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HYBRID AOA AND TDOA MEASUREMENTS BASED MULTI-SOURCE LOCALIZATION USING THE GENETIC ALGORITHM

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Abstract
The multi-source localization in sensor array networks is a major challenge for radar and sonar. Recent advances in array signal processing techniques and hardware technologies have resulted in a significant increase in the accuracy of angle of arrival or time difference of arrival estimation. So, the angle of arrival or time difference of arrival is widely used as the fundamental features in source localization. In this paper, we address the problem of locating multiple sources based on the angle of arrival and time difference of arrival obtained from small sensor arrays. We propose a new likelihood function for multi-source localization under the assumption that the measurements follow a normal distribution. And we propose a method to simultaneously estimate the source positions using the genetic algorithm. Compared the existing methods, the effectiveness of the proposed method is demonstrated through simulation experiments.

Keywords:
Multi-Source Localization, Angle of Arrival, Time Difference of Arrival, Sensor Network, Genetic Algorithm

1. INTRODUCTION

Source localization is widely used in signal processing fields, including sonar, radar, target tracking, and noise cancellation. Recent advances in array signal processing techniques and hardware technologies have resulted in a significant increase in the accuracy of angle of arrival (AOA) or time difference of arrival (TDOA) estimation in sensor arrays, and the AOA or TDOA has recently been used as a fundamental feature in source localization [1-12]. The localization method, which uses combined features of AOA and TDOA, can achieve the desired localization accuracy by reducing the number of sensors, and moreover, it can prevent the appearance of false targets in multitarget processing, which has been widely used in precision positioning, such as passive sonar [13,14].

Maximum likelihood estimation is widely used for localization because of its asymptotic efficiency, which in this case makes it difficult to obtain closed-form formulae and requires iterative methods. However, the numerical method can converge to the optimal solution only when the maximum likelihood function is convex, so it is difficult to find its global optimal solution when the likelihood function is nonconvex. To overcome this shortcoming, Cong et al ([15]) proposed a two-stage least squares estimation method for maximum likelihood estimation of the source localization by combining the AOA and the TDOA in a 2D plane. In [16], Huai-Jing Di et al. propose an extension of the method proposed in [15] to 3D localization and verify its performance through simulation experiments. In contrast to maximum likelihood estimation, in [13], the TLS method (see [29]) that was used in localization using only the TDOA or the AOA is applied to joint localization.

Multi-target/source localization is a field of research focus in the world because it still has several challenges in real life applications. The methods typically used for multitarget localization based on TDOA or AOA can be divided into two categories: clustering-based methods and data association-based methods.

The hyperbolas or azimuth lines intersect each other in a two-dimensional plane, forming a set of points of intersection around the sources. This set of intersection points can be partitioned with each other by a clustering algorithm and the location of multiple sources can be estimated based on each set of intersection points. This multi-source localization method is considered as clustering-based method. The paper [17] describes a method of estimating the locations of multiple sources as intersections of hyperbolas corresponding to the TDOA in 3D space, which is only possible in 3D and not applicable to the estimation of multiple source locations in 2D planes.

Unlike clustering-based methods, multi-source localization based on data association has become a typical method in recent years because multiple target locations can be estimated efficiently from it [18-23]. Suppose that we have estimated the AOAs for M targets at N sensors during a time interval. And then, $i(0<i<M)^{th}$ AOAs at each sensor are not all the AOAs corresponding to the same target. So, before estimating the position of the source, we must find all the AOAs corresponding to the same source for each sensor. This way, the problem of finding all estimates corresponding to the same source among the estimates obtained at each sensor is called the Data Association Problem. The data association problem, first mentioned in 1964, was originally used for target tracking. Since the rapidly growing applications of sensor networks, the problem of data association has been exploited for multi-source localization.

