

BER EVALUATION OF LDPC CODES WITH GMSK IN NAKAGAMI FADING CHANNEL

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Abstract

LDPC codes (Low Density Parity Check Codes) have already proved its efficacy while showing its performance near to the Shannon limit. Channel coding schemes are spectrally inefficient as using an unfiltered binary data stream to modulate an RF carrier that will produce an RF spectrum of considerable bandwidth. Techniques have been developed to improve this bandwidth inefficiency or spectral efficiency, and ease detection. GMSK or Gaussian-filtered Minimum Shift Keying uses a Gaussian Filter of an appropriate bandwidth so as to make system spectrally efficient. A Nakagami model provides a better explanation to less and more severe conditions than the Rayleigh and Rician model and provide a better fit to the mobile communication channel data. In this paper we have demonstrated the performance of Low Density Parity Check codes with GMSK modulation (BT product=0.25) technique in Nakagami fading channel. In results it is shown that average bit error rate decreases as the 'm' parameter increases (Less fading).

Keywords:

Channel Codes, LDPC, Density Evaluation, GMSK, Nakagami Fading

1. INTRODUCTION

Channel coding is the transformation of signals in a way that they become more immune to noise, interference and other channel impairments. Channel coding has become very popular in the past few years because of the reason that it provides about 10dB performance improvement at a low cost. In channel coding, redundant bits introduced into the input data, help in removing the errors that are introduced during transmission over the channel.

Low Density Parity Check (LDPC) codes also known as Gallager codes [1] were invented by Robert Gallager in his Ph.D thesis in 1962. They are one the best forward error correction codes because they approach the Shannon limit [2]-[4] and are to be used as a standard in next generation satellite digital video broadcasting DVB-S2 and also best contenders for hard discs and 4G mobile phones.

These codes have become so popular mainly due to the fact that the minimum distance of an LDPC code increases proportionally to the code length with high probability [5]. In addition to their excellent block error performance, highly parallel LDPC decoder architecture is realizable by directly instantiating the LDPC decoding algorithm to hardware, resulting in very high speed LDPC decoder hardware. Along with LDPC codes, many researchers have worked with different modulation schemes in multipath fading channels. As it is a well known fact that channel codes use more spectrum so GMSK techniques which is spectrally efficient scheme is employed. Gaussian Minimum-Shift keying (GMSK) was first proposed in

1981 [6] and has become very popular due to its compact power spectral density and excellent error performance. It is a bandwidth-efficient constant envelope digital modulation technique, and therefore, it is suitable for mobile communications in fading channels. It has been used in cellular systems such as GSM and DCS1800. Although much research has been done on GMSK [7]-[10] but a very little work has been done with channel coding with GMSK. By the use of error control coding, we can further improve the BER that can be achieved by GMSK alone. The little work that has been done on coded GMSK is in R-S codes [11] and Turbo codes [12], [13] compared the performance of GMSK modulated optimum viterbi algorithm receiver with LDPC coded GMSK. Simulation results have demonstrated that the BER performances of LDPC coded GMSK and Viterbi algorithm receiver based GMSK are same but LDPC coded GMSK has less complexity as compared to VA receiver. The theoretical analysis of GMSK using LDPC codes in Nakagami fading has been left a future research issue. Few papers [14]-[15] are available on the topic.

There are several distributions that can be considered to model the statistical characteristics of the fading channels. Nakagami-m distribution is the widely accepted statistical model due to its good fit with experimental results and its versatility. Much theoretical and numerical analysis of the performances of diverse communication systems operating in Nakagami fading has been reported in the literature [16]-[22]. In this type of fading, we have 'm' parameter by which we can model the signal fading conditions from severe to no fading at all. The Nakagami fading model was initially proposed because it matched empirical results for short ionosphere propagation. The Rayleigh and Rician distributions can be closely approximated by Nakagami fading by adjusting the value of the m-parameter [23]-[24].

