

# PERFORMANCE EVALUATION OF TURBO CODED OFDM SYSTEMS AND APPLICATION OF TURBO DECODING FOR IMPULSIVE CHANNEL

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## Abstract

A comparison of the performance of hard and soft-decision turbo coded Orthogonal Frequency Division Multiplexing systems with Quadrature Phase Shift Keying (QPSK) and 16-Quadrature Amplitude Modulation (16-QAM) is considered in the first section of this paper. The results show that the soft-decision method greatly outperforms the hard-decision method. The complexity of the demapper is reduced with the use of simplified algorithm for 16-QAM demapping. In the later part of the paper, we consider the transmission of data over additive white class A noise (AWAN) channel, using turbo coded QPSK and 16-QAM systems. We propose a novel turbo decoding scheme for AWAN channel. Also we compare the performance of turbo coded systems with QPSK and 16-QAM on AWAN channel with two different channel values- one computed as per additive white Gaussian noise (AWGN) channel conditions and the other as per AWAN channel conditions. The results show that the use of appropriate channel value in turbo decoding helps to combat the impulsive noise more effectively. The proposed model for AWAN channel exhibits comparable Bit error rate (BER) performance as compared to AWGN channel.

## Keywords:

Bit Error Rate, Orthogonal Frequency Division Multiplexing, Turbo Codes, 16-QAM, AWAN Channel

## 1. INTRODUCTION

TURBO codes were introduced initially by Berrou, Glavieux and Thitimajshima. The iterative property of turbo codes helps to achieve a large coding gain with respect to an un-coded system. It uses two or more concatenated or parallel codes on different interleaved versions of the original data. The decoders exchange soft decisions rather than hard decisions so that best results are achieved. A typical turbo encoder uses parallel concatenated convolutional codes (PCCC) in which data bits are coded by two or more recursive systematic convolutional (RSC) coders, each with input as interleaved versions of data. In a turbo system with two component decoders, the decisions of one component decoder are passed as input to another decoder and this process is iteratively done for several times to get more reliable decisions. The high error correction power of turbo code originates from the random interleaving at the encoder and iterative decoding using extrinsic information at the decoder [1-3].

Spectral efficiency of frequency division multiplexing (FDM) is very low as the carriers are spaced sufficiently far apart to ensure non-overlapping spectra. Multi carrier modulation (MCM) or Multi tone modulation (MTM) system uses multiple carriers to carry different bits of single higher rate information signal. Each channel in this MCM system is a low-rate signaling path, hence less susceptible to inter symbol interference unlike the case of a single carrier system.

Orthogonal Frequency Division Multiplexing (OFDM) is a discrete multi tone (DMT) system, making use of Inverse Fast Fourier Transform (IFFT) for modulation and Fast Fourier Transform (FFT) for demodulation. Input information is assembled into block of N complex numbers, considering one per channel. IFFT is performed on this block and the result is transmitted serially. At the receiver, FFT is performed block wise to recover the data. As FFT is computationally efficient, OFDM is a popular modulation scheme in high-speed communication systems. OFDM with forward error correction (FEC) methods is most suitable scheme to transmit information wirelessly - quickly and accurately [2]. Turbo coded OFDM (TCOFDM) system combines the good features of OFDM with that of turbo code.

Power line communication (PLC) is an innovative idea of transmitting the telecommunication signal through the public power network. What makes this new technology more appealing is that it utilizes the existing infrastructure and there is no need for new wires. One of the challenges in PLC technology is to combat the electromagnetic noise on power line channels, having impulsive characteristics, introduced by the electrical appliances. Middleton's Class A noise model [4], [5] for non-Gaussian noise channels is used for modeling of man-made impulsive noise channels, for example, wireless channels, power line channels, etc. This model corresponds to an independent and identically distributed (i.i.d.) discrete-time random process whose probability density function (pdf) is an infinite weighted sum of Gaussian densities, with decreasing weights and increasing variance for the Gaussian densities [5].

The organization of the paper is as follows. Section 2 describes turbo encoding while Section 3 explains the turbo decoding and various algorithms that are used in decoding. In Section 4, we give a brief introduction to OFDM and the advantage of using OFDM along with turbo coding. Section 5 explains the AWAN channel model while section 6 gives the performance evaluation with system model and numerical results obtained through simulation. Conclusions are given in Section 7.

## 2. TURBO ENCODER

The encoder of PCCC turbo system with two RSC coders is shown in Fig.1. The binary input data sequence is represented by  $d_k = (d_1, \dots, d_N)$ . The input sequence is passed into the input of the first RSC coder, ENC1 that generates a coded bit stream,  $x_k^{p1}$ . For the second RSC coder ENC2, the data sequence is interleaved using random interleaver in which the bits are output in a pseudo-random manner. The interleaved data sequence is passed to ENC2, and a second coded bit stream,  $x_k^{p2}$  is

generated. The output code sequence of the turbo encoder is a multiplexed (and possibly punctured) stream consisting of systematic code bits  $x_k^s$  along with the parity bits of first and second encoders,  $x_k^{p1}$  and  $x_k^{p2}$  [1],[6].