In [20], a probabilistic data association method for multi-source localization using only three sensors is proposed to perform data association under the assumption that the errors of AOA are normally distributed. This method is only available for three sensors and has the disadvantage of being difficult to apply when the number of sensors is higher. After that, how to overcome this drawback and perform data association using multiple sensors was also investigated. In a paper [21], based on the AOAs for several targets from a moving sensor, they discussed a probabilistic data association problem under the normal distribution of AOA errors. A method based on TDOA measurements have been proposed to implement data association in multitarget localization in [22], but this method have the disadvantage that they cannot be extended to other scenarios. To overcome this shortcoming, a paper [23] proposed a probabilistic data association method that can be commonly used for multi-source localization of AOA or TDOA measurements. In [18], an algorithm to speed up data association based on AOA measurements is proposed, while in [19] a method to estimate data association for all possible pairs of measurements is proposed. In contrast to the approach of first combining the data and then estimating the source location based on it, in [24] combined the TOA measurements and candidate target locations together and solved the OMT (Optimal Mass Transport) [25] [26] [27] [28] problem to estimate the source location.

We discuss the problem of estimating the location of multiple sources based on the AOA and target detection time information obtained on small sensor arrays. For this, we propose a new likelihood function for multi-source localization and propose a new method to estimate the locations of sources simultaneously using a genetic algorithm. We have constructed the following system to estimate the location of several targets. A small sensor array consisting of several sensors (the distances between each sensor in array network is negligibly small compared to the distance to the source, so that the sound wave entering the small sensor array is assumed to be a plane wave) estimates the reception time and the AOA from the multiple sources. These small sensor arrays are spatially distantly distributed, and the reception time and AOA information of the signal measured by the small sensor arrays are concentrated at the central station in wireless or wired communications. At the central station, the measured information from each of the small sensor arrays is aggregated to estimate the location of multiple sources simultaneously. We consider a small sensor array simply as a sensor and discuss the source localization problem in a 2D plane.

Let n be the number of sensors and m be the number of targets. Let Z_n^+ be the set of positive integers from 1 to n. Let us denote the coordinates of the point P of the plane by $(x(P),y(P))$ in the Cartesian coordinate system. Let $t_0(S_k)$, $(k \in \mathbb{Z}_m^+)$ be the time at which the signal emitted from the source S_k, $t_i(S_k)$, $(i \in \mathbb{Z}_n^+, k \in \mathbb{Z}_m^+)$ be the time at which the signal from the S_k was detected at the sensor P_i, $\alpha_i(S_k)$, $(i \in \mathbb{Z}_n^+, k \in \mathbb{Z}_m^+)$ be the azimuth of the source S_k measured from the sensor P_i, $\tau_{ij}(S_k)$ be the TDOA between the i^{th} sensor and the j^{th} sensor for the signal emitted from the source S_k, and C be the speed of signal propagation in air. Let us denote the distance between two points P and Q by $d(P,Q)$ and the angle that the line PQ makes with the reference axis by $\theta(P,Q)$.

And then, the estimators of signal detection time, AOA, and TDOA are as follows.

$$\hat{t}_i(S_k) = t_i(S_k) + \varepsilon_i^{(t)}(S_k)$$

$$\hat{\alpha}_i(S_k) = \alpha_i(S_k) + \varepsilon_i^{(\alpha)}(S_k)$$

$$\hat{\tau}_{ij}(S_k) = \tau_{ij}(S_k) + \varepsilon_{ij}^{(\tau)}(S_k)$$

where $t_i(S_k) = t_0(S_k) + \dfrac{d(P_i,S_k)}{C}$, $\tau_{ij}(S_k) = t_i(S_k) - t_j(S_k)$.

We assume that $\{\varepsilon_i^{(t)}(S_k)\}$ and $\{\varepsilon_i^{(\alpha)}(S_k)\}$ follow i.i.d $\mathrm{N}(0,\sigma_\alpha^2)$, $\mathrm{N}(0,\sigma_t^2)$. Then, according to the additivity of the normal distribution, $\{\varepsilon_{ij}^{(\tau)}(S_k)\}$ also follows $\mathrm{N}(0,\sigma_t^2)$. Under this assumption, we will discuss the problem of estimating multiple source locations separated from each other.

