ISSN: 2229-6948 (ONLINE)
ICTACT JOURNAL OF COMMUNICATION TECHNOLOGY, JUNE 2010, VOLUME: 01, ISSUE: 02
DOI: 10.21917/ijct.2010.0016

EVALUATION OF OPTIMAL TRANSMIT POWER IN WIRELESS SENSOR NETWORKS IN PRESENCE OF RAYLEIGH FADING

Arnab Nandi[1] and Sumit Kundu[2]
Department of Electronics & Communication, National Institute of Technology, Durgapur, Durgapur-713209, India
E-mail: nandi_arnab@yahoo.co.in[1], sumit.kundu@ece.nitdgp.ac.in[2]

Abstract
Design of an energy efficient wireless sensor network (WSN) has emerged as an important research area. Minimizing energy consumption is the primary objective for WSN. WSN is usually characterized by tiny size, low cost and low transmission power. So optimization of transmission power is of great importance. Optimal transmit power not only achieves better network lifetime but also reduces inter-node interference significantly. In this paper we carry out simulation studies to investigate the effects of Rayleigh fading on the performance of WSN and optimal transmit power in presence of Rayleigh fading is derived. The effects of bit rate and Rayleigh fading on optimal transmit power are investigated under several network conditions. In this paper the network performance is estimated in terms of a quality of service (QoS) constraint given by the maximum tolerable bit error rate (BER). The derived optimal transmit power maintains a minimum BER constraint. The impact of fading on critical bit rate i.e., the bit rate below which a desired BER can not be achieved with any amount of transmit power is also studied.

Keywords:
Sensor Networks, Power Control, Network connectivity, Rayleigh Fading.

1. INTRODUCTION

A wireless sensor networks (WSN) consists of many small devices, powered by batteries and operates unattended for protracted duration. So, minimizing energy consumption is the primary goal for WSN since the lifetime of a sensor network is determined by its power consumption rate. The connectivity of an ad hoc wireless network mostly depends on the transmission power of the source nodes. If the transmission power is not sufficiently high there may be single or multiple link failure. Again very high transmission power creates excessive amount of inter-node interference. So optimization of transmission power is needed to achieve a minimum power to assure uninterrupted network connectivity and longer network lifetime. Several approaches have been proposed in literature to prolong network lifetime. Sooksan et al. [1] evaluated Bit Error Rate (BER) performance and optimal power to preserve the network connectivity considering only path-loss and thermal noise. In [2] Bettstetter et al. derived the transmission range for which network is connected with high probability considering free-space radio link model. In [3] the relationships between transmission range, service area and network connectedness is studied in a free space model. Narayanaswamy et al. [4] proposed a protocol that extends battery life through providing low power routes in a medium with path loss exponent greater than 2. In [5] minimum uniform transmission power of an ad hoc wireless network to maintain network connectivity is proposed considering path loss only. However most of the previous works deal without considering fading environment. In practical situation there may be multiple reflective paths between source and sink leading to Rayleigh fading [6]. Hence it is important to investigate minimum transmission power in presence of fading. We derived the minimum common transmit power in presence of Rayleigh fading to maintain the network connectivity. The effects of bit rate and Rayleigh fading on optimal transmit power are investigated under several network conditions.

Obtaining minimum transmission power considering every link in an ad hoc network is difficult and burdensome [1]. In the absence of centralized system for controlling transmission power, it is very difficult to maintain the transmission power on link-by-link basis. Using a common transmission power satisfying desired QoS of the network requires a trade off between local power control and minimum common transmission power.

In this paper, we derive the optimal transmit power over Rayleigh fading channel in sensor networks. The minimum common transmission power in the presence of Rayleigh fading also depends on the routing and the medium access control (MAC) protocol used. Here we considered a very simple routing strategy following [1, 7]. We carry out simulation study to derive the optimal transmit power in presence of Rayleigh fading for square grid topology under some network conditions. The impact of network conditions such as bit rate and node spatial density on optimal transmit power is investigated. There exist a critical bit rate below which a desired BER can not be achieved with any amount of transmit power. The effects of Rayleigh fading on critical bit rate are also shown.