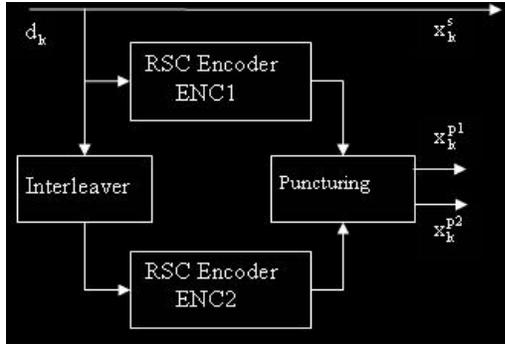


Fig.1. Turbo Encoder

### 3. TURBO DECODER

The Turbo decoder consists of two component decoders, DEC1 to decode sequences from ENC1, and DEC2 to decode sequences from ENC2 as shown in Fig. 2. It takes a sequence of received code values,  $r_k = \{y_k^s, y_k^p\}$ . Both the decoders are Maximum A Posteriori (MAP) decoders. DEC1 takes  $y_k^s$ , the systematic values of the received sequence along with  $y_k^{p1}$  which is the received sequence parity values belonging to the first encoder ENC1. Sequence of soft estimates  $L_{e1}(\hat{d}_k)$  of the transmitted data  $d_k$  that are available at the output of DEC1, are interleaved and passed to the second decoder DEC2 as a priori information. The same interleavers are used at both encoder and decoder. The three inputs to DEC2 are as follows: interleaved version of the systematic received values  $y_k^s$ , sequence of received parity values from the second encoder  $y_k^{p2}$  and interleaved version of the soft estimates  $L_{e1}(\hat{d}_k)$ . The a priori information for first decoder,  $L_{e2}(\hat{d}_k)$  is obtained from de-interleaving the soft estimates of DEC2. This procedure is repeated in an iterative manner for required number of iterations, and the final a posteriori output of the second decoder is de-interleaved to get the log likelihood representation of the estimate of  $d_k$ . Larger negative values of the likelihood ratio  $\wedge(d_k)$  represent a strong likelihood that the transmitted bit was a '0' and larger positive values represent a strong likelihood that the transmitted bit was a '1'. This output is sent to a hard decision device to get the binary stream of data [1],[6].

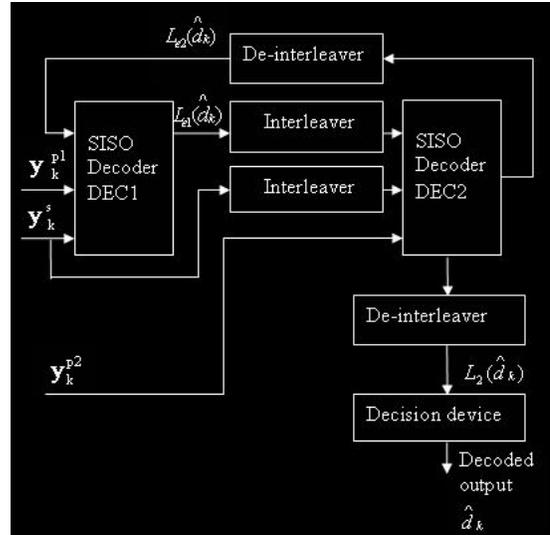


Fig.2. Turbo Decoder

Various algorithms are used for turbo decoding such as MAP algorithm, Log-MAP algorithm, Max\*-Log-MAP algorithm, Max-Log-MAP algorithm, Soft Output Viterbi Algorithm (SOVA).

#### 3.1 MAP ALGORITHM

Turbo code uses Bayes' theorem to describe a posteriori probability (APP) of a decision in terms of continuous valued random variable 'x' as follows for a set of M classes.

$$P(d=i/y) = \frac{p(y/d=i)P(d=i)}{p(y)} \text{ for } i=1,2,\dots,M$$

$$p(y) = \sum_{i=1}^M p(y/d=i)P(d=i) \quad (1)$$

where  $d$  = data,  $y$  = received data plus noise signal,  $P(d=i/y)$  is the APP with 'd' belonging to  $i^{\text{th}}$  signal class,  $p(y/d=i)$  is the pdf of 'y' conditioned on  $(d=i)$  and  $P(d=i)$  is the a priori probability accounting for the probability of occurrence of the  $i^{\text{th}}$  signal class. The computation of a priori probability gives refinement to the prior knowledge about the data.

Turbo-code decoding requires APP for each data bit. Then the data bit value corresponding to the maximum APP for that data bit is chosen. The MAP algorithm splits the paths into two sets: those that have an information bit as '1' at step  $k$  and those that have a '0', returning the log-likelihood ratio (LLR) of these two sets. The implementation of MAP algorithm is similar to performing Viterbi algorithm in two directions on code bits. MAP algorithm uses forward state metric  $\alpha_k(S_k)$ , reverse state metric  $\beta_k(S_k)$ , and a branch metric  $\gamma_k(S_k, S_{k+1})$  at time  $k$ , present state  $S_k$  and next state  $S_{k+1}$  [1],[3].

$$\alpha_k(S_k) = \sum_{S_{k-1}} \alpha_{k-1}(S_{k-1})\gamma_{k-1}(S_{k-1}, S_k) \quad (2)$$

$$\beta_k(S_k) = \sum_{S_{k+1}} \beta_{k+1}(S_{k+1})\gamma_k(S_k, S_{k+1})$$

$$\begin{aligned}
\alpha_0(S_{start}) &= 1 \\
\alpha_0(S_k) &= 0 \quad \forall \quad S_k \neq S_{start} \\
\beta_L(S_{end}) &= 1 \\
\beta_L(S_k) &= 0 \quad \forall \quad S_k \neq S_{end}
\end{aligned} \quad (3)$$

$$\Lambda(\hat{d}_k) = \frac{\sum_{s_k \xrightarrow{1} s_{k+1}} \alpha_k(S_k) \gamma_k(S_k, S_{k+1}) \beta_{k+1}(S_{k+1})}{\sum_{s_k \xrightarrow{0} s_{k+1}} \alpha_k(S_k) \gamma_k(S_k, S_{k+1}) \beta_{k+1}(S_{k+1})} \quad (4)$$

where  $\Lambda(\hat{d}_k)$  is the likelihood ratio.

### 3.2 LOG-MAP ALGORITHM

Logarithm of the likelihood ratio is the LLR which is a real number representing soft decision output of a decoder. LLR of a systematic decoder is represented with three elements – a channel measurement  $L_c(x)$ , a priori knowledge of the data

$L_a(d)$ , and an extrinsic LLR  $L_e(\hat{d})$ . The extrinsic LLR is the extra knowledge gained from the decoding process.