2. MULTIPLE SOURCE LOCALIZATION ALGORITHM

Let us first consider single source location estimation prior to multiple source localization. Let m=1, and the source location S_1 is estimated as follows.

$$\hat{S}_1 = \underset{S \in \mathbb{R}^2}{\arg\min} \left\{ \sum_{i=1}^{n} \left| \alpha_i(S) - \hat{\alpha}_i(S_1) \right| + \lambda \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \left| \tau_{ij}(S) - \hat{\tau}_{ij}(S_1) \right| \right\} \quad (1)$$

where $\alpha_i(S) = \theta(P_i,S)$, $\tau_{ij}(S) = \dfrac{d(P_i,S) - d(P_j,S)}{C}$ and λ is a constant.

Since $\{\varepsilon_i^{(\alpha)}(S_1)\}$ follows the uniform distribution $\mathrm{N}(0,\sigma_\alpha^2)$, according to the 3σ-law of the normal distribution, the probability $P\left(\left|\varepsilon_i^{(\alpha)}(S_1)\right| < 3\sigma_\alpha\right)$ that the absolute value of the azimuth estimation error in the i th sensor is less than $3\sigma_\alpha$ is given by 0.9973.(Later we use η=0.9973).

Thus, if we denote by $P\left(\left|\varepsilon_i^{(\alpha)}(S_1)\right| < 3\sigma_\alpha\right)$ the event that the absolute value of the azimuth estimation error at all stations will be smaller than $3\sigma_\alpha$, then satisfy following equation.

$$P\left(A^{(\alpha)}\right) = \prod_{i=1}^{n} P\left(\left|\varepsilon_i^{(\alpha)}(S_1)\right| < 3\sigma_\alpha\right) = \eta^n$$

Since $\{\varepsilon_{ij}^{(\tau)}(S_1)\}$ follows the $\mathrm{N}(0,\sigma_t^2)$, according to the 3σ-law of normal distribution, the probability that the absolute value of the TDOA estimation error at the $i,(i \in \mathbb{Z}_n^+)$ th and $j,(j \in \mathbb{Z}_n^+)$ th sensors is less than $3\sigma_t$ is equal to η, i.e. $P\left(\left|\varepsilon_{ij}^{(\tau)}(S_1)\right| < 3\sigma_t\right) = \eta$.

Therefore, if $A^{(\tau)}$ is the event that the absolute value of the arrival time difference instantaneous error in all possible sensor pairs is less than $3\sigma_t$, following equation will hold.

$$P\left(A^{(\tau)}\right) = \prod_{i=1}^{n-1} \prod_{j=i+1}^{n} P\left(\left|\varepsilon_{ij}^{(\tau)}(S_1)\right| < 3\sigma_t\right) = \eta^{\frac{n(n-1)}{2}}$$

So, we can get following equation:

$$P\left(A^{(\alpha)} \cap A^{(\tau)}\right) = \eta^{n+\frac{n(n-1)}{2}} = \eta^{\frac{n(n+1)}{2}}$$

Let us now define the following functions for point $S(x,y) \in \mathbb{R}^2$, which are nonlinear functions with a peak at some point near the source.

$$f_\alpha(x,y) := \begin{cases} 0, & \exists i : \left|\alpha_i(S) - \hat{\alpha}_i(S_1)\right| > 3\sigma_\alpha, \\ 0, & \exists i,j : \left|\tau_{ij}(S) - \hat{\tau}_{ij}(S_1)\right| > 3\sigma_t, \\ [2ex] \dfrac{3 \cdot \sigma_\alpha \cdot n(n-1)}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\left|\alpha_i(S) - \hat{\alpha}_i(S_1)\right| + \left|\alpha_j(S) - \hat{\alpha}_j(S_1)\right|\right)}, & \text{otherwise.} \end{cases} \quad (2)$$

$$f_\tau(x,y):=\begin{cases}0, & \exists i:\left|\alpha_i(S)-\hat{\alpha}_i(S_1)\right|>3\sigma_\alpha,\\ 0, & \exists i,j:\left|\tau_{ij}(S)-\hat{\tau}_{ij}(S_1)\right|>3\sigma_t,\\ \dfrac{3\sigma_t n(n-1)}{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|\tau_{ij}(S)-\hat{\tau}_{ij}(S_1)\right|}, & \text{otherwise.}\end{cases} \tag{3}$$

