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INSTANTANEOUS FREQUENCY ESTIMATION OF THE CHIRP SIGNAL BY COMBINING PROJECTION METHOD AND STOCHASTIC RESONANCE METHOD

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Abstract
In this paper, we propose a method to solve the instantaneous frequency estimation problem at each point of the digital signal sequence obtained by digitizing the chirped signal by combining the projection method with the stochastic resonance method. This problem is chosen as the estimation of the instantaneous frequency (instantaneous frequency at the midpoint of a frame) at the center of the frame, and then we can solve the above problem through the orthogonal projection of a space of which dimension is equal to the frame length into a two-dimensional subspace, combined with stochastic resonance theory. Assuming this digital signal frame as a vector lying in a space of which dimension is equal to the length of the frame, we estimate the instantaneous frequency of the frame center point by finding the basis vector of the corresponding frequency that constitutes the two-dimensional subspace in which the vector is placed. By moving the center point of the frame onto each point of the digital signal sequence corresponding to a period of frequency modulation, we can obtain the overall frequency curve accurately. At this time, the basis vectors that constitute the two-dimensional subspace are constructed by reflecting the frequency modulation characteristics. The estimation results obtained in the simulation are compared with the results using the short-time Fourier transform, Wigner-Bill distribution, which are often used in the presence of noise and Doppler effects such as Doppler radar and sonar, which shows that the proposed method has very high accuracy.

Keywords:
IF (Instantaneous Frequency), FM (Frequency Modulation), Stochastic Resonance, Signal Frame, Basis Vector

1. INTRODUCTION

Instantaneous frequency (IF) estimation in FM signals attracts more interests in various fields such as RADAR, SONAR signal processing applications, audio and music analysis, and monitoring [1-5].

In general, the instantaneous frequency estimation problem is defined as a finding the derivative of phase-change function $\phi(t)$ in a signal $f(t)=a(t)\cos(\phi(t))$ with respect to the time, where the amplitude $a(t)$ is assumed to change relatively slowly.

In practical applications such as radar, sonar and communication systems, frequency modulation techniques have been commonly used, and accurate estimation of the instantaneous frequency of these FM signals is a critical due to the essential influences on the performance of the overall system. Therefore, many studies have been reported in the context of instantaneous frequency estimation, such as Hilbert transform, short-time Fourier transform, wavelet transform, and estimation using Wigner-Bile distribution.

In [5-10], the problem of instantaneous frequency estimation and its application using Hilbert transform and its extension are addressed. Here, we describe the following complex trigonometric form by Hilbert transform of the signal to estimate the instantaneous frequency.

$$x(t)=Ae^{j\phi(t)}$$

And from the derivative of phase function $\phi(t)$:

$$\Omega(t)=\phi'(t) \tag{1}$$

We can obtain the frequency at the instant. The IF estimation using the Hilbert transform is mathematically rigorous, however it has a disadvantage that it is strongly affected by the presence of noise in practice.

Researchers proposed new algorithms [11-16] to calculate the instantaneous frequency of multi-component nonstationary signals using short-time Fourier transform (STFT), its variants and a combination of spectral spectra (FS=Adaptive Fractional Spectrogram).

The short-time Fourier transform (STFT) is mathematically defined as

$$\mathrm{STFT}(t,f)=\int_{-\infty}^{\infty} s(\tau)\,h(\tau-t)\,e^{-j2\pi f\tau}\,d\tau \tag{2}$$

where $h(\tau)$ is the window function used for the analysis. The resolution of the STFT depends on the shape and size of the window. The long window has a good frequency resolution, while the short one has a good time resolution. The S-transform, which is a alternation of STFT, may be also used, where the width of the Gaussian window is inversely proportional to the frequency. Using the above transformations, the optimized window length at each time step is calculated by the method proposed in [17]. Using the transformation based on Eq.(2) and optimizing the parameter definition, it was proved to be simple and effective for calculating the time-frequency distribution (TFD).

The high-resolution time-frequency distribution is defined by combining the FS calculated using the length and noise variations, and the IF of each signal component is calculated by applying the peak search and component extraction procedure. In [16], it is found that the least squares deviation of the IF estimations computed with the adaptive spectrum (AFS Adaptive Fractional Spectrogram) is smaller than that obtained with other time-frequency-distributions.

Studies on IF estimation using wavelets have also been conducted, which use a combination of the advantages of complex-shifted Morlet wavelets with the those of subspace approaches [18, 19]. In [20], the complex-shift Morlet wavelet (CSMW) method is modulated in the time domain by a Gaussian time window, providing an optimal solution in both the time- and frequency-domains simultaneously, leading to a continuous approximation in the time and frequency domains simultaneously compared to the discrete wavelet transform (DWT), and not related to the specific leakage effect, and has the advantage of allowing simultaneous computation of the instantaneous amplitude and frequency of the signal.

For IF estimation at low signal-to-noise ratios, Wigner-Ville (WVD) distribution is often used by detecting peaks in time frequency distribution [11, 21-25]. Higher order phase functions [26] and wavelet dictionary-based joint tracking methods [27] for IF estimation are also proposed, and a multi-component analysis method using Fourier Bessel series and time-varying autoregressive (FB-TVAR) models is also introduced in [28].