$$L(\hat{d}) = L_c(x) + L_a(d) + L_e(\hat{d}) \quad (5)$$

As given in [7], [8], at the  $k^{\text{th}}$  time instant,

$$L_a(d_k) = \ln \left( \frac{P(d_k = 1)}{P(d_k = 0)} \right) \quad (6)$$

If  $P(d_k = 1 / S_{k-1}, S_k) = 1$  then

$$P(S_k / S_{k-1}) = \left( \frac{\exp(L_a(d_k))}{1 + \exp(L_a(d_k))} \right) \quad (7)$$

$$L_a(d_k) = \ln \left( \frac{P(S_k / S_{k-1})}{1 - P(S_k / S_{k-1})} \right)$$

If  $P(d_k = 0 / S_{k-1}, S_k) = 1$  then

$$P(S_k / S_{k-1}) = \left( \frac{1}{1 + \exp(L_a(d_k))} \right) \quad (8)$$

$$L_a(d_k) = \ln \left( \frac{1 - P(S_k / S_{k-1})}{P(S_k / S_{k-1})} \right)$$

### 3.3 MAX\* -LOG-MAP ALGORITHM

In Max\* - Log-MAP algorithm, the Log-MAP algorithm is implemented based on *Jacobian algorithm* [9] as given below:

$$\begin{aligned}
\ln(e^x + e^y) &= \max(x, y) + \ln(1 + e^{-|x-y|}) \\
&= \max(x, y) + f_c(|x - y|)
\end{aligned} \quad (9)$$

The correction term  $f_c(x)$  can be pre-computed and stored for pre-defined quantization levels. This further reduces the computational burden of decoding.

### 3.3 MAX- LOG-MAP ALGORITHM

Decoding Turbo codes with the Max-Log-MAP Algorithm is a good compromise between performance and complexity. The performance is better than the standard SOVA algorithm and reaches closer to the optimal performance results of the

MAP/Log-MAP algorithm. The Decoding quality of the Max-Log-MAP decoder can be improved by using a scaling factor within the extrinsic calculation [10]. Max-Log-MAP algorithm reduces computational complexity of Log-MAP algorithm with a slightly poorer BER performance [9]. This algorithm looks at only two paths *per step*: the best with bit zero and the best with bit one at transition  $k$ , returning the difference of the log-likelihoods as LLR [11].  $\alpha$ ,  $\beta$  and  $\gamma$  could be computed as per the property that

$$\ln \sum_j e^{a_j} \approx \max_j (a_j) \quad (10)$$

$$\alpha_k(S_k) = \max_{S_{k-1}} (\alpha_{k-1}(S_{k-1}) + \gamma_{k-1}(S_{k-1}, S_k)) \quad (11)$$

$$\beta_k(S_k) = \max_{S_{k+1}} (\beta_{k+1}(S_{k+1}) + \gamma_k(S_k, S_{k+1}))$$

$$\begin{aligned}
L(\hat{d}_k) &= \max_{S_k \xrightarrow{1} S_{k+1}} [\alpha_k(S_k) + \gamma_k(S_k, S_{k+1}) + \beta_{k+1}(S_{k+1})] \\
&\quad - \max_{S_k \xrightarrow{0} S_{k+1}} [\alpha_k(S_k) + \gamma_k(S_k, S_{k+1}) + \beta_{k+1}(S_{k+1})]
\end{aligned} \quad (12)$$

### 3.5 LOG-MAP EXTENSION (simple Log-MAP)

Computation of the terms in log-MAP algorithm using *Jacobian logarithmic function* is cumbersome. Also, storing results of the correction term  $\log(1 + \exp(-|x|))$  in a look up table introduces quantization error and becomes infeasible if operating signal to noise ratio has a wide range. As this correction term is effective near zero, *MacLaurin Series expansion* can be employed to describe this logarithmic term [12]. As per this algorithm, if second and higher order terms are ignored, correction term is computed as follows:

$$\begin{aligned}
\ln(1 + e^{-|x|}) &\approx \ln(2) - x/2 \\
\ln(e^a + e^b) &\approx \max(a, b) + \ln(2) - \frac{(a-b)}{2}
\end{aligned} \quad (13)$$

Though the complexity of this algorithm is lower than that of Log-MAP algorithm, it achieves almost the same performance.

### 3.6 SOVA DECODING

SOVA decoding works with the same metric as Max-Log-MAP algorithm, but the information returned about the reliability of decoded bit  $d_k$  is computed in a different way. The SOVA considers only one competing path per decoding step. That is to say, for each bit  $d_k$  it considers only the survivor path of the Viterbi algorithm among all the competing paths. The MAP algorithm can outperform SOVA decoding by 0.5 dB or more [1].

## 4. TCOFDM SYSTEM

OFDM is a MCM technique in which a single high rate data-stream is divided into multiple low rate data-streams and is modulated using sub-carriers, which are orthogonal to each other. Some of the main advantages of OFDM are its multi-path delay spread tolerance and efficient spectral usage by allowing overlapping in the frequency domain. Also one other significant advantage is that the modulation and demodulation can be done using computationally efficient IFFT and FFT operations. OFDM has become a popular modulation method in high-speed

wireless communications.

The combination of turbo codes with the OFDM transmission, so called TCOFDM can yield significant improvements in terms of lower energy needed to transmit data [13]. There is a large potential gain in using the iterative property of turbo decoders where soft bit estimates are used together with the known pilot symbols. The performance of such an iterative estimation scheme proves to be of particular interest when the channel is strongly frequency-selective and time-selective. Similar to every other communications scheme, coding can be employed to improve the performance of overall system. Several coding schemes, such as block codes, convolutional codes and turbo codes have been investigated within OFDM systems. An interleaving technique along with coding can guarantee the independence among errors by affecting randomly scattered errors.

## 5. TURBO DECODING WITH AWAN CHANNEL

As class A noise statistics is much different from additive white Gaussian noise (AWGN) channel, the conventional receivers which are optimized for AWGN channel are not suitable for AWAN channels.

Impulsive noise is one of the major problems in power line channel. Middleton's class A noise model defines the probability density function with impulsive index A, Gaussian-to-impulsive noise power ratio  $\Gamma$ , Gaussian noise power  $\sigma_G^2$ , and impulsive noise power  $\sigma_I^2$  as follows [4],[14]:

$$p_A(x) = \sum_{m=0}^{\infty} \left( \frac{e^{-A} A^m}{m!} \right) \left( \frac{1}{\sqrt{2\pi}\sigma_m} \right) \exp\left(-\frac{x^2}{2\sigma_m^2}\right) \quad (14)$$

where  $\sigma_m^2 = \frac{\sigma^2 \left( \frac{m}{A} + \Gamma \right)}{1 + \Gamma}$  and  $\Gamma = \frac{\sigma_G^2}{\sigma_I^2}$ . The total noise power is given as  $\sigma^2 = \sigma_G^2 + \sigma_I^2$ . Sources of impulsive noise exhibit Poisson distribution  $\frac{e^{-A} A^m}{m!}$  with each source contributing to noise with Gaussian PDF and variance  $\frac{\sigma_I^2}{A}$ . At a given time, assuming m such sources, the receiver noise exhibits Gaussian PDF with variance  $\sigma_m^2 = \sigma_G^2 + \frac{m \sigma_I^2}{A}$ . For larger A, class a noise is approaching Gaussian noise.