The Fig.1 shows the shapes of these functions. The Fig.1 is a graph of $f_\alpha(x,y)$, Fig 1-2 is a graph of $f_\tau(x,y)$, Fig.1-Fig.3 is a graph of $f_\alpha(x,y) + f_\tau(x,y)$, Fig.1-Fig.4 is a graph of a function with a unique peak at the source. These graphs are generated in the case where the distance from the source to the sensor is about 10 km, the maximum distance between sensors is about 4 km, and σ_α = 0.02(rad), σ_t = 100(ms). The detailed coordinates of the sensors are as follows. These graphs are generated in the case where the distance from the source to the sensor is about 10 km, the maximum distance between sensors is about 4 km, and σ_α = 0.02 (rad), σ_t = 100(ms). The detailed coordinates of the sensors are as follows.

S_1 = (19000, 7000),
P_1 = (12309, 4731),
P_2 = (11037, 6282),
P_3 = (9265, 8639),
P_4 = (8639, 9707)

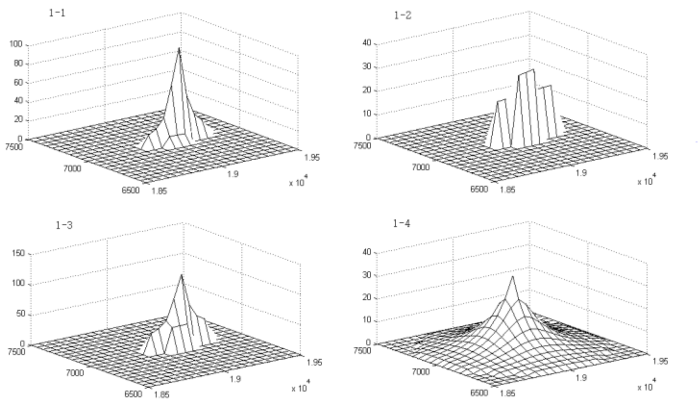

Fig.1. Graph of functions with maximum value around of a source

From these two functions, we can construct the likelihood function at point $S(x,y)\in\mathbb{R}^2$ as follows:

$$f_1(x,y):=f_\alpha(x,y)+\lambda f_\tau(x,y) \tag{4}$$

In the above expression, λ is the weighting factor. At this time, $S_1(x_1,y_1)$ is estimated as follows.

$$\hat{S}_1(x_1,y_1)=\underset{(x,y)}{\arg\max}\, f_1(x,y) \tag{5}$$

It can also be seen that, as discussed above, the probability that $S_1(x_1,y_1)$ is estimated is $\eta^{\frac{n^2(n-1)}{2}}$. Let us extend this idea to multi-source localization. If we know the azimuth estimates $\{\hat{\alpha}_i(S_k)\}$, $i\in\mathbb{N}_n^+$ for the $k\left(k\in\mathbb{N}_m^+\right)$ th source and the target signal detection time $\{\hat{t}_i(S_k)\}$, $i\in\mathbb{N}_n^+$ then the function given by Eq.(8) for $S(x,y)\in R^2$ has a peak in the neighbourhood of the k th source S_k and satisfied the Eq.(9).

Let us define the following functions.

$$f_\alpha^{(k)}(x,y):=\begin{cases}0, & \exists i:\left|\alpha_i(S)-\hat{\alpha}_i(S_k)\right|>3\sigma_\alpha,\\ 0, & \exists i,j:\left|\tau_{ij}(S)-\hat{\tau}_{ij}(S_k)\right|>3\sigma_t,\\ \dfrac{n(n-1)3\sigma_\alpha}{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(\left|\alpha_i(S)-\hat{\alpha}_i(S_k)\right|+\left|\alpha_j(S)-\hat{\alpha}_j(S_k)\right|\right)}, & \text{otherwise.}\end{cases} \tag{6}$$

$$f_\tau^{(k)}(x,y):=\begin{cases}0, & \exists i:\left|\alpha_i(S)-\hat{\alpha}_i(S_k)\right|>3\sigma_\alpha,\\ 0, & \exists i,j:\left|\tau_{ij}(S)-\hat{\tau}_{ij}(S_k)\right|>3\sigma_t,\\ \dfrac{n(n-1)3\sigma_t}{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|\tau_{ij}(S)-\hat{\tau}_{ij}(S_k)\right|}, & \text{otherwise.}\end{cases} \tag{7}$$

$$f_k(x,y):=f_\alpha^{(k)}(x,y)+\lambda f_\tau^{(k)}(x,y) \tag{8}$$

And then, the source location is estimated as follows.