Although the proposed methods have achieved a little improvement in accuracy and computational cost, the accuracy of the estimated instantaneous frequency is still not satisfactory, due to the unavoidable effect of noise on the input signal. In addition, these methods are difficult to apply in the general methodology of Fourier series by constructing orthogonal basis vectors that satisfy a strict orthogonality with digital signal sequence frames in the form of harmonic signals such as sine and cosine function when processing digital signals in frame units.

In this paper, we propose a novel method to estimate the frequency variation of a frame by applying orthogonal projection method to a two-dimensional subspace by means of basis vectors with frequency modulation characteristics, by removing the noise from the original signal of a noisy digital signal source using stochastic resonance method and then considering the frame as a vector in a finite dimensional space of dimensions equal to its length. This allows us to obtain the frequency variation of the frame reflecting the frequency modulation characteristics under constant noise removal and, based on it, to estimate the instantaneous frequency of the center point, thus increasing the accuracy of the instantaneous frequency estimation. We address the problem of frequency estimation at the center point of a frame $R=(r_1,r_2,\ldots,r_N)$ with a digitized signal sequence. Here, the time interval corresponding to a frame is considered relatively small, and the amplitudes S_i of the effective signal in the signal sequence within the frame are considered constant S.

As for using Doppler effects, such as in Doppler RADAR and SONAR, the frequency shift coefficient ($\tilde{k}_\omega$) in the received signal, depending on the object's motion, has a value different from the k_ω that when it is radiated and varies with the moving speed.

In Section 2, we consider the digital signal frame $R=(r_1,r_2,\ldots,r_N)$ as an element of the N-dimensional space Y^N and discuss the theoretical considerations of the method of estimating the center frequency by orthogonal projection into a two-dimensional subspace generated by two vectors with the characteristics of chirped. In Section 3, the proposed method is compared and tested by simulation, and in Section 4, the analysis of the proposed method is presented.

The comparative verification is compared with the results of the application of the short-time Fourier transform method and the Wigner-Bile distribution, given that the proposed method is an improved form of the short-time Fourier transform.

2. DIGITAL SIGNAL FRAME CENTER FREQUENCY ESTIMATION AND ORTHOGONAL PROJECTION METHOD

In general, the linearly frequency modulated receiving signal may have a mathematical formulation as following.

$$r(t) = S(t)\sin\left(\omega_0 t + \frac{1}{2}k_{\omega_0}t^2 + \phi_0\right) + n(t), \tag{3}$$

where ω_0 is a constant frequency, k_{ω_0} frequency modulation linearity coefficient, making the frequency changed linearly with respect to the time as $\omega_0 + k_{\omega_0}\cdot t$, ϕ_0 the initial phase, n(t) the additive WGN. Sampling the signal of Eq.(3) will lead to as following.

$$r(t_i) = S(t_i)\sin\left(\omega_0 t_i + \frac{1}{2}k_{\omega_0}t_i^2 + \phi_0\right) + n(t_i), \quad i = 0,1,2,\ldots$$

Let the sampling period as Δt , $t_i = \Delta t\cdot i$, $i = 0,1,2,\ldots$, then $S_i = S(t_i)$, $\omega = \omega_0\cdot\Delta t$, $k_\omega = k_{\omega_0}\cdot\Delta t^2$, $n_i=n(t_i)$, and the digital signal sequency for the above expression can be obtained as

$$r_i = S_i\sin\left(\omega i + \frac{1}{2}k_\omega i^2 + \phi_0\right) + n_i, \quad i = 0,1,2,\ldots \tag{4}$$

where ω denotes the initial angular frequency, ϕ_0 denotes the initial phase at time t=0.

As for the phase change of the Eq.(4)

$$\omega\cdot i + \frac{1}{2}\cdot k_\omega\cdot i^2 + \phi_0,\ i = 0,1,2,... \tag{5}$$

The change relationship for the arbitrary frame $i=j_0+j$, $j=1,2,\ldots,N$ corresponding to the above-mentioned digital signal frame $R=(r_1,r_2,\ldots,r_N)$ can be as following.

$$\begin{aligned}&\omega\cdot(j_0+j) + \frac{1}{2}\cdot k_\omega\cdot(j_0+j)^2 + \phi_0\\ &= \left(\omega\cdot j_0 + \frac{1}{2}\cdot k_\omega\cdot j_0^2 + \phi_0\right) + \left(\omega + k_\omega\cdot j_0\right)\cdot j + \frac{1}{2}k_\omega\cdot j^2,\end{aligned}$$

As can be seen in the above expression, the phase change in the frame $R=(r_1,r_2,\ldots,r_N)$ has the same formulation as the Eq.(5), where the initial phase is $\left(\omega_0\cdot j_0 + \frac{1}{2}\cdot k_\omega\cdot j_0^2 + \phi_0\right)$, the initial angular frequency $\left(\omega_0 + k_\omega\cdot j_0\right)$. Therefore, we let $\left(\omega + k_\omega\cdot j_0\right)$, $\left(\omega\cdot j_0 + \frac{1}{2}\cdot k_\omega\cdot j_0^2 + \phi_0\right)$ as the initial angular frequency ω, the initial phase ϕ_0 respectively, and consider that the phase change will have the form as following:

$$\omega\cdot i + \frac{1}{2}\cdot k_\omega\cdot i^2 + \phi_0,\ i = 0,1,2,...,N$$

From which we can estimate ω, and then calculate the angular frequency at the midpoint of the frame $\omega + k_\omega\cdot\frac{N}{2}$.