In Turbo decoding, for an AWGN channel, the channel value is derived for BPSK (or QPSK) modulation as

$$L_c(y_k) = 4 \cdot \frac{E_s}{N_0} \cdot y_k \quad (15)$$

where  $E_s$  is the energy per symbol and  $N_0$ , the Gaussian noise power spectral density. This channel value is not suitable for AWAN channel.

With a correction in [14], the computation of the channel value  $L_c(y_k)$  for turbo coded BPSK system is given below:

$$L_c(y_k) = \ln \left( \frac{P(y_k / d_k = +1)}{P(y_k / d_k = -1)} \right) = \ln \left( \frac{p_A(y_k - 1)}{p_A(y_k + 1)} \right) \\ = \ln \left( \frac{\sum_{m=0}^{\infty} \left( \frac{A^m}{m!} \right) \sqrt{\frac{A(1+\Gamma)}{m+A\Gamma}} \exp\left(-\frac{E_s}{N_0} \left( \frac{A\Gamma}{m+A\Gamma} \right) (y_k - 1)^2\right)}{\sum_{m=0}^{\infty} \left( \frac{A^m}{m!} \right) \sqrt{\frac{A(1+\Gamma)}{m+A\Gamma}} \exp\left(-\frac{E_s}{N_0} \left( \frac{A\Gamma}{m+A\Gamma} \right) (y_k + 1)^2\right)} \right) \quad (16)$$

where  $\frac{E_s}{N_0} = \frac{1}{2\sigma^2}$ . If  $\Gamma \ll 1$ , then (16) reduces to

$$L_c(k) = \ln \left( \frac{\sum_{m=0}^{\infty} \left( \frac{A^m}{m!} \right) \sqrt{\frac{A\Gamma}{m+A\Gamma}} \exp\left(-\frac{E_s}{N_0} \left( \frac{A\Gamma}{m+A\Gamma} \right) (y_k - 1)^2\right)}{\sum_{m=0}^{\infty} \left( \frac{A^m}{m!} \right) \sqrt{\frac{A\Gamma}{m+A\Gamma}} \exp\left(-\frac{E_s}{N_0} \left( \frac{A\Gamma}{m+A\Gamma} \right) (y_k + 1)^2\right)} \right) \quad (17)$$

For turbo coded QPSK system,  $L_c(k)$  computation remains same except that  $y_k$  in (17) is replaced by  $y_1(k)$  for LLR of first bit of QPSK mapping and  $y_0(k)$  for the second bit.

## 6. PERFORMANCE EVALUATION AND RESULTS

### 6.1 PERFORMANCE COMPARISON OF HARD AND SOFT-DECISION TCOFDM SYSTEMS

System model used in the simulation is shown in Fig. 3. The input bits are converted to frames and given as input to a rate  $\frac{1}{2}$  (or  $\frac{1}{3}$ ) PCCC turbo encoder. The generators used in the system are  $G_1=[7,5]_8$  or  $G_2=[15,17]_8$ . The encoded data stream is then modulated using QPSK/16-QAM as per gray coded constellation mapping. The complex symbols are again modulated using 64 sub carrier OFDM modulator and passed on AWGN channel. Out of the 64 sub carriers, only 52 are data sub carriers. A cyclic prefix of 16 samples is considered. An assumption is made that cyclic prefix completely eliminates the inter symbol interference. At the receiver soft output de-mapping is used for QPSK/16-QAM demodulation [6].

$$\text{LLR}(b_i) = \ln \left( \frac{\sum_{b_i=+1} P(s_i(b_i) / x)}{\sum_{b_i=-1} P(s_i(b_i) / x)} \right) \text{ for } b_1 b_2 \quad (18)$$

$$\text{LLR}(b_i) = \ln \left( \frac{\sum_{b_i=+1} P(s_q(b_i) / x)}{\sum_{b_i=-1} P(s_q(b_i) / x)} \right) \text{ for } b_3 b_4$$

Gray mapped signal constellation of 16-QAM system is shown in Fig.4. The corresponding input bits are marked in the order  $b_1 b_2 b_3 b_4$ . Simplified soft-output de-mapper used at the receiver reduces complexity of 16-QAM demodulation largely and improves the coding gain [15]. As per [15], if  $b_1 b_2 b_3 b_4$  are the bits representing 16-QAM constellation where  $b_1 b_2$  are mapped to the I-component and  $b_3 b_4$  are mapped to the Q-component, then we estimate the soft de-mapped bits  $D_{1,k}$  and  $D_{Q,k}$  as given below:

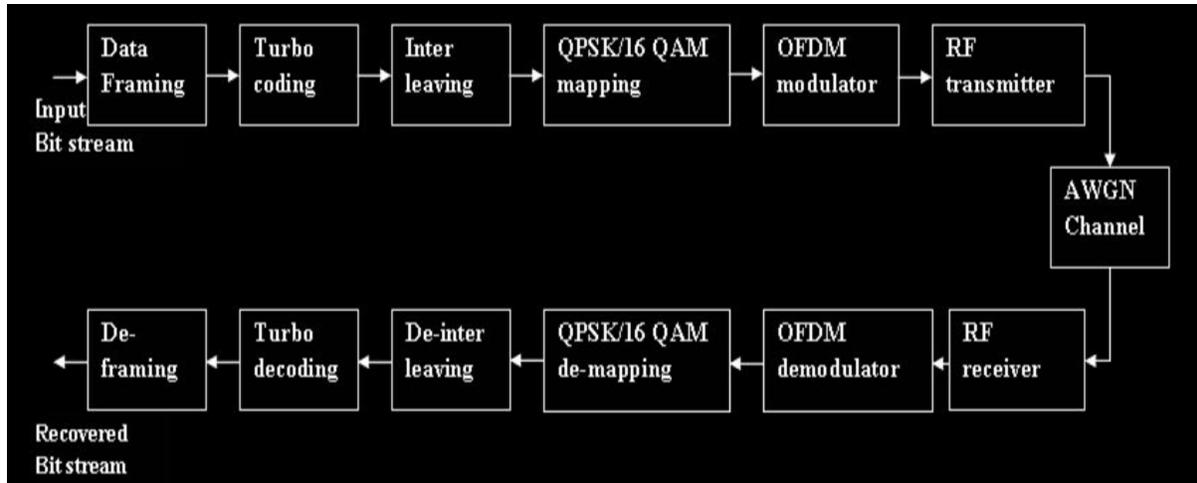


Fig.3. Turbo coded OFDM system model

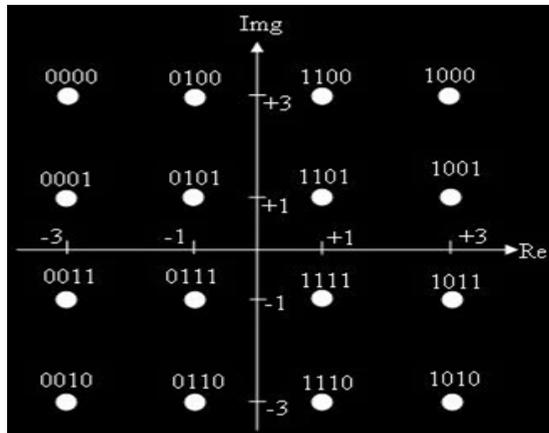


Fig.4. 16-QAM gray mapped constellation

Evaluation of the terms  $D_{I,k}$  for the in-phase bits and  $D_{Q,k}$  for the quadrature bits of a 16-QAM symbol yields[15]:

$$\begin{aligned}
 D_{I,1} &= \begin{cases} y_I(i), & \text{for } |y_I(i)| \leq 2 \\ 2(y_I(i) - 1), & \text{for } y_I(i) > 2 \\ 2(y_I(i) + 1), & \text{for } y_I(i) < -2 \end{cases} \\
 D_{I,2} &= -|y_I(i)| + 2 \\
 D_{Q,1} &= \begin{cases} -y_Q(i), & \text{for } |y_Q(i)| \leq 2 \\ -2(y_Q(i) - 1), & \text{for } y_Q(i) > 2 \\ -2(y_Q(i) + 1), & \text{for } y_Q(i) < -2 \end{cases} \\
 D_{Q,2} &= -|y_Q(i)| + 2
 \end{aligned} \quad (19)$$

where  $Y_I(i)$  and  $Y_Q(i)$  are the real (in-phase) and imaginary (quadrature) components of the  $i^{\text{th}}$  received symbol propagated through AWGN channel.  $D_{I,1}$ ,  $D_{I,2}$ ,  $D_{Q,1}$  and  $D_{Q,2}$  represent the LLR values corresponding to bits  $b_1$  through  $b_4$ .

The results obtained by MATLAB simulations are given in terms of BER versus  $E_b/N_0$ , where  $E_b$  is the energy per information bit. The performance of hard and soft decoding of the TCOFDM system is verified for two possible first modulation formats, using QPSK and 16-QAM while the second

modulation is OFDM. Data frame size of 2000 is used and 500 such frames are considered in the simulation.

Fig.5 gives a comparison of un-coded QPSK-OFDM system with TCOFDM system using QPSK. At a BER of  $10^{-4}$ , the soft-decoding gain compared to hard-decoding is equal to 2.2 dB for a rate  $\frac{1}{2}$  system. In Fig. 6, comparison of un-coded 16-QAM-OFDM system with TCOFDM system using 16-QAM is shown. At a BER of  $10^{-4}$ , the soft-decoding gain compared to hard-decoding is equal to 2.6 dB for rate  $\frac{1}{2}$  as well as rate  $\frac{1}{3}$  systems. Also the soft-decoding gain of a rate  $\frac{1}{3}$  system is 5.7 dB with respect to hard-decoded rate  $\frac{1}{2}$  system. The details of coding gains are given in Table 1.

Table.1 Coding gain achieved over un-coded OFDM systems

TCOFDM	Coding gain in dB at:		
	BER= $10^{-2}$	BER= $10^{-3}$	BER= $10^{-4}$
16_QAM, rate $\frac{1}{2}$ (hard) TCOFDM	5.5	7.6	8.8
16_QAM, rate $\frac{1}{3}$ (hard) TCOFDM	8.4	10.6	11.9
16_QAM, rate $\frac{1}{2}$ (soft) TCOFDM	7.8	10.0	11.4
16_QAM, rate $\frac{1}{3}$ (soft) TCOFDM	10.4	13.0	14.5
QPSK, rate $\frac{1}{2}$ (hard) TCOFDM	4.7	6.6	8.0
QPSK, rate $\frac{1}{2}$ (soft) TCOFDM	6.5	8.6	10.2

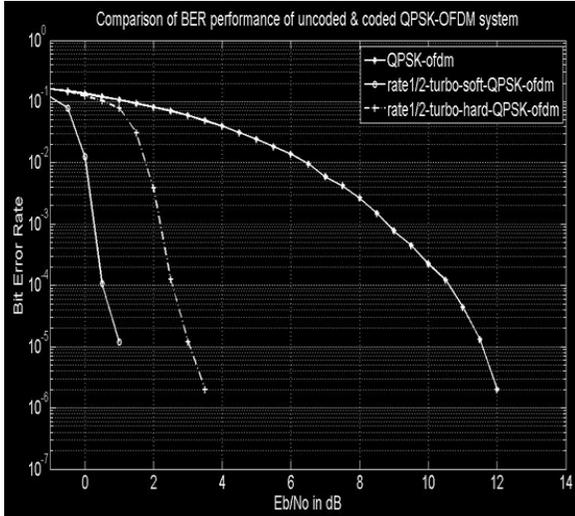


Fig.5. Comparison of un-coded and Turbo coded (rate  $\frac{1}{2}$ ) QPSK modulated OFDM systems

