

# PERFORMANCE ANALYSIS OF CHAOS BASED INTERLEAVER IN IDMA SYSTEM

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## Abstract

Chaos are wide spread in nature, furthermore its use in the field of communication drew attention in early 90's and recently chaos based spread spectrum (SS) communication become an interesting area of research. Based on the research; this paper presents the study of chaos theory and then leads the discussion towards the chaos based SS system. Interleave Division Multiple Access (IDMA) based spread spectrum has been considered for performance analysis. First it reviewed the role of chaos in spread spectrum communication and then discussion extends to chaos based IDMA, which is relatively a new and promising technique in this area. Simulation results verify that chaos based IDMA can achieve good BER performance as well as offers less computational complexity.

## Keywords:

Chaos, Spread Spectrum System, CBDSS, Bifurcation, IDMA

## 1. INTRODUCTION

In IDMA scheme, interleavers are used to distinguish the different users. This scheme is the improved version of classical CDMA scheme. IDMA inherits all the advantages of CDMA scheme especially it overcomes the major limiting factor of CDMA such as: multiple access interference (MAI) and Inter Symbol Interference (ISI). Interleavers are used to spread the information and to protect these bits from error bursts due to multipath propagation and noise sources. Hence, efficient interleaver can increase the throughput of iterative MUD receivers [1].

Interleavers should be easy to generate and not to consume large bandwidth and memory. Interleavers should also be non-colloidal in nature [2]. Many Interleavers were studied and proposed in the literature. Rumsey (1970) presented the study of basic interleaver design. Randomly Interleaved sequence can be the good choice of interleaver but the memory requirement in Random Interleaver (RI) is very high and this limitation motivates for further research. Pupeza et al. (2006) suggested the study of Nested Interleaver (NI). The limitation of this interleaver was the extra memory requirement [2]. Kusume et al. (2008) proposed the designing of Shifting Interleaver (SI). The bit error rate performance of this interleaver was good but not suitable for multi user detection (MUD) system [3]. M. Shukla et al. (2009) presented the designing of Tree based Interleaver (TBI). This interleaver was also having the scope of improvement in memory requirement [5].

On the other hand Chaotic signals are deterministic, limited and non-periodic as well as highly sensitive to the initial condition [4]. Chaotic signals are noise like and wide-band and hence may be good candidate for interleaving sequence. Furthermore, the cross correlation property is also encouraging. Chaos-based systems are having significant advantages over traditional spread spectrum systems in terms of security and synchronization [5].

Chaos is in itself a very universal and robust phenomenon in many nonlinear systems with certain characteristics. According to chaotic dynamics these characteristics are (a) highly sensitive to initial conditions (b) wideband frequency spectrum (c) noise like behavior (d) high complexity [6]. These properties made chaos useful in communication engineering specifically for security of information.

This paper meets the following objectives; firstly it provides introduction to chaos theory and its role in spread spectrum communication engineering and secondly the chaos based interleaver employed in IDMA based communication system for performance enhancement. It will also explore the new area of developments on the basis of signal processing capability.

The content of the paper is organized as follows. In section 2, the introduction of chaos theory is discussed. Section 3, defines the IDMA system and algorithms for interleaver generation. Section 4, develops the performance analysis of chaos based IDMA schemes and finally section 5 conclude the paper.

## 2. CHAOS THEORY

This could be noted that sinusoidal carriers may be a better choice in communication systems. When a sinusoidal signal is used to transmit information, the power spectral density concentrates in a narrow range of frequencies. Whereas chaotic signals, can occupy a large bandwidth, their autocorrelations and cross correlation properties are also favorable. These characteristics made chaotic signals a better choice in communication systems. Chaos-based SS systems have several properties, namely (i) Difficult to interfere with any unauthorized user; (ii) information transfer is more secure than any other communication system (iii) resistant to jamming. Chaos can be better understood with the help of difference equations. The logistic population model is very popular to understand chaos.

**Definition 1:** The logistic map is given as:

$$f(x_n) = x_{n+1} = rx_n(1-x_n) \quad (1)$$

where,  $r$  is growth rate of population and an important parameter to discuss. Here parameter  $r$  is elaborated for different value ranges.