$$\hat{S}_k(x_k,y_k)=\underset{(x,y)}{\arg\max}\, f_k(x,y) \tag{9}$$

Thus, the function $f(x,y)$, defined as Eq.(10), has a peak only in the neighbourhood of the source and taking 0 in the rest.

$$f(x,y):=\sum_{k=1}^{m} f_k(x,y) \tag{10}$$

In other words, the multi-source localization problem leads to the problem of finding the maximum points of Eq.(10).

Let $\left\{\left(\hat{\alpha}_{i,j},\hat{t}_{i,j}\right)\right\}, i\in\mathbb{N}_n^+, j\in\mathbb{N}_m^+$ be the AOA and target detection time estimates for m targets obtained from n sensors, and Eq.(10) can be approximated as follows.

$$F_\alpha^{(S)}(i,j_i,k,j_k):=\begin{cases}0, & \begin{array}{l}\left|\alpha_i(S)-\hat{\alpha}_{i,j_i}\right|>3\sigma_\alpha\\ \forall\left|\alpha_k(S)-\hat{\alpha}_{k,j_k}\right|>3\sigma_\alpha,\end{array}\\ 0, & \left|t_i(S)-t_k(S)-\left(\hat{t}_{i,j_i}-\hat{t}_{k,j_k}\right)\right|>3\sigma_t,\\ \begin{array}{l}\left|\alpha_i(S)-\hat{\alpha}_{i,j_i}\right|\\ +\left|\alpha_k(S)-\hat{\alpha}_{k,j_k}\right|,\end{array} & \text{otherwise.}\end{cases} \tag{11}$$

$$F_\tau^{(S)}(i,j_i,k,j_k):=\begin{cases}0, & \begin{array}{l}\left|\alpha_i(S)-\hat{\alpha}_{i,j_i}\right|>3\sigma_\alpha\\ \forall\left|\alpha_k(S)-\hat{\alpha}_{k,j_k}\right|>3\sigma_\alpha,\end{array}\\ 0, & \left|\begin{array}{l}t_i(S)-t_k(S)\\ -\left(\hat{t}_{i,j_i}-\hat{t}_{k,j_k}\right)\end{array}\right|>3\sigma_t,\\ \left|\begin{array}{l}t_i(S)-t_k(S)\\ -\left(\hat{t}_{i,j_i}-\hat{t}_{k,j_k}\right)\end{array}\right|, & \text{otherwise.}\end{cases} \tag{12}$$

$$\Omega:=\left\{(i,j_i,k,j_k)\mid F_\alpha^{(S)}(i,j_i,k,j_k)\neq 0\right\}$$

$$f(x,y)\approx\frac{3\sigma_\alpha n(n-1)}{\sum_{\omega\in\Omega}F_\alpha^{(S)}(\omega)}+\lambda\frac{3\sigma_t n(n-1)}{\sum_{\omega\in\Omega}F_\tau^{(S)}(\omega)} \tag{13}$$

Eq.(13) can be thought as the maximum likelihood of a point $S(x,y)$ for a given measurement data. The algorithm for constructing the function $f(x,y)$, given by Eq.(13), for a $S(x,y)\in R^2$

is given below. The algorithm for computing the maximum likelihood

Step 1: $i=0, k_1=0, k_2=0, N_\alpha=0, N_t=0, f_\alpha=0, f_t=0$

Step 2: $i \leftarrow i+1, j \leftarrow i, k_1 \leftarrow 0$, compute $\alpha_i(S)$, $t_i(S)$.

Step 3: $k_1 \leftarrow k_1+1$

Step 4: $j \leftarrow j+1, k_2 \leftarrow 0$, compute $\alpha_j(S)$, $t_j(S)$.