This paper is organized as follows. In section 2.1, a brief explanation of LDPC codes is given. The modulation technique GMSK is presented in section 2.2. The Nakagami fading is discussed in section 2.3. BER performance of LDPC codes are discussed in section 3. Results and observations have been discussed in section 4. Finally conclusion has been presented in section 5.

2. SYSTEM MODEL

The system model used in the simulation is shown in Fig.1. An un-coded data bit streams are used as an input to the system. These bits are coded using LDPC coding scheme. To generate the GMSK signal, a pre-modulation Gaussian low pass filter

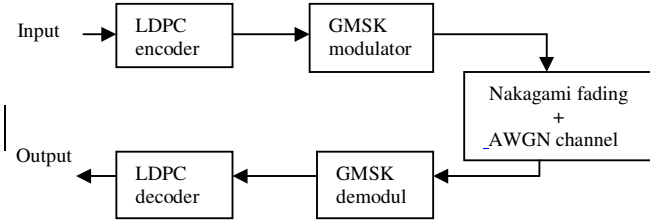


Fig.1. Block diagram of proposed system

(LPF) is adopted, through which LDPC coded data bits are passed. The input data is thus filtered with Gaussian filter and its shape is stabilized. The width of the Gaussian filter is determined by the normalized bandwidth product BT (e.g., BT = 0.3 for GSM and BT = 0.5 for Cellular Digital Packet Data (CDPD)). The filtered signal is fed into FSK based MSK transmitter to make its output power more compact. The modulated output is assumed to be transmitted through multipath fading channel with Nakagami fading distribution in presence of Additive White Gaussian Noise (AWGN). The Nakagami distribution is considered better as it accounts for both Rayleigh distribution (NLOS, m=1) and Rician distribution (LOS, m=1.25). At the receiver end, the received signal is demodulated and decoded to recover the corrected data bits. The detailed description of the various blocks of the system model is explained next.

2.1 LDPC CODE

Low density parity check codes are linear block code that provides the performance near the Shannon limit. LDPC codes were invented by Robert Gallager in his PhD dissertation. Based on the sparse parity check matrix, render a improved bit error rate on a large collection of data transmission and storage channels while simultaneously acknowledging implementable decoders. Thus the structure of a code is completely described by the generator matrix \mathbf{G} or the parity check matrix \mathbf{H} . The \mathbf{H} matrix is very sparse and has very few 1's in each row and each column. In LDPC codes, coded bits (u bits) obtained by multiplying un-coded (s bits) data with parity check matrix \mathbf{H} ($\mathbf{u} = \mathbf{s} * \mathbf{H}$). Unlike classical error correcting codes that are decoded with ML detection, LDPC codes are decoded iteratively using a graphical representation of their parity-check matrix and so are designed with the properties of \mathbf{H} as a focus.

Decoding message is computed on each variable and checks nodes and update the respective node. Then it makes a tentative decision of coded bit =1 if the bit node update is greater than 1 otherwise made a decision in favour of zero (0).

2.2 GMSK

An unfiltered binary data stream modulated with an RF carrier produces an RF spectrum of considerable width. Techniques have been developed to minimize this bandwidth, improve spectral performance with ease detection. MSK or Minimum Shift Keying is a digital modulation scheme in which the phase of the carrier remains continuous while the frequency changes. MSK is a continuous phase FSK (CPFSK) where the frequency changes occur at the carrier zero crossings. MSK is unique due to the relationship between the frequency of a logic 0

and 1. The difference between the frequencies is always $\frac{1}{2}$ the data rate. This is the minimum frequency spacing that allows 2 FSK signals to be coherently orthogonal. The fundamental problem with MSK is that the spectrum has side-lobes extending well above the data rate. For wireless systems which require more efficient use of RF channel bandwidth, it is necessary to reduce the energy of the upper side-lobes. The solution is to use a pre-modulation Gaussian Filter Impulse response defined by a Gaussian Distribution, in which BT product refers to the filter's 3dB bandwidth. GMSK or Gaussian-filtered Minimum Shift Keying thus differs from MSK in that a Gaussian Filter of an appropriate bandwidth is used before the modulation stage. In GSM standards BT=0.3 standardized because it provides the best compromise between increased bandwidth occupancy and resistance to inter symbol interference. GMSK with BT= ∞ is equivalent to MSK. The spectral property of MSK by using a pre-modulation Gaussian filter is given by.