2.1 THEORETICAL INTERPRETATION

In this section, we present the considerations concerning the central frequency estimation of the aforementioned digital signal sequence frame $R=(r_1,r_2,\ldots,r_N)$. Eq.(4) may be rewritten as following.

$$r_i = S\cdot\sin\left(\omega\cdot i+\frac{1}{2}k_\omega\cdot i^2\right)\cdot\cos(\phi_0) + S\cdot\cos\left(\omega\cdot i+\frac{1}{2}k_\omega\cdot i^2\right)\cdot\sin(\phi_0)+n_i,\quad i=1,2,\ldots,N$$

$$S_i = S\cdot\sin\left(\omega\cdot i+\frac{1}{2}k_\omega\cdot i^2\right)\cdot\cos(\phi_0) + S\cdot\cos\left(\omega\cdot i+\frac{1}{2}k_\omega\cdot i^2\right)\cdot\sin(\phi_0),\quad i=1,2,\ldots,N \tag{6}$$

In the above equations, ω is the frequency at the beginning of the frame, the first is the received signal, and the second is the effective signal except for noise. To estimate the digital signal frame center frequency (frequency at frame center), we consider one frame $R=(r_1,r_2,\ldots,r_N)$ of digital signal as an element of an N-dimensional space Y^N and look at the correlation between this frame and the two-dimensional subspace $Y_{\tilde\omega}^2$ generated by two independent vectors $\xi_s(\tilde\omega)$, $\xi_c(\tilde\omega)$ for a certain frequency $\tilde\omega$.

$$\xi_s(\tilde\omega)=(\sin(\tilde\omega\cdot 1+\frac{1}{2}k_\omega\cdot 1^2),\sin(\tilde\omega\cdot 2+\frac{1}{2}k_\omega\cdot 2^2),\ldots,\sin(\tilde\omega\cdot N+\frac{1}{2}k_\omega\cdot N^2))^T$$

$$\xi_c(\tilde\omega)=(\cos(\tilde\omega\cdot 1+\frac{1}{2}k_\omega\cdot 1^2),\cos(\tilde\omega\cdot 2+\frac{1}{2}k_\omega\cdot 2^2),\ldots,\cos(\tilde\omega\cdot N+\frac{1}{2}k_\omega\cdot N^2))^T \tag{7}$$

In other words, the effective signal of Eq.(6) will be expressed by linear combination of the following sequences:

$$\xi_s(\omega)=(\sin(\omega\cdot 1+\frac{1}{2}k_\omega\cdot 1^2),\sin(\omega\cdot 2+\frac{1}{2}k_\omega\cdot 2^2),\ldots,\sin(\omega\cdot N+\frac{1}{2}k_\omega\cdot N^2))^T$$

$$\xi_c(\omega)=(\cos(\omega\cdot 1+\frac{1}{2}k_\omega\cdot 1^2),\cos(\omega\cdot 2+\frac{1}{2}k_\omega\cdot 2^2),\ldots,\cos(\omega\cdot N+\frac{1}{2}k_\omega\cdot N^2))^T \tag{8}$$

with respect to frequency ω, which means that the following one

$$S_i = S\cdot\sin\left(\omega\cdot i+\frac{1}{2}k_\omega\cdot i^2\right)\cdot\cos(\phi_0) + S\cdot\cos\left(\omega\cdot i+\frac{1}{2}k_\omega\cdot i^2\right)\cdot\sin(\phi_0),\quad i=1,2,\ldots,N$$

can be considered as the N-dimensional vector underlying on the 2D subspace Y_ω^2 with respect to the frequency ω.

The N-dimensional space Y^N can be considered to be composed of two-dimensional subspace Y_ω^2 and orthogonal subspace $Y_{\tilde\omega}^{N-2}$, so that the digital signal frame R also consists of the projective component $R_{\tilde\omega}$ into the two-dimensional subspace Y_ω^2 and the projective component $R_{\tilde\omega}^{\perp}$ into the orthogonal subspace $Y_{\tilde\omega}^{N-2}$ (see Fig.1).