The rest of this paper is organized as follows: In Section II, we describe the system model and the parameters to be used in the derivation of the optimal transmit power in the presence of Rayleigh fading. Relevant results and discussions are given in section III. Finally paper is concluded in section IV.

2. SYSTEM MODEL

In this section, we describe the system model used in this paper. Fig. 1 shows a two-tier sensor network using square grid topology. The distance between two nearest neighbor is d_{link}. When the node density increases, the distance between two consecutive nodes decreases following eqn. (1). We considered a scenario where N numbers of nodes are distributed over region of area A obeying a square grid topology. The node spatial density ρ_{sq} is defined as the number of nodes per unit area i.e., $\rho_{sq} = N/A$. The minimum distance between two consecutive neighbors is given by [1]

$$d_{link} = \frac{1}{\sqrt{\rho_{sq}}} \tag{1}$$

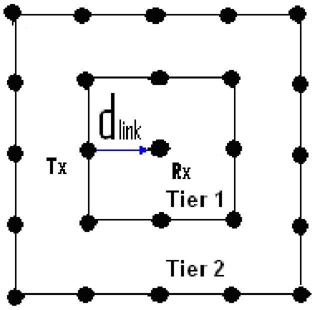

Fig.1. Sensor nodes in square grid topology

Here we assume a simple routing strategy such that a packet is relayed hop-by-hop through a sequence of nearest neighboring nodes, until it reaches the destination. Again we assume that a source node discovers a route prior to data transmission [1]. Discovery of a multihop route from a source to a destination is a crucial phase in a wireless networking scenario with regular architecture. The focus of this paper is on the characterization of the steady state behavior of on-going peer-to-peer multihop communications. Therefore, we will assume that a route between source and destination exists as in [8].

Here we consider a simple reservation based MAC protocol as introduced in [7] and called as reserve-and-go (RESGO). In this protocol, a source node first reserves intermediate nodes on a route for relaying its packets to the destination. A transmission can begin after a route is discovered and reserved. The main idea of the protocol is that a source node or a relay node generates an exponential random back-off time before it transmits or relays each packet. After the random back-off time expires, a node can start transmitting a packet. The random back-off time helps to reduce interference among nodes in the same route and also among nodes in different routes. Throughout this paper, we assume that the random back-off time is exponential with mean $1/\lambda_t$. Where λ_t is the packet transmission rate.

We know that major perturbation in wireless transmission is path-loss, large scale fading and small scale fading. Large-scale fading arises due to motion over large areas and affected by prominent terrain contours (hills, forests, clumps of buildings, etc.) between the transmitter and receiver, which generally follows a lognormal distribution [11]. Further small-scale fading exhibits rapid changes in signal amplitude and phase as a result of small changes (as small as a half-wavelength) in the spatial separation between a receiver and transmitter. The rate of change of these propagation conditions accounts for the fading rapidity. Small-scale fading is also called Rayleigh fading because if the multiple reflective paths are large in number and there is no line-of-sight signal component, the envelope of the received signal is statistically described by a Rayleigh pdf given below

$$\begin{aligned} p(r) &= r/\sigma^2 \exp\left[-r^2/2\sigma^2\right] \text{ for } r \geq 0 \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{2}$$

where r is the envelope amplitude of the received signal and $2\sigma^2$ is the pre-detection mean power of the multipath signal. When there is a dominant non-fading signal component present, the small scale fading envelope is described by a Rician pdf.

In [6] the effects of pathloss and thermal noise are considered to derive the optimal transmission power. However it is important to extend the analysis in presence of Rayleigh fading.