$$h(t) = \frac{1}{\sigma T \sqrt{2\pi}} \exp\left(-\frac{t^2}{2\sigma^2 T^2}\right) \quad (1)$$

where

$$\sigma = \frac{\sqrt{\ln(2)}}{2\pi B_b T} \quad (2)$$

where the σ T $h(t)$ B_b are bandwidth time product.

2.3 NAKAGAMI DISTRIBUTION

The Nakagami distribution has gained widespread application in the modelling of wireless fading channels. The primary justification for the use of the Nakagami distribution is its good fit to measured fading data. This distribution can model signal fading conditions that range from severe to moderate, to light or no fading. Despite the fact that Nakagami fading is widely used in theoretical analysis of the performances of different communication systems, very few results pertaining to the computer simulation of Nakagami fading have been reported in literature. For $m=1/2$, the distribution is a one sided Gaussian distribution, for $m=1$, the distribution corresponds to Rayleigh fading, for fading more severe than Rayleigh fading, $1/2 \leq m \leq 1$ and for no fading at all $m=\infty$. When $m=1.5$ the distribution approximates Rician distribution. The probability density function of the received signal r with Nakagami distribution is given as:

$$f(r) = \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{mr^2}{\Omega}\right) \quad (3)$$

$$m \geq 1/2 ; r \geq 0$$

m is Nakagami parameter or shape factor, describing the fading degree due to scattering and multipath interference

$$m = \frac{E^2\{r^2\}}{E\{[r^2 - E(r^2)]^2\}} \quad (4)$$

Ω is the average power of multipath scatter field

$\Omega = E\{r^2\}$ and, $\Gamma(m)$ is the gamma function.

3. BER ANALYSIS USING LDPC CODES

LDPC decoding can be either Hard decision decoding or soft decision decoding. In this paper, a soft decoding has been implemented in terms of loglikelihood ratio. The analysis of

iterative systems is nontrivial, especially since most systems are nonlinear.

The probability of received signal presented in terms of LLR can be written as

$$L(b_i) = \ln \left(\frac{P(b_i=+1)}{P(b_i=-1)} \right) \quad (5)$$

The sign of this quantity is the hard decision of the estimated value, while its absolute value is the reliability of that decision. From this definition, we can rewrite

$$e^{L(b_i)} = \frac{P(b_i=+1)}{P(b_i=-1)} = \frac{P(b_i=+1)}{1-P(b_i=+1)} \quad (6)$$

or

$$P(b_i = +1) = \frac{e^{L(b_i)}}{1+e^{L(b_i)}}$$

and

$$P(b_i = -1) = \frac{1}{1+e^{L(b_i)}}$$

When decisions are taken conditioned to received vector Y, the LLR can be re-written as

$$L(b_i/Y) = \ln \left(\frac{P(b_i=+1/Y)}{P(b_i=-1/Y)} \right) = \ln \left(\frac{P(b_i=+1)}{P(b_i=-1)} \right) + \ln \left(\frac{P(Y_i/b_i=+1)}{P(Y_i/b_i=-1)} \right) \quad (7)$$