For the range  $0 \leq r \leq 4$ .

**Proposition:** For the above range logistic map sends  $[0, 1]$  to itself.

**Proposition:** For the value of  $r < 1$ , fixed and stable point is 0. For the value  $r > 1$  it is unstable. One more point is stable i.e.  $x = 1 - (1/r)$ , but only for  $0 < r < 3$  and unstable for  $r > 3$ .

**Proof:** If we solve the equation  $rx(1-x) = x$ , the fixed point yields  $x_1 = 0$  and  $x_2 = 1 - (1/r)$  and from the derivative of equation i.e.  $f'(x) = r(1-2x)$  we get  $f'(0) = r$ . Hence 0 is stable point for the specified range. Similarly second point is also stable for above said range.

**Proposition:** The logistic map has 2 cycles for  $r > 3$  and stable if  $r < 1 + \sqrt{6}$ .

**Proof:** In 2 cycle, logistic map has set of points such that  $u \neq v \in [0,1]$  and  $f(u) = v$  and  $f(v) = u$  and hence  $f^2(u) = u$  and  $f^2(v) = v$ . Now solving the equation  $f^2(x) = r^2u(1-x)[1-rx(1-x)] = x$  yields four solutions and these solutions proves that the map forms 2-cycles if  $r > 3$ .

Now another interesting property is yet to be discussed i.e. sensitivity on initial conditions, which is a much needed requirement for a spread spectrum system to be chaotic

**Definition:** Let  $\alpha_0$  be initial condition and consider the orbit of a nearby point  $\alpha_0 + \theta_0$  where  $\theta_0$  is very small. Let  $\theta_n$  is the separation in two orbits after  $n$  iterations. If  $|\theta_n| \approx |\theta_0|e^{n\lambda}$ , then  $\lambda$  is called the Liapunov exponent. A positive value of it shows dependency on initial condition.

Above discussion shows that as the value of  $r$  increases, the stability coefficients of the fixed points decreases and at  $r = 3$ , a stable 2-cycle formed. So this value of  $r$  is popularly known as period doubling bifurcation. For higher values of  $r$ , again 2-cycle has also become unstable but simultaneously a stable 4-cycle started at  $r = 3.4494897$ .

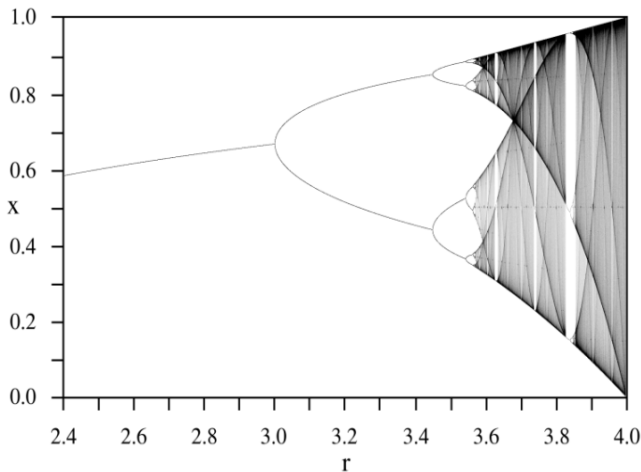


Fig.1. Bifurcation diagram

In Fig.1, the entire discussions can be pictorially depicted that for  $r < 4$  an orbit eventually converges to the stable fixed point. In Fig.1, the situation is completely different at  $r = 4$  i.e. it may be considered as chaotic. At this value of  $r$  the stable point becomes unstable. The use of chaos into spread spectrum communication systems offers many advantages as well as several opportunities for further improvement. Random nature and the sensitivity on initial conditions of chaotic systems may help to generate large number of uncorrelated, random-like, yet deterministic and reproducible signals or sequences [7]. Many chaotic maps are available which help in the generation of chaotic sequences and these sequences can be used at the place of p-n codes and interleaving sequences. The popular chaotic maps are shown in Table.1, such as Logistic map, Tent map, Bernoulli’s map, Baker Map, Henon map etc.