As discussed earlier, the optimal common transmit power is the minimum power sufficient to preserve network connectivity. Conceptually, an ad hoc wireless network is often viewed as a graph, where vertices represent the nodes and edges represent the links connecting neighboring nodes. However, using this notion of connectivity for an ad hoc wireless network, where a communication channel is error-prone, can be misleading. Since the wireless links are susceptible to errors, the QoS in terms of route BER deteriorates as the number of hops in a route increases. Consequently, the performance may be unacceptable, although there is a sequence of links to the destination.

Hence it is necessary to consider network connectivity from communication theoretic viewpoint, where a network is said to be connected if any source node can communicate with a BER lower than a prescribed value BER_{th} to a destination node placed at the end of a multihop route with an average number of hops. Here we consider an ideal worst-case scenario where an information bit is relayed on each link of a route toward a destination without retransmissions. However, the use of retransmission techniques can make the situation better.

We can assume without any loss of generality that a source node is at the center of the network (see Fig. 1). If a destination node is selected at random, the minimum number of hops to reach the destination can range from 1 to $2i_{max}$, where i_{max} is the maximum tier order. In other words, it takes 1 hop to reach a destination, which is a neighbor of a source node in tier 1 and it takes $2i_{max}$ hops to reach the farthest node from the center in tier i_{max}. Counting the number of hops on a route from the source to each destination node and finding the average value can obtain the average number of hops. Assuming that each destination is equally likely, the average number of hops on a route can be written as [1]

$$\bar{n}_{hop} = \frac{1}{N-1}\left[4\sum_{i=1}^{i_{max}} i + 4\sum_{i=1}^{i_{max}} 2i + 8\sum_{i=1}^{i_{max}}\sum_{j=1}^{i-1}(i+j)\right] \tag{3}$$

It can be approximated as

$$\bar{n}_{hop} \cong \sqrt{N}/2 \tag{4}$$

The average number of hops, $\bar{n}_{hop}$ is used to obtain the route BER from the link BER. The network connectivity is defined in terms of BER quality at the end of a multihop route. In this section, we analyze the link BER and the route BER in the presence of Rayleigh fading using a detection-theoretic approach. The received signal at the receiver is the sum of three components (i) the intended signal from a transmitter, (ii) the interfering signals from other active nodes and (iii) the thermal noise. Since the interfering signals come from other nodes, we assume that the total interfering signal can be treated as an additive noise process independent of the thermal noise process. The received signal S_{rcv} during each bit period can be expressed as

$$S_{rcv} = S_{Ray} + \sum_{j=1}^{N-2} S_j + n_{thermal} \tag{5}$$

where S_{Ray} is the desired signal in the presence of Rayleigh fading, S_j is the interference from the other nodes and $n_{thermal}$ is the thermal noise signal

Considering source node and sink/relay node are separated by a distance of d_{link} as shown in Fig. 1. The power received at the receiving end is given by Frii's transmission equation [9, 10]

$$P_{rcv} = \frac{P_t G_t G_r c^2}{(4\pi)^2 f_c^2 d_{link}^{\alpha}} \qquad (6)$$

where P_t is the transmit power, G_t is the transmitting antenna gain, G_r is the receiving antenna gain, f_c is the carrier frequency, α is the path-loss exponent and c is the velocity of light. Here we considered omni directional (G_t=G_r=1) antennas at the transmitter and receiver. The carrier frequency is in the unlicensed 2.4 GHz band. P_{Ray} is the received signal power in presence of Rayleigh fading and is given as

$$P_{Ray} = \gamma P_{rcv} \qquad (7)$$

where γ is the Rayleigh fading factor signifying the severity of the Rayleigh fading. Assuming a binary phase shift keying (BPSK) modulation, there can be two cases for the amplitude of the S_{Ray}

$$\begin{aligned} S_{Ray} &= \sqrt{\frac{P_{Ray}}{R_{bit}}} = \sqrt{E_{bit}} \text{ for a } +1 \text{ transmission} \\ &= -\sqrt{\frac{P_{Ray}}{R_{bit}}} = -\sqrt{E_{bit}} \text{ for a } -1 \text{ transmission} \qquad (8) \end{aligned}$$

where $\sqrt{E_{bit}}$ is the bit energy of the received signal in presence of Rayleigh fading.