$$R=\left(R_{\tilde\omega}\cdot R_{\tilde\omega}^{\perp}\right) \tag{9}$$

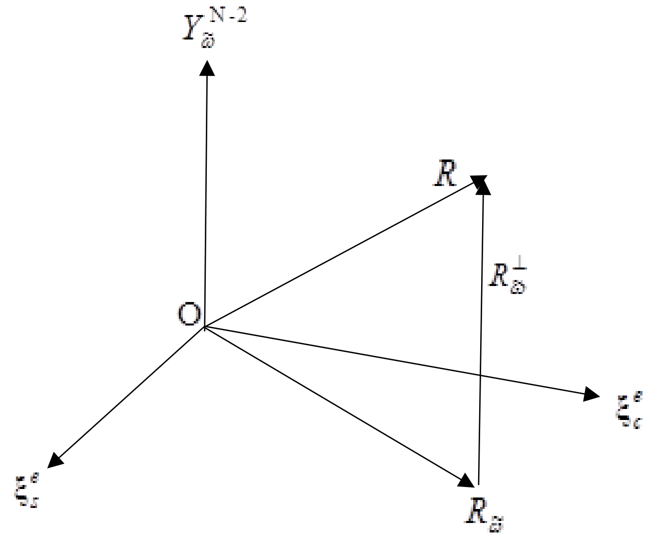

Fig.1. Vector R in N-dimensional space Y^N

As can be seen in Fig.1, two orthogonal subspaces $Y_{\tilde\omega}^2$ and $Y_{\tilde\omega}^{N-2}$ of Y^N will be changed according to the frequency $\tilde\omega$, hence the two projections $R_{\tilde\omega}$ and $R_{\tilde\omega}^{\perp}$ of R will be too. Clearly, the norm $\|R_{\tilde\omega}\|_{Y^N}$ of the vector $R_{\tilde\omega}$ is not larger than the norm $\|R\|_{Y^N}$ of the digital signal frame R.

If the noise components n_i, i=0,1,2,…,N are equal to zero, then the norm $\|R_{\tilde\omega}\|_{Y^N}$ is smaller than the norm $\|R\|_{Y^N}$ and when $\tilde\omega$ equal ω in Eq.(5), $\|R_{\tilde\omega}\|_{Y^N}$ and $\|R\|_{Y^N}$ are equal, R is completely located in the subspace $R_{\tilde\omega}$, and R is represented by a linear combination of the independent vectors of Eq.(8).

$$\begin{aligned} &\|R_{\tilde\omega}\|_{Y^N} < \|R\|_{Y^N},\quad \omega\neq\tilde\omega \\ &\|R\|_{Y^N} = \|R_{\tilde\omega}\|_{Y^N},\quad \omega=\tilde\omega \\ &\therefore\quad R=R_{\tilde\omega},\quad \omega=\tilde\omega \end{aligned} \tag{10}$$

Then the norm $\|R_{\tilde\omega}\|_{Y^N}$ of the vector $R_{\tilde\omega}$ in the subspace $Y_{\tilde\omega}^2$ has the maximum, while the norm $\|R-R_{\tilde\omega}\|_{Y^N}$ of the vector $R-R_{\tilde\omega}$ the minimum. Therefore, the IF estimation problem can be replaced into the finding the angular frequency ω which makes the norm $\|R-R_{\tilde\omega}\|_{Y^N}$ the minimum. Alternatively, we can solve the problem to find out the frequency ω which make the norm $\|R_{\tilde\omega}\|_{Y^N}$ the maximum.

2.2 NOISE REMOVAL USING STOCHASTIC RESONANCE METHOD

The general formulation of stochastic resonance method in the monostatic damping system can be written as following [29]:

$$\frac{d^2x}{dt^2}=-V'(x)-\gamma\frac{dx}{dt}+\{s(t)+n(t)\} \tag{11}$$

where $x(t)$ is the output signal, γ is the attenuation coefficient, $s(t)$ is the initial weak signal, and $n(t)$ is the noise. Therefore, $\{s(t)+n(t)\}$ is the noisy signal input a given system.

The potential curve may be expressed by

$$V(x) = V_0 - V_d\left[\exp\left(-\frac{(x+x_0)^2}{L^2}\right) + \exp\left(-\frac{(x-x_0)^2}{L^2}\right)\right] \quad (12)$$

So, the derivative of the potential curve may lead to

$$V'(x) = \frac{dV(x)}{dx} = V_d\left[\begin{array}{l}\frac{2(x+x_0)}{L^2}\exp\left(-\frac{(x+x_0)^2}{L^2}\right)\\ +\frac{2(x-x_0)}{L^2}\exp\left(-\frac{(x-x_0)^2}{L^2}\right)\end{array}\right] \quad (13)$$

Considering the symmetry of SNR curvature surface, if we substitute x_0=0 into Eq.(13), then we get

$$V'(x) = \frac{4V_d}{L^2}x\exp\left(-\frac{x^2}{L^2}\right) \quad (14)$$

Assuming that L has the relatively large value in the above expression, then the exponential term will be approached to 0, thus Eq.(14) will be simplified into the following [30]

$$V'(x) \approx \frac{4V_d}{L^2}x \quad (15)$$

Substituting Eq.(15) into Eq.(11), then we get

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \frac{4V_d}{L^2}x = \{s(t)+n(t)\} \quad (16)$$