Table.1. Chaotic maps used in SS system

Sl, No	Chaotic Map	Equation
1	Logistic Map	$X_{n+1} = AX_n(1 - X_n)$
2	Tent Map	$F_m(x) = \begin{cases} mx_n & \text{if } 0 \leq x \leq \frac{1}{2} \\ m(1 - x_n) & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$
3	Baker Map	$F(x, y) = \begin{cases} (2x, y)/2 & 0 < x < 1/2 \\ ((2x-1), (y+1/2))/2 & 1/2 < x < 1 \end{cases}$
4	Henon Map	$X_{n+1} = 1 - aX_n^2 + Y_n$ $Y_{n+1} = bX_n$

### 3. SYSTEM OVERVIEW

In IDMA system interleavers play a vital role as they are used to distinguish the data from different users. Many interleavers are suggested by researchers such as random Interleaver, orthogonal interleaver, pseudo random interleavers and tree based Interleaver etc. Although all of these interleavers ensure good interleaving performance, memory requirement is always less. But some limiting factors motivate for further research. These limiting factors are computational complexity, memory requirement for the storage of the interleaving pattern and detection at receiver.

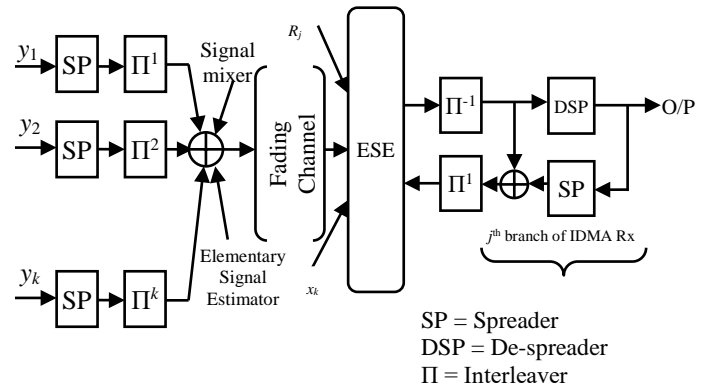


Fig.2. Transmitter and Receiver structure of iterative IDMA scheme

As shown in Fig.2, we consider an Interleave Division Multiple Access (IDMA) system with  $K$  users. At the transmitter, a  $n$ -length input data sequence  $y_k = [y_{k1}, y_{k2}, \dots, y_{ki}, \dots, y_{kn}]$  of  $k^{\text{th}}$  user is spread and encoded into chips  $c_k = [c_{k1}, c_{k2}, \dots, c_{kj}]$ , where  $j$  is the Chip length. Now chips are interleaved by a specific interleaving pattern (based on chaotic maps) to produce transmitted chip sequence  $x_k = [x_{k1}, x_{k2}, \dots, x_{kj}]$ . For performance evaluation multipath Rayleigh fading channel is opted [1]. In receiver section, the received signal from the  $K^{\text{th}}$  users with channel coefficient  $h_k$  for  $k^{\text{th}}$  user and for the samples of fading  $\{\zeta_j\}$  can be written as:

$$R_j = \sum_{k=1}^K h_k x_{kj} + \zeta_j \tag{2}$$

The iterative MUD detector also consists Elementary signal Estimator (ESE) and a posteriori decoder. The output of ESE is known log likelihood ratios and defined as:

$$e_{ESE}(x_k(j))_w = 2h_k \frac{R_{j+w} - E(\zeta_{k,w})}{\text{var}(\zeta_{k,w}(j))} \quad (3)$$

The output of APP-DEC is called as extrinsic log likelihood ratio. Based on the iterative process, the final outcome is decided. It is already discussed that along with iterative process the effective interleaver can enhance throughput of the IDMA system. In view of that, a chaos based interleaver is proposed. Flow chart and detailed algorithm is presented in further subsections. The popular Logistic map is used for interleaver generation.