Step 5: $k_2 \leftarrow k_2+1$

Step 6: $\Delta\alpha_i = \left|\alpha_i(S)-\hat{\alpha}_{i,k_1}\right|,\ \Delta\alpha_j = \left|\alpha_j(S)-\hat{\alpha}_{j,k_2}\right|,$

$$\Delta\tau_{ij} = \left|t_i(S)-t_j(S)-\left(\hat{t}_{i,k_1}-\hat{t}_{j,k_2}\right)\right|$$

Step 7: If $\Delta\alpha_i < 3\sigma_\alpha \wedge \Delta\alpha_j < 3\sigma_\alpha \wedge \Delta\tau_{ij} < 3\sigma_t$ then

$$f_\alpha \leftarrow f_\alpha + \Delta\alpha_i + \Delta\alpha_j,\ f_t \leftarrow f_t + \Delta\tau_{ij}$$

$$N_\alpha \leftarrow N_\alpha + 2,\ N_t \leftarrow N_t + 1$$

Step 8: If $k_2 < m$ then go to step 5, else go to step 9

Step 9: If $j < n$ then go to step 4, else go to step 10

Step 10: If $k_1 < m$ then go to step 3, else go to step 11

Step 11: If $i < n$ then go to step 2, else go to step 12

Step 12: If $N_\alpha \geq N$ then $f_\alpha(x,y) = \dfrac{n(n-1)3\sigma_\alpha}{f_\alpha}$ else $f_\alpha(x,y)=0$

Step 13: If $N_t \geq N/2$ then $f_t(x,y) = \dfrac{\frac{n}{2}(n-1)3\sigma_t}{f_t}$, else $f_t(x,y)=0$

Step 14: $f(x,y) = f_\alpha(x,y) + \lambda f_t(x,y)$

In the above algorithm, N should normally be equal to $n(n+1)/2$. However, in practical applications, it is common to use values less than $n(n+1)$ because some sensors cannot obtain measurements. By applying a genetic algorithm using the likelihood function computed in this way, we can estimate multiple source locations simultaneously.

The length l of the individual binary sequence was allowed to vary variably, and gray coding was performed. The population size M=50 was chosen, and the selection operation was performed on a random sample. The mating operation was single point mating, the mating probability P_c=0.7, and the mutation operation was single mutation, the mutation probability P_m=0.01. In the following, we describe the search region establishment and coding, initial population selection, and final result processing, which are important issues in applying genetic algorithm.

2.1 SETTING UP THE SEARCH AREA

Setting up the search area well is very important for speeding up the operation. Since the length of a gene varies with the size of the search area, it should be set to reduce the length of the gene with as much accuracy as possible. Let $I := \left\{I_{(i,k_1)}^{(j,k_2)}\right\}$, $\left(i,j \in \square_n^+, k_1,k_2 \in \square_m^+, i \neq j\right)$ be the set of intersection points of two lines (the azimuth lines at each sensor) with azimuth angles of $\alpha_{(i,k_1)}, \alpha_{(j,k_2)}$ respectively across P_i, P_j, and we can use these intersection points to change the search region variably.

Let,

$$X_{\min} := \min_x \left\{I_{(i,k_1)}^{(j,k_2)}\right\},\ X_{\max} := \max_x \left\{I_{(i,k_1)}^{(j,k_2)}\right\}$$

$$Y_{\min} := \min_y \left\{I_{(i,k_1)}^{(j,k_2)}\right\},\ Y_{\max} := \max_y \left\{I_{(i,k_1)}^{(j,k_2)}\right\}$$

$$\left(i,j \in \square_n^+, k_1,k_2 \in \square_m^+, i \neq j\right)$$

Then we can set the search region Ω as follows:

$$\Omega := \left\{(x,y) \mid x \in (X_{\min}-L, X_{\max}+L), y \in (Y_{\min}-L, Y_{\max}+L)\right\}$$

In the above expression, L is a positive constant, generally set to around 1000. Once the search region Ω is set, any point in the search region can be encoded with reference to the starting point of the region $(X_{\min}-L, Y_{\min}-L)$.