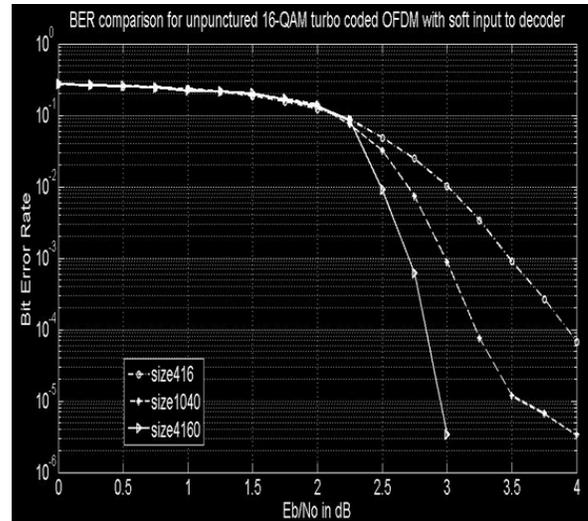


Fig.7. BER comparison for rate  $\frac{1}{3}$ , 16-QAM TCOFDM for different frame lengths after 9 iterations

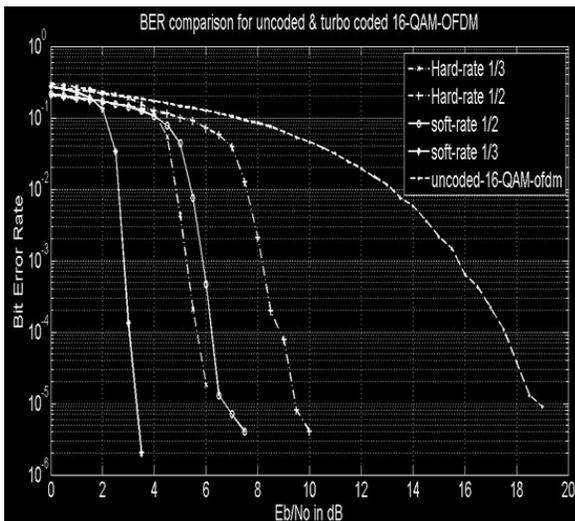


Fig.6. Comparison of un-coded and turbo coded 16-QAM OFDM systems

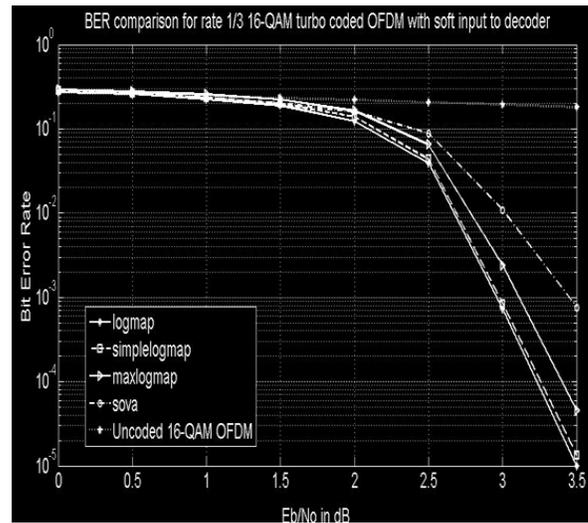


Fig.8. Comparison of un-coded and Turbo coded 16-QAM OFDM system with different decoding algorithms

The improvement in performance of a rate  $\frac{1}{3}$  TCOFDM system with increased frame length is shown in Fig. 7. A gain of 1.05 dB is achieved when frame size is changed from 416 to 4160. The performance of various decoding algorithms is shown in Fig. 8, which indicates a close match between LOGMAP and extended LOGMAP algorithms. Performance of SOVA decoding is much lower than the others. But for smaller memory, computations are much less in SOVA decoding compared to other algorithms [8].

In Fig. 9, response for different generators of RSC coders is shown. There exists a trade-off between constraint length and computational complexity. Fig. 10 shows the performance of the two systems on Rayleigh flat fading channel assuming channel state information is available at the receiver.

Finally, Fig. 11 shows the received image for different iterations of a rate  $\frac{1}{2}$  soft decoded TCOFDM system with QPSK modulation, which is comparable with rate  $\frac{1}{3}$  hard decoded TCOFDM system with QPSK modulation as shown in Fig.12. Hence, for a given SNR, soft decoding achieves better data rate as compared to hard decoding.

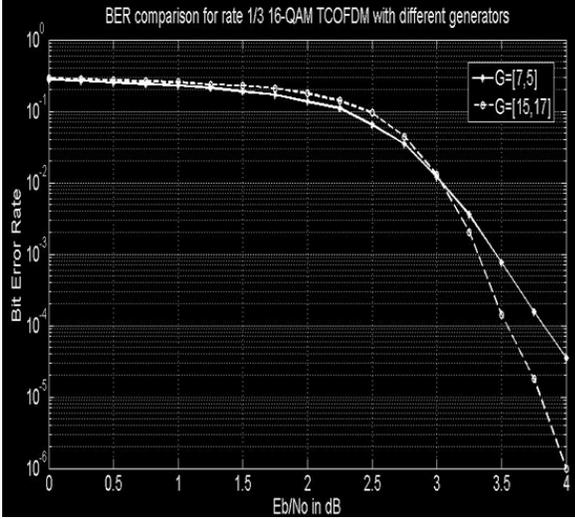


Fig.9. BER comparison for rate  $\frac{1}{3}$ , 16-QAM TCOFDM system with different generators after 3 iterations

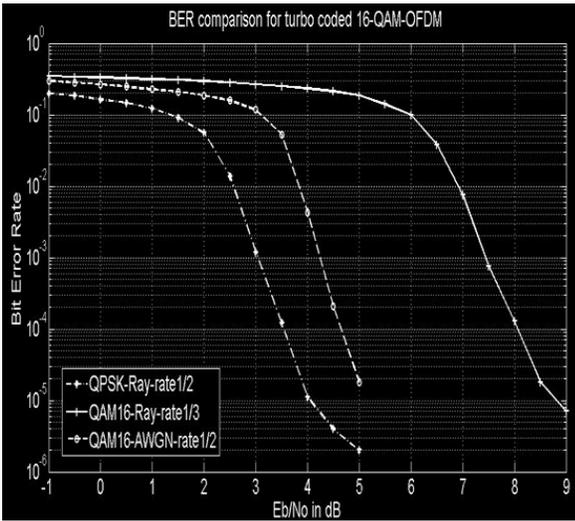


Fig.10. Comparison of Turbo coded 16-QAM OFDM system on Rayleigh (flat fading) and AWGN channels