The interference power from node j can be written as

$$P_{\text{int}\,j} = \frac{P_t G_t G_r c^2}{(4\pi)^2 f_c^2 (\nu_j d_{link})^{\alpha}} = \frac{P_{rcv}}{\nu_j^{\alpha}} \qquad (9)$$

where ν_j is a multiplicative factor depends on the position of the interfering node. For example, the node at the corner of the second tier has a distance $2\sqrt{2}d_{link}$. So in this case $\nu_j = 2\sqrt{2}$. It is observed that the significant part of the inter-node interference comes from the first two tiers only. Here we considered inter-node interference from first two tiers only.

For each interfering node j, the amplitude of the interfering signal can be of three types [1]:

$$\begin{aligned} S_j &= \sqrt{\frac{P_{\text{int}\,j}}{R_{bit}}} \text{ for a } +1 \text{ transmission} \\ &= -\sqrt{\frac{P_{\text{int}\,j}}{R_{bit}}} \text{ for a } -1 \text{ transmission} \\ &= 0 \text{ for no transmission of node j} \qquad (10) \end{aligned}$$

The probability that an interfering node will transmit and cause interference depends on the MAC protocol employed. Considering the RESGO MAC protocol and assuming that each node transmits packets with fixed length L_{packet}, the interference probability is equal to the probability that an interfering node transmits during the vulnerable interval of duration L_{packet}/R_{bit}, where R_{bit} is the bit rate. The probability can be written as [6]

$$p_{transmission} = 1 - e^{-\frac{\lambda_t L_{packet}}{R_{bit}}} \qquad (11)$$

So, S_j appears with different probability of transmission given below

$$\begin{aligned} S_j &= \sqrt{\frac{P_{\text{int}\,j}}{R_{bit}}} \text{ with probability } \frac{1}{2}P_{transmission} \\ &= -\sqrt{\frac{P_{\text{int}\,j}}{R_{bit}}} \text{ with probability } \frac{1}{2}P_{transmission} \\ &= 0 \text{ with probability } (1 - P_{transmission}) \qquad (12) \end{aligned}$$

The thermal noise power can be written as

$$P_{thermal} = FkT_0B \qquad (13)$$

where F is the noise figure, $k = 1.38 \times 10^{-23} J/K$ is the Boltzmann's constant, T_0 is the room temperature and B is the transmission bandwidth.

The received thermal noise signal is simply

$$n_{thermal} = \sqrt{FkT_0B} \qquad (14)$$

Assuming that a bit detected erroneously at the end of a link is not corrected in successive links, the BER at the end of a route with $\overline{n}_{hop}$ links, denoted as BER_{route}, can be written as

$$BER_{route} = 1 - (1 - BER_{link})^{\overline{n}_{hop}} \qquad (15)$$

Size of the interference vector $\vec{S}_j$ increases as the number of nodes increases in the network. But it is found that interference from the first two tiers is significant. So without any loss of generality we considered the interference from the first two tiers only.

3. RESULTS AND DISCUSSION

We developed a simulation test bed in MATLAB for evaluating the performance of WSN. Important parameters used in simulation are given in Table.1. For Fig.2, 3 and 4the node density is varied. Similarly for Fig.5 and 6 bit rate is varied. Other parameters remain same for all the simulations as shown below:

Table.1. Network Parameters Used in the Simulation

Parameter	Values
Path loss exponent (α)	2
Number of nodes in the network (N)	289
Node special density (ρ_{sq})	10^{-7}m^{-2}
Packet length (L_{packet})	10^3bit
Packet arrival rate at each node (λ_t)	0.5 pkt/s
Carrier frequency (f_c)	2.4 GHz
Noise figure (F)	6dB
Room Temperature (T_0)	300K
Transmission Power (P_t)	1 mW