2.2 CODING

Since the likelihood function we discussed above is a function that takes a positive value only in the vicinity of the source, we must increase the local search ability in the vicinity of the source. To this end, we performed Gray coding. In addition, since the position of the source in the 2D plane has to be determined, the variables x,y are each Gray coded and they are combined in a given order and represented as a binary code sequence according to the multivariate coding method. When the length of the segment is less than $\delta(m)$, the length l of the gene is determined as follows.

$$l_x := \min\left\{n_x \mid 2^{n_x}\delta > X_{\max} - X_{\min} + 2L\right\}$$

$$l_y := \min\left\{n_y \mid 2^{n_y}\delta > Y_{\max} - Y_{\min} + 2L\right\}$$

$$l = l_x + l_y$$

2.3 INITIAL POPULATION

The initial population consists of randomly selected points in the search domain, which is a common method in genetic algorithms. However, if we construct the initial population in this way, the characteristics of the likelihood function we propose (positive values only in the vicinity of the source and zero in the rest) makes it a long time to converge to the maximum points and it is not possible to estimate the sources simultaneously even if they converge. We thus constructed the initial population as follows. First, for any point in the set I is consisting of the intersection points of the azimuth lines, we construct a new set I' obtained by adding arbitrarily chosen points in some neighborhood (about δ=100) of that point. Then, for every point of I', we compute the fitness according to the algorithm that constructs the function $f(x,y)$ given by Eq.(13). Then, based on this fitness, we select N elements of I' by stochastic sampling method. The population thus obtained is taken as the initial population. Constructing the initial population in this way takes the initial approximation points in the vicinity of the sources and thus converges quickly to the maximum points.

3. NUMBER OF GENERATIONS TO BE INHERITED

The purpose of using the genetic algorithm based on the survival environment is to estimate N source locations simultaneously and with high accuracy, so we used the following approach to satisfy both these conditions. First, the number of generations to be inherited, T, is not large (less than 10). Then, among the $M+N$ individuals obtained through T genetic operations, we select individuals whose fitness is larger than a certain specified value. Among the selected individuals, the individuals corresponding to the same source are weighted average according to the fitness. In this way, based on the AOA and TDOA at each sensor, the GA can simultaneously estimate the locations of multiple sources with high accuracy.

4. SIMULATION EXPERIMENTS

To evaluate the performance of the simulation multi-source localization algorithm, we have written the following simulation scheme.

4.1 EXPERIMENT SCHEME 1

If the first reference sensor location is fixed, the sensors are placed on the same line, and the sources are randomly selected in the neighbourhood of a fixed point in the detection zone, in this simulation, the sensors are placed at equal intervals on a line parallel to the x-axis, and the sources are assumed to be randomly selected in a circle centered at a fixed point in the detection zone and with a radius of 1 km.

We then evaluate the source detection accuracy and multi-source localization accuracy by varying the spacing between sensors. In the Cartesian coordinate system, the position of the first reference sensor is set to [5000, 5000], and the position of the fixed point to [10,000, 25,000]. (The unit is m).

4.2 EXPERIMENT SCHEME 2

If the position of the first reference sensor is fixed, the sensors are collinear, and the sources are randomly selected within the detection zone, in this simulation we assume that the sensors are equally spaced on a straight line parallel to the x-axis, and the sources are randomly selected within the detection zone We then evaluate the target detection accuracy and multi-source localization accuracy by varying the distance between sensors.

In the Cartesian coordinate system, the position of the first reference sensor is set to [5000,5000], and the detection region is set to [-5000,15000] x [10,000, 30,000]. All the simulations with multi-source localization assume that the number of sources is 10 and the time to generate the signal at each source is randomly chosen between [80 ms, 100 ms]. It is also assumed that only four sensors are used and all sensor locations are known correctly. It is also assumed that the number of trials in each simulation is 200 times, the AOA measurement error is $N(0,0.0175^2)$, the unit is radian (rad), the TDOA measurement error is $N(0,0.1^2)$ and the unit is seconds (s).