If bits b_1 and b_2 are generated by independent random sources, then

$$\begin{aligned} P((b_1 \oplus b_2) = +1) &= \frac{1+e^{L(b_1)}e^{L(b_2)}}{(1+e^{L(b_1)})(1+e^{L(b_2)})} \\ P((b_1 \oplus b_2) = -1) &= \frac{e^{L(b_1)}+e^{L(b_2)}}{(1+e^{L(b_1)})(1+e^{L(b_2)})} \\ L((b_1 \oplus b_2)) &= \ln \left[\frac{P((b_1 \oplus b_2) = +1)}{P((b_1 \oplus b_2) = -1)} \right] = \ln \left[\frac{1+e^{L(b_1)}e^{L(b_2)}}{e^{L(b_1)}+e^{L(b_2)}} \right] \\ &\approx \text{sign}(L(b_1)) \text{sign}(L(b_2)) \min(|L(b_1)|, |L(b_2)|) \end{aligned} \quad (8)$$

By using

$$\tanh \left(\frac{b}{2} \right) = \frac{e^b - 1}{e^b + 1}$$

And after mathematical manipulation (8) can be re written as

$$\begin{aligned} \sum_{j=1}^J [\oplus] L(b_j) &= L(\sum_{j=1}^J \oplus b_j) = \ln \left[\frac{1+\prod_{j=1}^J \tanh(L((b_j)/2))}{1-\prod_{j=1}^J \tanh(L((b_j)/2))} \right] \\ &= 2 \tanh^{-1} \left(\prod_{j=1}^J \tanh(L((b_j)/2)) \right) \end{aligned} \quad (9)$$

which can be approximately calculated as

$$\begin{aligned} \sum_{j=1}^J [\oplus] L(b_j) &= L \left(\sum_{j=1}^J \oplus b_j \right) \\ &\approx \left(\prod_{j=1}^J \text{sign}(L(b_j)) \right) \min_{j=1, \dots, J} |L(b_j)| \end{aligned} \quad (10)$$

So we are using density evolution method to evaluate the BER performance. Probability of BER is given by [26]

$$BER = \int L^{[l]}(b) db \quad (11)$$

4. SIMULATION DETAILS

The main focus is to use Monte Carlo simulations to demonstrate the BER performance of LDPC coded system using GMSK modulation technique. In computer simulations a (500,600) **irregular** LDPC with the mean column weight of 3 is employed. A frame consisting of 200 coded bits is sent from transmitter with all the bits having random values 0 and 1. For GMSK simulation parameters are BT product is 0.25 and oversampling period is 8. The signal is divided into in-phase and Quadrature phase components Simulation is run over 1000 such iterations for each value of SNR varying from 0 to 6 dB. The channel is modeled as Nakagami faded. For a given time instant, the faded received signal in which effect of motion is included taking into account the Doppler effect is considered. The path amplitudes were taken to be Weibull-distributed random variables and generated using the function from the Statistics Toolbox. The two-parameter Weibull distribution allowed the variation of the mean and variance of the scattering amplitudes. The phases were taken to be uniform in $[0, 2\pi]$ and were generated using the function from the Statistics Toolbox. The signal was then GMSK demodulated to get the in- phase and Quadrature components, using the command from the Signal Processing Toolbox. The noise is considered as AWGN. Soft decoding with density evolution method determines the threshold above which the code performs well. The demodulated signal is sent to the LDPC decoder. After decoding, un-coded signal which is taken as reference signal and decoded signal is compared to evaluate BER. Fig.2 demonstrates the BER performance of uncoded GMSK modulated in Nakagami fading environment system. Fig.3 shows the BER performance of the LDPC coded GMSK modulated signal for Nakagami m parameter $m=1$ (corresponding to Rayleigh fading) and $m=2$ (corresponding to Rician fading). Results shows that there is significant improvement in BER when Nakagami distribution tends to $m=2$ (Rician distributed). Fig.4 shows the comparison of uncoded and LDPC coded system. It has been shown that for BER of 10^{-2} LDPC coded requires only 1.5 dB signal power for $m=2$ (Rician) and 2 dB signal power for $m=1$ (Rayleigh) while uncoded system require 4.5 dB and 4.7 dB with $m=1$ and 2 respectively. A normalized graph is shown in fig 5 that clearly shows the substantial coding gain is achieved with LDPC coding. Nakagami fading envelope with $m=1$ and $m=2$ is shown in Fig.6 and Fig.7 where the fading conditions has been changing from very severe to low fading. As the value of the m

parameter increases, the severity of fading conditions goes on decreasing and the envelope reaches very low fading conditions for values of m greater than 10.