Thus, the frequency of the output signal $x(t)$ can be obtained from the natural frequency of the one freedom vibrating system as

$$\omega = 2\pi f = \sqrt{\frac{4V_d}{L^2}} \quad (17)$$

Therefore,

$$f = \sqrt{\frac{4V_d}{\pi^2 L^2}} \quad (18)$$

2.3 INSTANTANEOUS FREQUENCY ESTIMATION USING ORTHOGONAL PROJECTION METHOD

2D subspace spanning vector ξ_s, ξ_c in Eq.(7-8) may be normalized as

$$\xi_s^e = \frac{\xi_s}{\|\xi_s\|_{Y^N}}, \quad \xi_c^e = \frac{\xi_c}{\|\xi_c\|_{Y^N}} \quad (19)$$

Then, the projection $R_{\tilde{\omega}}$ can be expressed by the linear combination of the above normalized vectors ξ_s, ξ_c as following:

$$R_{\tilde{\omega}} = k_s \cdot \xi_s^e + k_c \cdot \xi_c^e \quad (20)$$

Now, we consider the linear combination coefficients k_s, k_c. With scalar production of both sides of Eq.(20) by the normalized vector ξ_s^e, then we get

$$(R_{\tilde{\omega}}, \xi_s^e) = k_s(\xi_s^e, \xi_s^e) + k_c(\xi_c^e, \xi_s^e) \quad (21)$$

Considering $(R_{\tilde{\omega}}^{\perp}, \xi_s^e) = 0$, then

$$(R, \xi_s^e) = (R_{\tilde{\omega}}, \xi_s^e) + (R_{\tilde{\omega}}^{\perp}, \xi_s^e) = (R_{\tilde{\omega}}, \xi_s^e),$$

$$(\xi_s^e, \xi_s^e) = \|\xi_s^e\|^2_{Y^N} = 1 \quad (22)$$

so, Eq.(21) will be rewritten as

$$k_s + k_c(\xi_c^e, \xi_s^e) = (R, \xi_s^e) \quad (23)$$

In the above expression, $(\xi_c^e, \xi_s^e) = (\xi_s^e, \xi_c^e) = c_{sc}$ denotes the correlation of the normalized span vectors ξ_s^e, ξ_c^e of the subspace $Y_{\tilde{\omega}}^2$, if two vectors are orthogonal, it will be zero. Eventually, Eq.(21) can be expressed as

$$k_s + k_c \cdot c_{sc} = (R, \xi_s^e) \quad (24)$$

Similarly, scalar production of both sides of Eq.(20) with the normalized vector ξ_c^e will lead to the following

$$k_s \cdot c_{sc} + k_c = (R, \xi_c^e) \quad (25)$$

Therefore, we can obtain the simultaneous equation with respect to the linear combination coefficients k_s, k_c as following

$$\begin{cases} k_s + k_c\, c_{sc} = (R, \xi_s^e) \\ k_s\, c_{sc} + k_c = (R, \xi_c^e) \end{cases} \quad (26)$$

If we set value of the frequency $\tilde{\omega}$, then we can calculate $\xi_s(\tilde{\omega})$, $\xi_c(\tilde{\omega})$, hence we can get (R, ξ_s^e), (R, ξ_c^e), so we can easily solve the Eq.(20) to obtain the linear combination coefficients k_s, k_c.

In the determinant of simultaneous in Eq,(26)

$$\begin{pmatrix} 1 & c_{sc} \\ c_{sc} & 1 \end{pmatrix} \quad (27)$$

If ξ_s^e, ξ_c^e are orthogonal, then c_{sc} will be zero, so the above matrix becomes the identity matrix.

From the above descriptions, we can summarize the IF estimation by means of the following formulation

$$J(\tilde{\omega}) = \|R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e)\|_{Y^N} : \min \quad (28)$$

Similarly, we can get the initial angular frequency ω of the digital signal frame sequence $R=(r_1,r_2,\ldots,r_N)$ by finding out the maximum of the norm $\|R_{\tilde{\omega}}\|_{Y^N}$.

$$J(\tilde{\omega}) = \|k_s \cdot \xi_s^e + k_c \cdot \xi_c^e\|_{Y^N} : \max \quad (29)$$

3. COMPUTATIONAL ALGORITHM AND EVALUATION OF COMPUTATIONAL LOAD

Let us consider the solution algorithm of the minimization problem Eq.(28). For computational convenience, instead of minimization problem Eq.(28), we solve the following equivalent problem

$$J^2(\tilde{\omega}) = \left\|R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e)\right\|^2_{Y^N} : \min \quad (30)$$

First, given the received signal frame $R=(r_1,r_2,\ldots,r_N)$, we evaluate the computational algorithm and the computational load of corresponding to the arbitrary choice of frequency value $\tilde{\omega}$, and the above-mentioned minimization problem algorithm and the computational amount.