The performance evaluation is performed by the localization accuracy (%d), Cramer-Rao lower bound (CRLB) and target detection rate (number of estimated source / total number of source). The proposed method is denoted as (GA-MSL) in the sense of multi-source location estimation using genetic algorithm and is compared with the proposed method (IPDA) in [24] and CRLB-Hybrid in [23]. The Fig.2 and Fig.3 show the target detection rate and the localization accuracy of each method according to the experimental scheme 1. In this simulation, we evaluate the accuracy of multi-source localization in the case of eight sources per second. It can be seen from Fig.2 that the method proposed in [24] has a low detection accuracy for source locations. This is due to the fact that this method uses only the AOAs to combine the data. However, since our method uses the AOA and TDOA information simultaneously to compute the likelihood function, it can achieve the high accuracy in these scenarios. The Fig.4 and Fig.5 show the multi-source localization performance in scenario 2.

The main purpose of this experimental scheme is to observe how the multi-source localization accuracy can be achieved if the sources are not dense.

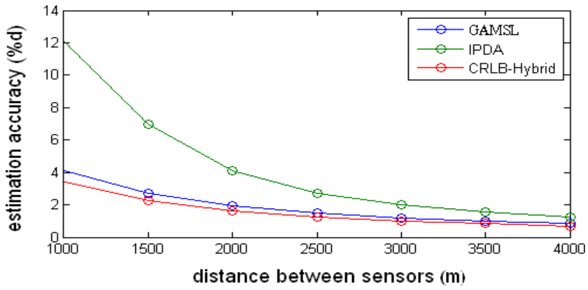

Fig.3. Accuracy with distance between sensors (Experiment 1)

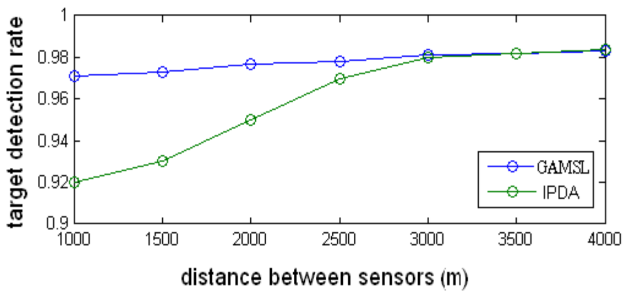

Fig.4. Target detection rate with between sensors (Experiment 2)

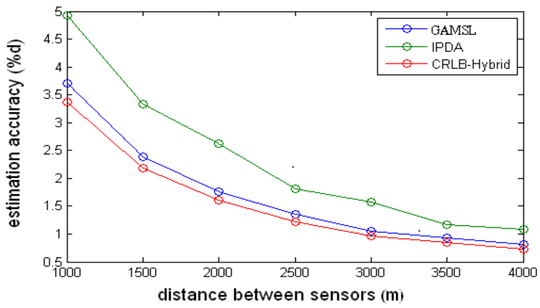

Fig.5. Accuracy with distance between sensors (Experiment 2)

As shown in the figure, if the sources are not dense, we can see that the proposed method has high target detection rate and high localization accuracy than the method proposed in [24]. If we compare the results of Experiment Scheme 1 and 2, we can notice that the result of Experiment Scheme 1 is outperform than the Experiment Scheme 2. Because the sources are far enough apart, the accuracy of the data association is high, and the accuracy of the localization is also high.

Through simulation experiments for the performance evaluation of multi-source localization, it can be seen that our proposed method can be successfully used even when the sources are clustered in a narrow region.

5. CONCLUSION

In this paper, we discuss how to make multi-source localization using the AOA and TDOA measurements obtained from the small sensor arrays.

The method of finding all pairs of measurements corresponding to a single source from measurements obtained from multiple sources using probabilistic data association and perform the source localization is a widely used and effective method for multi-source localization, but the error of data association has a great influence on the location estimation accuracy. In order to improve the accuracy of multi-source localization under these conditions, we derive a likelihood function that can estimate multiple sources simultaneously and propose a method to solve this problem by applying genetic algorithms.

The contributions of this paper are as follows. First, under the assumption that the measured data are normally distributed, a new likelihood function for multi-source localization is proposed. Second, we propose a method to efficiently perform maximum likelihood estimation for multi-source localization by applying genetic algorithms. Third, simulation experiments have shown the effectiveness of the proposed method.

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