In Fig. 2, we compare the link BER obtained from simulation for different bit rates in the presence of Rayleigh fading and without considering Rayleigh fading. It shows that link BER performance degrades in presence of Rayleigh fading. This is because in presence of Rayleigh fading the desired signal strength decreases. Simulation result shows that beyond a certain node density the BER does not change with increased node spatial density and a floor in BER, denoted as $\mathrm{BER}_{\mathrm{floor}}$ appears. This is expected because, increasing node spatial density beyond a certain limit no longer improves the signal to noise ratio (SNR), as the interfering nodes also become close enough to the receiver. For a bit rate of 100 Kb/s we obtain the floor at around node density ρ_{sq} of 7×10^{-2} in the presence of Rayleigh fading, whereas it is around $\rho_{sq}=5\times10^{-3}$ without Rayleigh fading. We get the $\mathrm{BER}_{\mathrm{floor}}$ for higher values of node density in presence of Rayleigh fading. For example at 2 Mb/s bit rate, the $\mathrm{BER}_{\mathrm{floor}}$ starts from node density of $\rho_{sq}=10^{-4}$ where as it starts from $\rho_{sq}=10^{-5}$ if Rayleigh fading is not considered. So, link BER performance degrades severely in presence of Rayleigh fading.

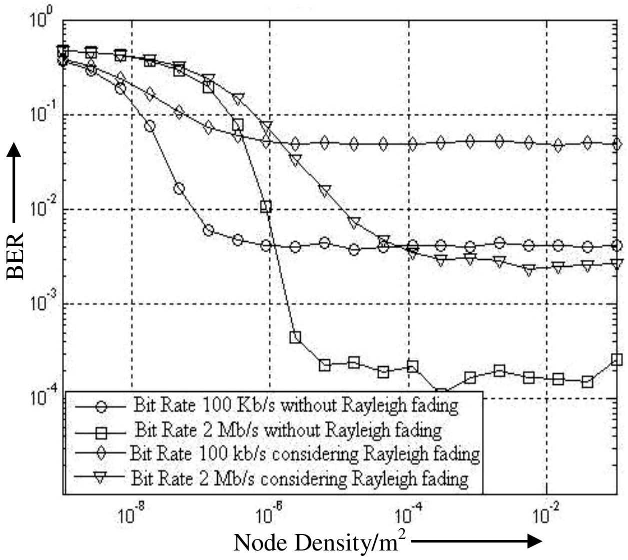

Fig.2. Link BER as a function of node spatial density, comparing the case in Rayleigh fading and without Rayleigh fading for different bit rates

In Fig. 3, the effect of Rayleigh fading on route BER is seen. Due to Rayleigh fading the BER is higher compared the case of no Rayleigh fading. We compare the route BER obtained from simulation for different bit rates in the presence of Rayleigh fading and without considering Rayleigh fading. It is seen that in presence of Rayleigh fading the route BER performance degrades. The desired signal power as well as the inter-node interference increases with the increase of node density. As a result we obtain the floor in the figure. For a bit rate of 2 Mb/s we obtain the floor at around $\rho_{sq}=10^{-4}$ in the presence of Rayleigh fading, while it is at around $\rho_{sq}=10^{-5}$ without Rayleigh fading. Figure shows that when the node density is greater than a certain value, the $\mathrm{BER}_{\mathrm{route}}$ attains a floor.