3.1 COMPUTATION OF $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$ ACCORDING TO $\tilde{\omega}$

In the phase change corresponding to the frame $\omega \cdot i + \frac{1}{2} k_\omega \cdot i^2 + \phi_0$, $i = 0,1,2,\ldots,N$ k_ω and ϕ_0 are the fixed ones, so we can preliminarily calculate the term $\frac{1}{2} k_\omega \cdot i^2 + \phi_0$ and utilize this one each computational step through storage. Therefore, we couldn't this one in the evaluation of computational load. Let computational load as c_n, multiplication as *multiple*, addition and subtraction as *add*, computation of triangular functions as *sin* and *cos*, computation of inversion of matrix Eq.(27) as *inversematrix*, then the computational steps and loads of $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$ can be as following:

- Calculation of $\xi_s^e = \xi_s^e(\tilde{\omega})$, $\xi_c^e = \xi_c^e(\tilde{\omega})$ according to the frequency $\tilde{\omega}$.

$$\text{Cost: cn} = N \cdot \text{multiply} + N \cdot \text{add} + N \cdot \sin + N \cdot \cos$$

- Calculation of $(\xi_s^e, \xi_c^e) = c_{sc}$ and $A = \begin{pmatrix} 1 & c_{sc} \\ c_{sc} & 1 \end{pmatrix}^{-1}$.

$$\text{Cost: cn} = N \cdot \text{multiply} + (N-1) \cdot \text{add} + \text{inverse matrix}$$

- Calculation of vector $R_\xi = \begin{pmatrix} (R, \xi_s^e) \\ (R, \xi_c^e) \end{pmatrix}$.

$$\text{Cost: cn} = 2 \cdot (N \cdot \text{multiply} + (N-1) \cdot \text{add})$$

- Calculation of $\begin{pmatrix} k_s \\ k_c \end{pmatrix} = A \cdot R_\xi$.

$$\text{cn} = 4 \cdot \text{multiply} + 2 \cdot \text{add}$$

- Calculation of $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$.

$$\text{cn} = 2 \cdot N \cdot \text{multiply} + 2 \cdot N \cdot \text{add} + N \cdot \text{square}$$

Therefore, the total computational load for calculating $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$ is given by:

$$\text{sumcn} = (6 \cdot N + 4) \cdot \text{multiply} + 5 \cdot N \cdot \text{add} + N \cdot \sin + N \cdot \cos + \text{inverse matrix} + N \cdot \text{square}$$

3.2 SOLUTION OF $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2 : \min$

The change curve of $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$ according to frequency $\tilde{\omega}$ is as following.

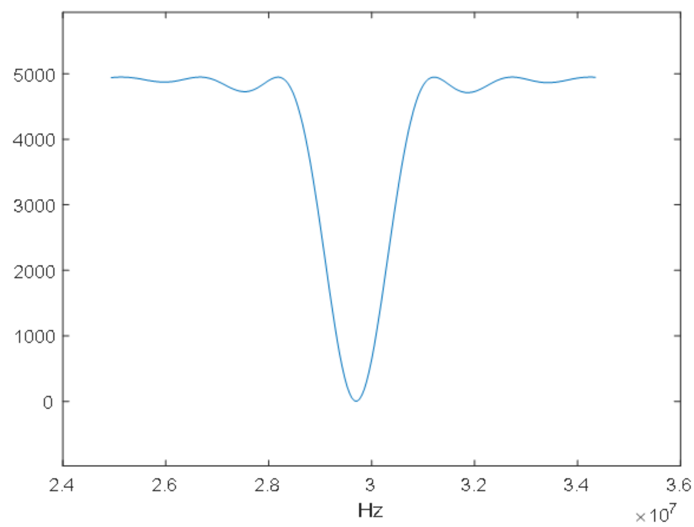

Fig.2. Change of $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$ according to $\tilde{\omega}$

We use the following computational algorithm to reduce the computational cost of the solution of the minimization problem Eq.(22) by using the above variation in the frequency variation range $2 \cdot B$ (including the frequency modulation band B).

Uniformly divide the frequency change range $(2 \cdot B)$ into M subsections, and calculate $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$ k=1,2,…,M at each $\tilde{\omega}_k$, $k = 1,2,\ldots,M$, and then find out the maximum point $\tilde{\omega}_{k_0}$. The frequency resolution of this time is $\delta_1 = \frac{2 \cdot B}{M}$.

And then, divide the neighboring section about maximum point $(\tilde{\omega}_{k_0-1}, \tilde{\omega}_{k_0+1})$ into M points $\tilde{\omega}_i$, i=1,2,…,M uniformly and calculate $J^2(\tilde{\omega}) = \left\| R - (k_s \cdot \xi_s^e + k_c \cdot \xi_c^e) \right\|_{Y^N}^2$, k=1,2,…,M, and find out the maximum point $\tilde{\omega}_{i_0}$. The frequency resolution for that time is $\delta_2 = \frac{2 \cdot \delta_1}{M} = \frac{4 \cdot B}{M^2}$.

Repeating this procedure K times, then the frequency resolution will be $\delta_K = \frac{2^K \cdot B}{M^K}$, and we will get the optimal solution of the minimization problem Eq.(30) with this accuracy. And the total computational amount will be given by

$$\text{sumcn} = K \cdot M \begin{pmatrix} (6 \cdot N + 4) \cdot \text{multiply} + 5 \cdot N \cdot \text{add} + N \cdot \sin \\ + N \cdot \cos + \text{inverse matrix} + N \cdot \text{square} \end{pmatrix}$$

Estimating the frequency ω like this, and then based on this, we can obtain the angular frequency of the frame midpoint $\omega + \frac{k_\omega \cdot N}{2}$. The above method is almost the same as in Eq.(29), which is the problem of finding the maximum.