5. CONCLUSION

It is concluded that BER performance of LDPC coded GMSK modulated signal over fading conditions ranging from very severe to less severe is excellent. There is improvement of 3 dB and 2.7 dB for $m=1$ and $m=2$ respectively. Also the average bit error rate decreases as m increases (less fading) as expected. The LDPC coding improves the average probability of error in the two cases of m . The coding gain decreases as m increases since fading decreases.

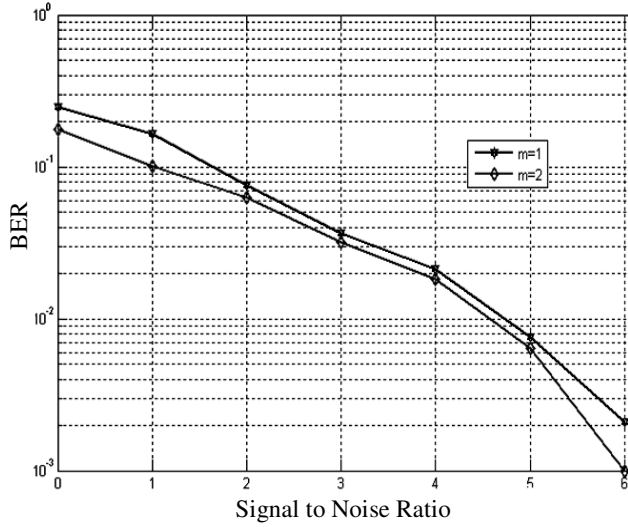


Fig.2. BER evaluation of uncoded-GMSK modulated in Nakagami fading

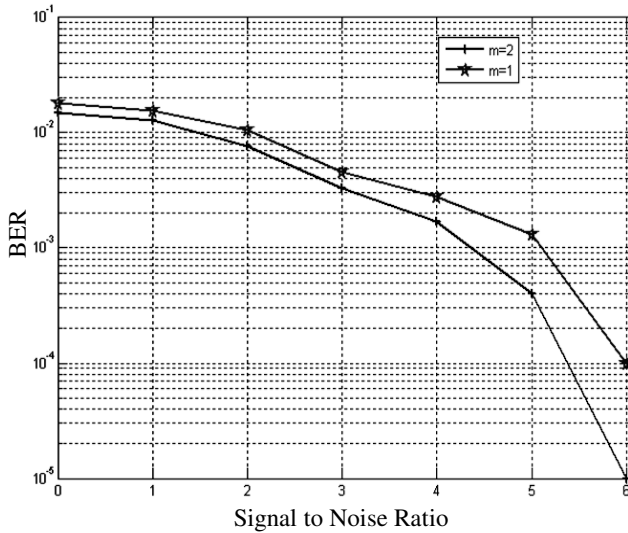