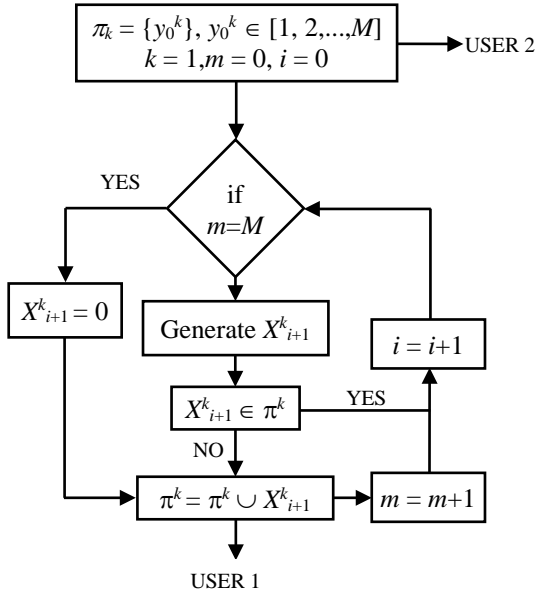


Fig.3. Flow Chart of Logistic Map Interleaver

In Fig.3, the flow chart is proposed which describe the algorithm for first interleaver generation. For the second user the whole process is repeated with some different initial interleaver. The Foot step  $\tau$  is used to modify the initial value of interleaver. Steps required for the interleaver pattern generation is described below.

### 3.1 ALGORITHM OF INTERLEAVER DESIGN BASED ON LOGISTIC MAP

#### Step 1: Initialization

- $\lambda > 3.58$ ,  $N$  = Interleaver length,  $I$  = no. of users
- $X_j^I = I^{\text{th}}$  user:  $0 < X_j^I < N$
- $\tau$  = Foot step
- $F_0^I = \lceil X_0^I \rceil$ : the first element ( $\Pi^I = F_0^I$ ),  $j = 0$  and  $n = 0$

#### Step 2: Main operation

- a) If  $n < N$
- Calculate  $F^I_{j+1} = \lfloor X^I_{j+1} \rfloor$
- Now Check:
- If  $F^I_{j+1}$  is in the set  $\Pi^I$
- Increment  $j$  by 1 and repeat the main operation
- Otherwise
- $\Pi^I \equiv \Pi^I \cup F^I_{j+1}$
- b) If  $n > N$ :

$$X^I_{j+1} = \{ \}$$

$$\Pi^I \equiv \Pi^I \cup \lceil X^I_{j+1} \rceil$$

## 4. PERFORMANCE OF LOGISTIC MAP BASED IDMA

### 4.1 ERROR-RATE ANALYSIS

In this section, the quality of transmission of conventional IDMA is evaluated for chaos based logistic map interleaver. Random Interleaver is also considered for bit error rate (BER) analysis of conventional IDMA system. For simplicity, BPSK modulation is considered in multipath Rayleigh fading channel. The spreading code for all users is simply a repetition code with the length  $S = 64$ . The Fig.4 shows the simulation results for data length = 256 bits. Simulation results authenticate the performance of chaos based IDMA system. This is clear in figure that the chaos based logistic map interleaver achieves similar BER performance compared to RI in case of similar and somewhat better for higher values of  $E_b/N_0$ .