4. VERIFICATION BY SIMULATION

In this section, we compare the proposed method presented in Section 2 by applying a linear frequency modulated signal from 25 MHz to 30 MHz in a period of 10 ms to a digital signal sampled at 150 MSPS for Doppler radar, as shown in Fig.3. The number of sample points corresponding to a 10 ms cycle is 150,000,000.

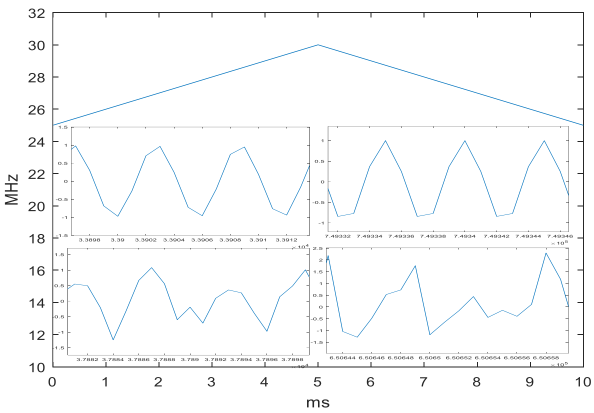

Fig.3. Signal waveform and its frequency curve

The Fig.3 shows the signal waveforms in the absence of noise near frequencies of 25 and 30 MHz, and the frequency curves and the noise-induced signal waveforms corresponding to 5 dB SNR.

In Doppler radar, the following relationship is assumed for the Doppler frequency and the target velocity in terms of ω_0, k_{ω_0} in Eq.(3):

$$f_d(t) = \frac{v}{c} f_e(t), \quad f_e(t) = f_0 + \frac{k_{\omega_0}}{2\pi} t$$

$$\therefore f_d(t) = \frac{v}{c} f_0 + \frac{k_{\omega_0}}{2\pi} \cdot \frac{v}{c} t = f_{d_0} + \frac{k'_{\omega_0}}{2\pi} t \tag{23}$$

where c is the propagation speed of electromagnetic wave, v the moving speed of the object, f_0 is the fundamental radiating frequency, $f_{d_0} = \frac{v}{c} f_0, k'_{\omega_0} = k_{\omega_0} \cdot \frac{v}{c}$.

And the frequency of the receiving signal is given by $f_r(t) = f_e(t) + f_d(t) = \left(f_0 + f_{d_0}\right) + \left(k_{\omega_0} + k'_{\omega_0}\right) t$.

As can be seen from the equation, the time-dependent coefficient of variation with the moving velocity of the object v will be changed as $\tilde{k}_{\omega_0} = k_{\omega_0} + k'_{\omega_0}$. Therefore, ω and k_ω of the digital signal given as Eq.(4) and Eq.(5) will be changed according to the Doppler frequency (moving speed of the object). Given a various condition for k_ω, the change curve of $J(\tilde{\omega}) = \| R - R_{\tilde{\omega}} \|_{Y^N}, J(\tilde{\omega}) = \| R_{\tilde{\omega}} \|_{Y^N}$ of Eq.(28), Eq.(29) according to $\tilde{\omega}$ is shown in Fig.4 for different frame lengths.

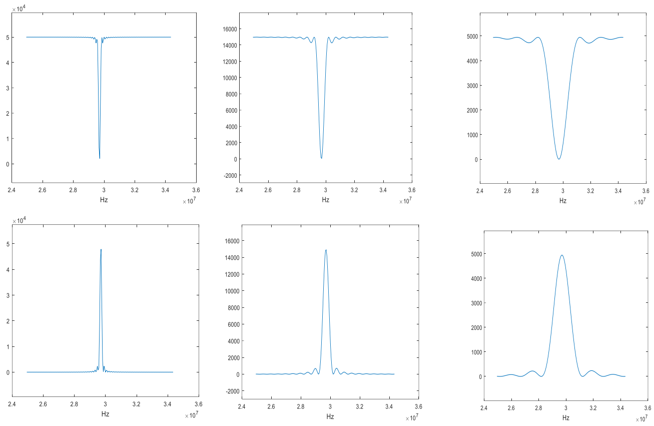

Fig.4. Change curve of $\|R - R_{\tilde{\omega}}\|_{Y^N}$ when frame length N is 1 000, 300, 100

As shown in Fig.4, the minimum and maximum points of the two norms coincide exactly. In the simulation algorithm applying the proposed method, the Doppler effect is not available, i.e., in frequency linear modulation, with the frequency change characteristic changed with the Doppler frequency (moving speed of the object) change, that is, with the phase change of Eq.(5)-Eq.(6) due to the change in the signal. The radiation frequency was assumed to be 9.15 GHz. Hence, we have estimated the error based on the frequency at the center of the frame. As can be seen in Fig.4, the larger the frame length, the sharper the change curve, and the higher the frequency selectivity.

This property can also be used for estimating the IF for a multi-components curve.