Fig.3. BER evaluation of LDPC coded GMSK modulated in Nakagami fading

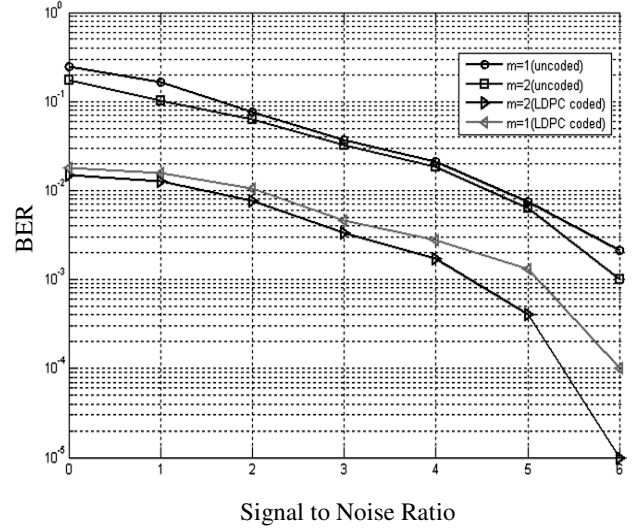


Fig.4. BER evaluation of LDPC coded GMSK modulated in Nakagami fading

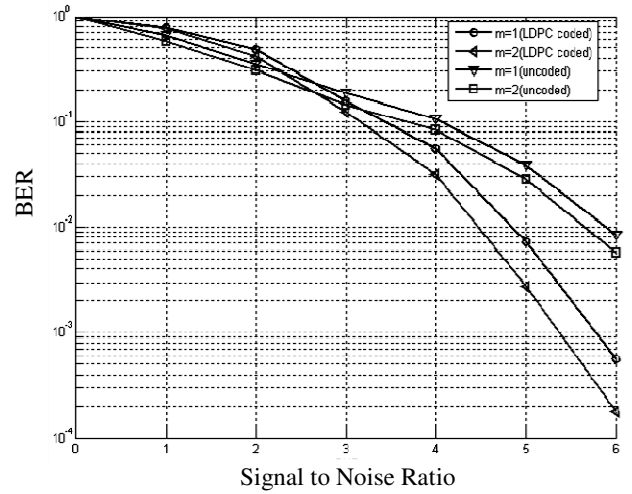


Fig.5. BER evaluation of Normalized LDPC coded GMSK modulated in Nakagami fading

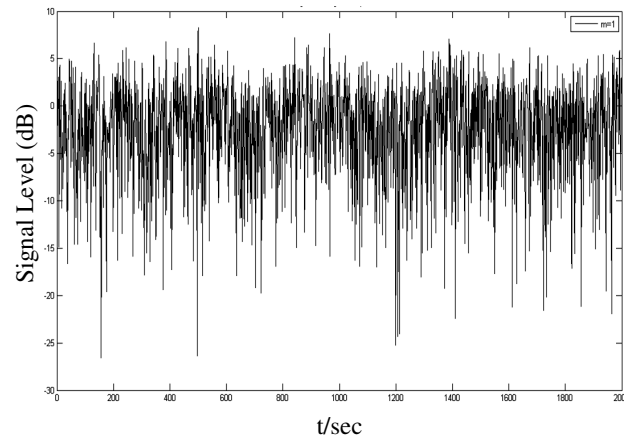


Fig.6. Nakagami fading envelope corresponding to $m=1$ (Rayleigh Faded)

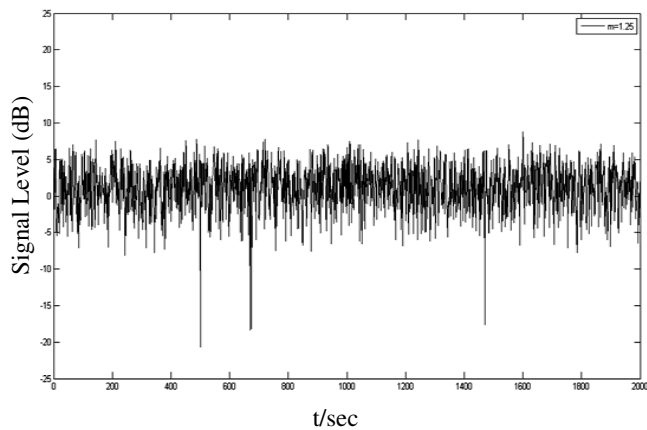


Fig.7. Nakagami fading envelope corresponding to $m=2$ (Rician faded)

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