Fig.11. Image transmission using TCOFDM (rate  $\frac{1}{2}$ ) with soft decoded QPSK modulation at  $E_b/N_0=0$  dB for iterations 1, 2, 5, and 9 respectively

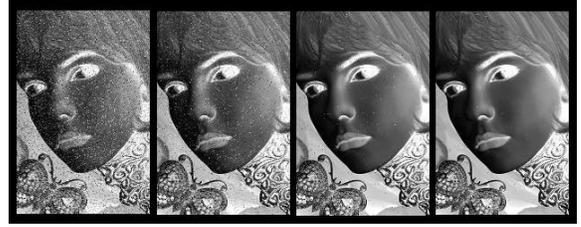


Fig.12. Image transmission using TCOFDM (rate  $\frac{1}{3}$ ) with hard decoded QPSK modulation at  $E_b/N_0=0$  dB for iterations 1, 2, 5, and 9 respectively.

## 6.2 Turbo coded QPSK/16-QAM system on AWAN channel

System model used in the simulation is shown in Fig. 13. The input bits are converted to frames and given as input to a rate  $\frac{1}{3}$  PCCC turbo encoder. The generator used in the system is  $G=[7,5]_8$ . The encoded data stream is then modulated using QPSK/16-QAM as per gray coded constellation mapping. The complex symbols are passed on AWAN channel. At the receiver soft output de-mapping is used for QPSK/16-QAM demodulation along with appropriate channel values or LLRs for the turbo decoding. Max\*-Log-MAP algorithm is used in turbo decoding.

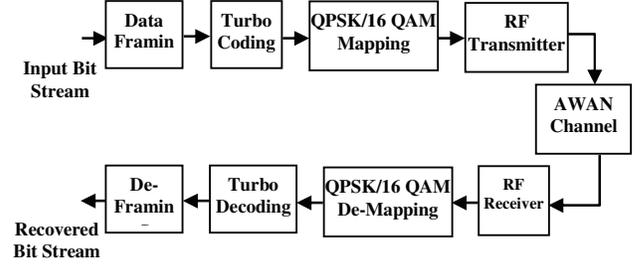


Fig.13. Turbo coded system

For a 16-QAM system with constellation as in Fig.3, the likelihood functions with AWGN noise model are given below:

$$\frac{p(y/b_1=1)}{p(y/b_1=0)} = \frac{\exp(-(y_i-1)^2/(2\sigma^2)) + \exp(-(y_i-3)^2/(2\sigma^2))}{\exp(-(y_i+1)^2/(2\sigma^2)) + \exp(-(y_i+3)^2/(2\sigma^2))}$$

$$\frac{p(y/b_2=1)}{p(y/b_2=0)} = \frac{\exp(-(y_i-1)^2/(2\sigma^2)) + \exp(-(y_i+1)^2/(2\sigma^2))}{\exp(-(y_i-3)^2/(2\sigma^2)) + \exp(-(y_i+3)^2/(2\sigma^2))}$$

and

$$\frac{p(y/b_3=1)}{p(y/b_3=0)} = \frac{\exp(-(y_o+1)^2/(2\sigma^2)) + \exp(-(y_o+3)^2/(2\sigma^2))}{\exp(-(y_o-1)^2/(2\sigma^2)) + \exp(-(y_o-3)^2/(2\sigma^2))}$$

$$\frac{p(y/b_4=1)}{p(y/b_4=0)} = \frac{\exp(-(y_o-1)^2/(2\sigma^2)) + \exp(-(y_o+1)^2/(2\sigma^2))}{\exp(-(y_o-3)^2/(2\sigma^2)) + \exp(-(y_o+3)^2/(2\sigma^2))}$$

(20)

where  $y$  is the received symbol at that instant.

The LLR of four bits of a given symbol are computed in the proposed model using (14), (17) and (20). Let  $L_c(y_{k,i})$  represent the LLR of  $i^{\text{th}}$  bit of the  $k^{\text{th}}$  received QAM symbol where  $i=1,2,3,4$  in the 16-QAM system. Also, let  $y_{l,k}$  and  $y_{o,k}$  represent the real and the imaginary parts of received symbol  $y_k$

respectively. LLR computation for first bit, i.e.  $b_1$  of the  $k^{\text{th}}$  received symbol is given in (21). Similarly, LLR for the other three bits of the  $k^{\text{th}}$  symbol can be computed. Computation complexity can be reduced as per the description in [14].

$$L_t(y_{k,l}) = \ln \left( \frac{\sum_{m=0}^{\infty} \left( \frac{A^m}{m!} \right) \sqrt{\frac{A\Gamma}{m+A\Gamma}} (\text{num})}{\sum_{m=0}^{\infty} \left( \frac{A^m}{m!} \right) \sqrt{\frac{A\Gamma}{m+A\Gamma}} (\text{den})} \right) \quad (21)$$

where

$$\text{num} = \exp\left(-\frac{E_s}{N_0} \left(\frac{A\Gamma}{m+A\Gamma}\right) (y_{l,k}-1)^2\right) + \exp\left(-\frac{E_s}{N_0} \left(\frac{A\Gamma}{m+A\Gamma}\right) (y_{l,k}-3)^2\right)$$

$$\text{den} = \exp\left(-\frac{E_s}{N_0} \left(\frac{A\Gamma}{m+A\Gamma}\right) (y_{l,k}+1)^2\right) + \exp\left(-\frac{E_s}{N_0} \left(\frac{A\Gamma}{m+A\Gamma}\right) (y_{l,k}+3)^2\right)$$

The MATLAB simulation results are given in terms of BER versus  $E_b/N_0$ , where  $E_b$  is the energy per information bit. The performance of rate  $\frac{1}{3}$  Turbo coded system over AWAN channel is verified for two possible modulation formats, QPSK and 16-QAM. Data frame size of 2000 is used and 500 such frames are considered in the simulation. We consider  $A=0.1$  and  $\Gamma=0.1$  for the AWAN channel. For turbo decoding, results of 9<sup>th</sup> iteration are considered.