Simulation results under different SNR and frame length conditions are compared with the frequency estimation method using our proposed method (denoted Project Method (PM), short-time Fourier transform (STFT) for frames and Wigner-Bill distribution (WVD) (Table.1). In the short-time Fourier transform (STFT), the window function is used to use a rectangular window function equal to the frame length. For the input digital signal frame, the instantaneous frequency estimation at the frame center point is first determined from the initial angular frequency candidate value and the corresponding search neighborhood from the chirp curve in Fig.3, depending on the position of the frame in the frequency modulation period 1. Then, the optimum point should lie within the search of the initial angular frequency candidate value. Then, we apply the computational algorithm of Section 3 for the desired frequency resolution.

The results of the frame center frequency estimation simulations for different frame lengths are given in Table.1, with the 500,000 sampling points from the beginning of the frequency modulation cycle as the frame center position, and with the moving speed of (30) 28351643.5230952 Hz, the signal-to-noise ratio of 10 dB, 5 dB, and 2 dB. To estimate the frequency at the correct midpoint, the frame length is odd and given with the corresponding time length.

We set B=5MHz, M=100, K=5 for applying the computational algorithm is Sec. 3.

Table 1. Result of estimation of frame central frequency

No	Frame length (ms)	SNR (dB)	Method	Estimation of Central Frequency (Hz)	Error (Hz)
1	751 (0.005)	10	PM	28354382.545	-2739.212
			STFT	28354388.478	-2738.478
			WVD	28348360.169	3289.830
2	1501 (0.01)	10	PM	28352603.860	-960.527
			STFT	28352626.043	-976.043
			WVD	28347917.855	3732.144
3	7501 (0.05)	10	PM	28351662.435	-19.102
			STFT	28351669.457	-19.457
			WVD	28351661.519	-11.519
4	15001 (0.1)	10	PM	28351641.114	2.219
			STFT	28331619.881	20030.118
			WVD	28360147.074	-8497.074
5	75001 (0.5)	10	PM	28351644.608	-1.27495
			STFT	28125883.061	225766.938
			WVD	28353898.020	-2248.020
6	150001 (1)	10	PM	28351643.523	-0.189
7	751 (0.005)	5	PM	28355095.687	-3452.354
			STFT	28355068.946	-3418.946
			WVD	28348489.019	3160.980
8	1501 (0.01)	5	PM	28352002.331	-358.998
			STFT	28352019.174	-369.174
			WVD	28346517.828	5132.171
9	7501 (0.05)	5	PM	28351662.798	-19.465
			STFT	28351698.609	-48.609
			WVD	28351722.365	-72.365
10	15001 (0.1)	5	PM	28351606.354	36.978
			STFT	28331467.619	20182.380
			WVD	28360283.504	-8633.504
11	75001 (0.5)	5	PM	28351642.354	0.978
			STFT	28465295.886	-113645.886
			WVD	28434496.704	-82846.704
12	150001 (1)	5	PM	28351643.122	0.211
13	751 (0.005)	2	PM	28360046.673	-8403.339
			STFT	28360046.864	-8396.864
			WVD	28347883.568	3766.431
14	1501 (0.01)	2	PM	28353116.505	-1473.171
			STFT	28353190.177	-1540.177
			WVD	28348017.832	3632.167
15	7501 (0.05)	2	PM	28351714.192	-70.859
			STFT	28351672.719	-22.719
			WVD	28351674.362	-24.362
16	15001 (0.1)	2	PM	28351602.630	40.702
			STFT	28331864.060	19785.939
			WVD	28352716.172	-1066.172
17	75001 (0.5)	2	PM	28351644.245	-0.912
			STFT	28125018.437	226631.562
			WVD	28399270.252	-47620.252
18	150001 (1)	2	PM	28351646.177	-2.844

The simulation results showed that frequency estimation with similar accuracy to Table.1 can be achieved for each point of the chirp 1 cycle signal sequence and for any Doppler frequency condition. Based on the above results, the frequency curve can be obtained by finding the center frequency of the digital signal frames centered at each point of the 1-cycle digital signal sequence.

5. CONCLUSION

In this paper, we propose a method to remove noise using stochastic resonance method and estimate the instantaneous frequency at each sampling point by processing the chirped signal in frame units using projection method and compare its performance by simulation. For the sake of our propose, the short-time Fourier transform of frame length is improved to suit the frequency modulation characteristics.

We have considered a digital signal frame as a vector of space of dimensions equal to its length and projected it into a two-dimensional subspace based on the digital signal sequence frame vectors of the harmonic signal type, whose orthogonality is not satisfied. In other words, estimating the instantaneous frequency of a digital signal frame can be viewed as finding the linear combination of the signal sequence of the corresponding frequency sin and cos type, and all elements of the linear subspace are represented by linear combinations of basis vectors, so the instantaneous frequency estimation problem is solved by finding orthogonal projections into a two-dimensional subspace.

Simulation results show that the accuracy of frequency estimation at the center point can be greatly improved by treating the signal with stochastic resonance method and setting the frame length to a certain size even under different Doppler shift frequency conditions. This method can be applied to instantaneous frequency estimation problems for FM signals other than chirp by constructing a basis vector that reflects the frequency modulation pattern. It is also considered applicable to instantaneous frequency estimation of multi-component signals.

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