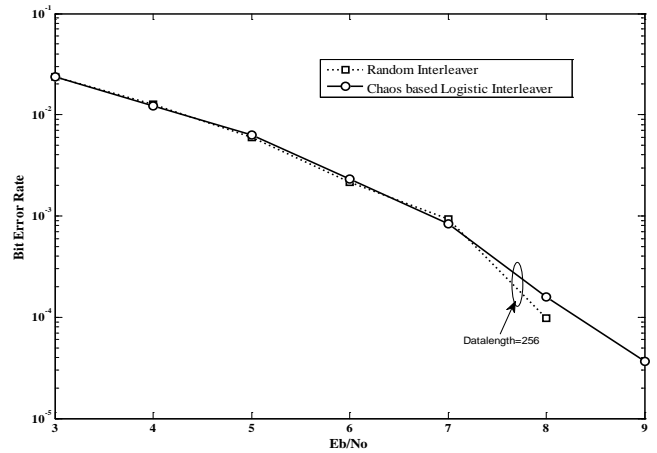


Fig.4. Performance of chaos based logistic interleaver

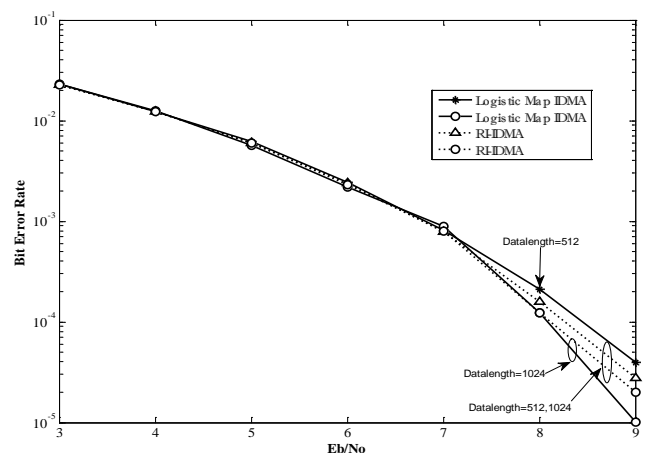


Fig.5. Performance of chaos based IDMA at different data length

In Fig.5, simulation results show the comparative BER performance of RI and logistic map based interleaver. The data length is assumed 512 and 1024 bits. Number of iterations is 10 and spread length  $S$  is considered to be 64. Simulation figure

concludes that the performance of chaos based IDMA have nearly same performance compared to RI-IDMA.

**4.2 COMPUTATIONAL COMPLEXITY**

Other than BER analysis, Computational complexity is also the important parameter to decide the quality of communication. Here, complexity means that number of cycles required for the generation of spreading codes and interleaver matrix in terms of users. In this section, complexity is calculated for chaos based logistic map interleaver and for other popular algorithms used for interleaver generation.

The Table.2 shows that the computational complexity increases with the number of user  $k$  for orthogonal, Nested and Tree based interleaver (TBI). It means complexity is dependent on users for all mentioned schemes except logistic map interleaver. In fact, the complexity is  $O(1)$  which means that computational complexity of logistic map interleaver is independent from the number of users [6].

Table.2. Computational complexity of interleaver generation of different algorithms

No. of Users	OI	Nested Interleaver	TBI	Logistic map Interleaver
1	1	1	1	1
2	2	2	1	1
4	4	4	2	1
16	16	16	4	1
50	50	50	5	1

**4.3 CORRELATION ANALYSIS**

The low value of correlation among interleavers is very important criterion and provides a way to reduce the value of multiple access interference (MAI). Pupeza et al. [2] proposed the correlation values for random interleaver, pseudo random and orthogonal interleaver. Akbil et al. [6] extended the discussions and also include the correlation values for chaos based interleavers. Here the correlation values for 5 users of random interleavers and chaos based interleavers are presented in Table.3 and Table.4 respectively [2][6].

Table.3. Peak correlation values for Random Interleaver

Row/ Column	1	2	3	4	5
1	16384	1814	1742	1708	1674
2	1822	16384	1765	1797	1701
3	1802	1786	16384	1766	1740
4	1813	1783	1762	16384	1769
5	1743	1772	1758	1765	16384

Table.4. Peak correlation values for Logistic map Interleaver

Row/ Column	1	2	3	4	5
1	16384	1872	1694	1698	1660
2	1574	16384	1790	1460	1952
3	1720	1920	16384	1712	1656
4	1520	1632	1594	16384	1730
5	1820	1490	1864	1578	16384

**5. CONCLUSIONS**

In this paper, the performance of chaos based interleaver division multiple access has been examined. Simulation results show that the use of chaos based interleaver in IDMA scheme enhances the bit error rate performance of IDMA system. The computational complexity of the generating algorithm is also independent from users. Correlation values of chaos based interleaver are nearly same as compared to random interleaver and hence good resistance to MAI. Chaos based interleaver also offers other advantages such as less requirement of memory and small implementation complexity. Future work may involve the improvements in logistic map such that the logistic map can be modified to make it less Ergodic, so that IDMA based on Logistic map can be more suitable for secure communication.

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