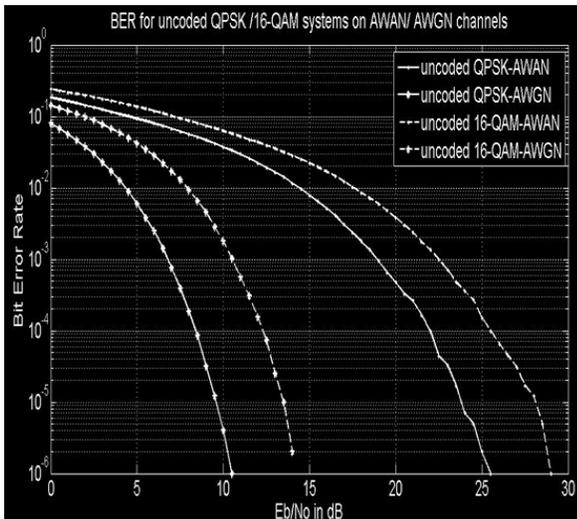


Fig.14. Uncoded QPSK/16-QAM system on AWGN/ AWAN channel

In Fig.14, we show the performance of uncoded QPSK/16-QAM system on AWGN as well as AWAN channels. We observe that uncoded QPSK/16-QAM system. In Fig.15, we compare the performance of Turbo coded QPSK system on AWAN channel for two different LLRs. We compute the channel value as per two conditions – one with AWGN scenario and the other with AWAN. Observation plots are given for system with these LLR computations. At a BER of  $10^{-3}$ , the coding gain of system with channel value computed as per AWAN channel conditions is around 10.9 dB compared to system with channel value computed as per AWGN channel conditions.

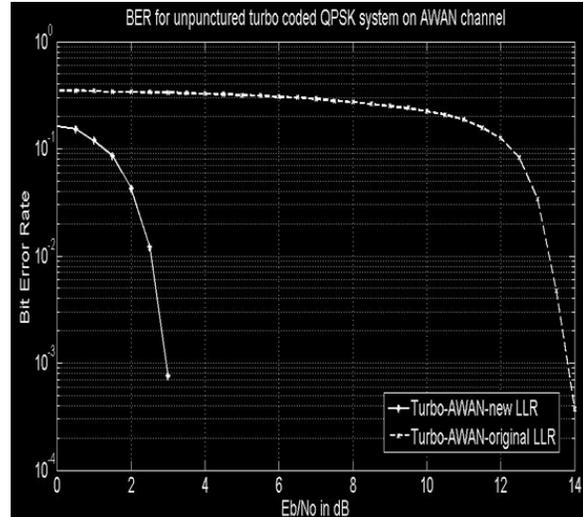


Fig.15. Turbo coded QPSK system on AWAN channel

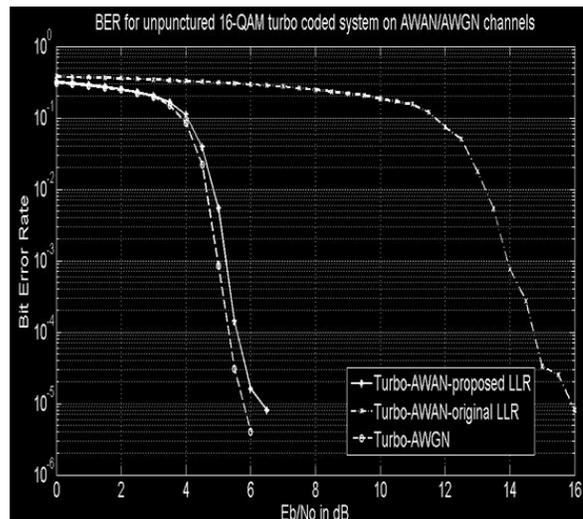


Fig.16. Turbo coded 16-QAM system on AWAN/AWGN channels

In Fig. 16, performance comparison of un-coded 16-QAM system on AWGN channel with Turbo coded 16-QAM system on AWAN channel with different channel values for the turbo decoder is given. Response of 16-QAM system on AWGN channel is also plotted. We observe that at a BER of  $10^{-5}$ , the coding gain of the system with channel value computed as per AWAN channel conditions is around 9.6 dB compared to system with channel value computed as per AWGN channel conditions. Also, the deviation of AWAN response from AWGN response at a BER of  $10^{-5}$  is only 0.5 dB.

## 7. CONCLUSION

From the results obtained by the comparison of hard and soft-decision TCOFDM Systems, it is observed that turbo coding is more advantageous in higher order constellations. It has been shown that, it is possible to transmit images on AWGN

channel even at  $E_b/N_0$  of 0 dB with rate  $\frac{1}{2}$  TCOFDM system using QPSK modulation. Also, rate  $\frac{1}{2}$  TCOFDM system with soft decoding achieves almost the same results as that of rate  $\frac{1}{2}$  TCOFDM system with hard decoding. Rate  $\frac{1}{3}$  TCOFDM system using QPSK modulation is preferred at very low SNR like  $E_b/N_0$  of -1 dB, while rate  $\frac{1}{2}$  16-QAM TCOFDM system achieves higher data rate and better BER performance ( $BER = 10^{-5}$ ) at around 6 dB. Hence a suitable modulation scheme could be chosen for specific channel conditions adaptively.

Also, in the later part of the paper, it is shown that class A noise can be effectively filtered out with the proposed model. Turbo coded QPSK system with proposed LLR as compared to original LLR, gives performance improvement by around 11 dB at a BER of  $10^{-3}$ . Similarly, Turbo coded 16-QAM system with proposed LLR as compared to original LLR, gives performance improvement by around 10 dB at a BER of  $10^{-5}$ . We have also shown that BER performance of the proposed model is very close to that of AWGN channel. Hence, we conclude that the proposed model is suitable for class A noise filtering at the receiver.

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