator is required which follows random physical phenomenon. Since, quantum systems are random inherently, they are able to generate truly random numbers. A good demonstration for construction of a cheap, simple, and easy to use quantum random number generator. This prototype is small (68×150×188 mm) and fast enough to be implemented for cryptography. It is possible to make a quantum random number generator based on a beam splitter that generates true binary random signals at a rate of 1 Mbit/s having an autocorrelation time of 11.8 ns [50]. For prospects, it is feasible to develop a random number generator based on quantum mechanism that will produce truly random numbers at a rate of 1 Gbit/s or even above that [44].

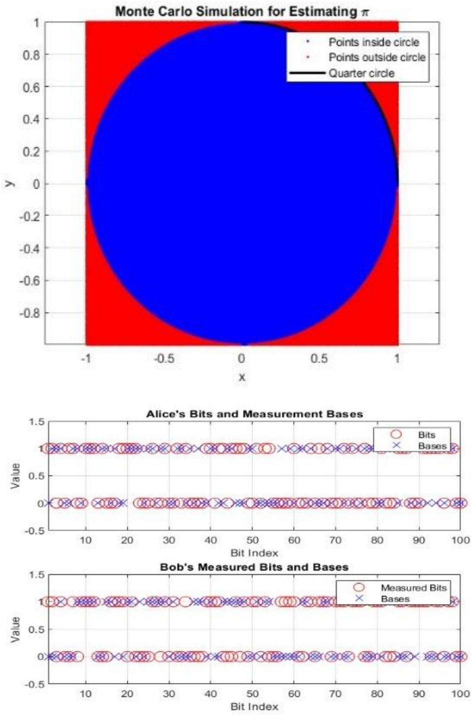

Fig.9. Monte-Carlo Simulation and Random measured bits

4.3 SATELLITE COMMUNICATION

Although quantum computer is in its initial state, but many algorithms and protocols have been developed, which can help in communication. At present, many applications are reliant on satellite and play a vital role in our day-to-day life. These are television, telephones, weather, navigation, business and finance, earth observation, space station, military and security purposes. Its applications have become an integral part of our life. A quantum channel is a communication channel meant to transfer classical or quantum information to a satellite. A free quantum space is required for communication to be made possible. In future, it is possible that free-space quantum key distribution applications can have direct communication: free space, satellite-to-satellite, and ground-to-satellite communication will be possible on low earth orbit, middle earth orbit, and geostationary orbits [45].

Fig.10. Satellite communication and the transmission of photons

5. CONCLUSION

Quantum computing represents a transformative leap in computational power and capability compared to classical computing. At its core, quantum computers leverage the principles of quantum mechanics—superposition, entanglement, and quantum interference—to process information in fundamentally different ways. This enables them to potentially solve certain types of problems much faster than classical computers. For instance, tasks like factoring large numbers (crucial for cryptography) or simulating complex quantum systems could be executed exponentially faster with quantum algorithms. This speed and efficiency stem from the ability of quantum bits (qubits) to exist in multiple states simultaneously, exploring many solutions at once. Beyond speed, quantum computing could revolutionize fields such as drug discovery, materials science, and artificial intelligence by tackling problems that are computationally prohibitive for classical machines. However, realizing this potential requires overcoming substantial technical challenges, such as qubit stability and error correction, before quantum computers can reliably outperform classical systems across a broad range of applications. Nonetheless, the promise of quantum computing underscores its potential to reshape computing and scientific discovery in profound ways in the coming decades.

APPENDIX

It is demonstrated that quantum computers have a significant advantage. There are several quantum algorithms that provide an edge over classical algorithms. Quantum simulations continue to attract researchers in quantum computations for several years, because of its wide possibilities. Novel and practical use cases for existing quantum algorithms is a useful future research direction. Quantum computers are expected to be made available via cloud computing in the future, which will make their integration with our existing classical computers easier. Quantum computations and its applications will be an exciting field for research because of its endless possibilities in future.

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