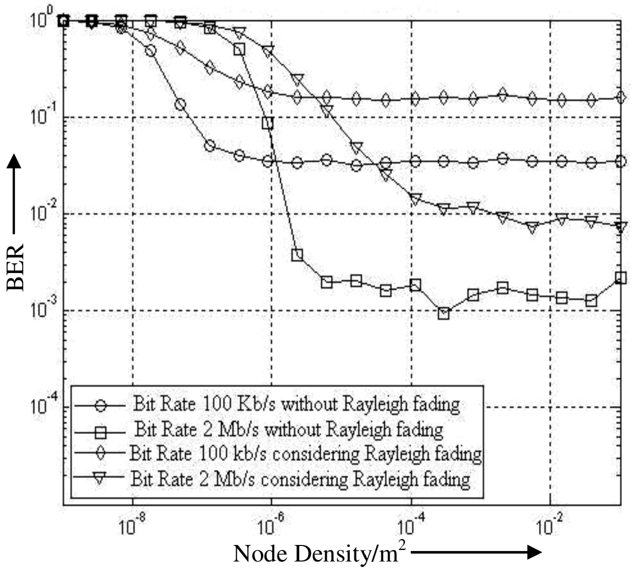

Fig.3. Route BER as a function of node spatial density

In Fig. 4, we study the impact of fading severity on the WSN. We compare the obtained $\mathrm{BER}_{\mathrm{route}}$ as a function of node spatial density, comparing the case in Rayleigh fading with several values of variance for a bit rate of 100 Kb/s. As we increase the severity of the Rayleigh fading, the BER performance degrades. Figure shows the BER performance for different values of variance of the Rayleigh fading. It is observed that BER performance degrades when severity of fading increases but they attain floor for almost same value of node density. Here BER floor starts from $\rho_{sq}=10^{-5}$ for all cases of variances for bit rate of 100 Kb/s.

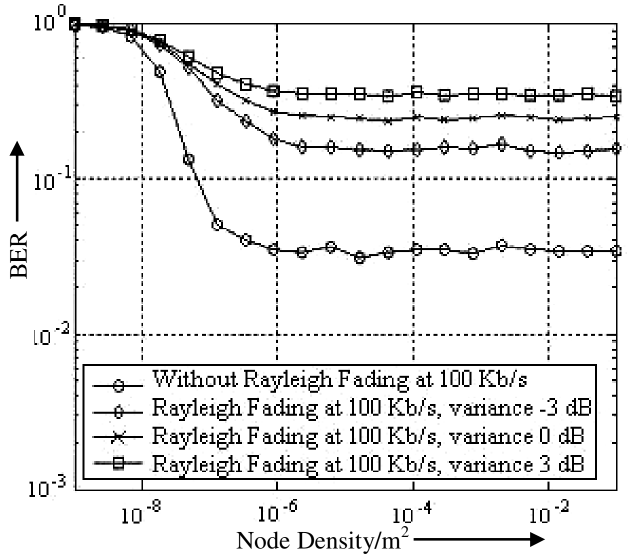

Fig.4. Route BER as a function of node spatial density in presence of Rayleigh fading

In Fig. 5, we compare the optimal common transmission power as a function of bit rate in the presence of Rayleigh fading and without Rayleigh fading. The optimal powers vs. data rate curves are shown for various values of BER_{th}. It is observed that the optimal transmit power increases as the data rate increases. In presence of Rayleigh fading, the required optimal transmission power is very high compared to the case considering only path loss and thermal noise. Although transmitting packets at a higher data rate reduces the vulnerable time (and, hence, smaller interference), increasing the data rate (i.e., bandwidth) also increases the thermal noise. Therefore, the minimum transmit power required to sustain the network connectivity increases. It is observed from Fig. 5 that there is a critical data rate, below which the desired BER_{th} cannot be satisfied for any transmit power. The critical bit rate occurs at the point where the BER_{floor} for that particular data rate becomes higher than the desired BER_{th}. Curves show critical bit rate value get worse in presence of Rayleigh fading. This is because in presence of Rayleigh fading signal to interference noise ratio (SINR) degrades and consequently BER_{floor} value degrades. For example, when we consider the transmission in Rayleigh fading environment the critical bit rate increases to 6 Mb/s whereas it is only 4 Mb/s for the case without Rayleigh fading for $BER_{th}=10^{-3}$. Consequently, no amount of transmission power can achieve the desired BER_{th} below the critical bit rate. The optimal transmit power is also minimized at the data rate near the critical point. This suggests that the data rate also plays an important role in the design of wireless ad hoc and sensor networks, i.e., for a given node spatial density, if the data rate is carefully chosen, the transmit power can be minimized, prolonging the network's lifetime.

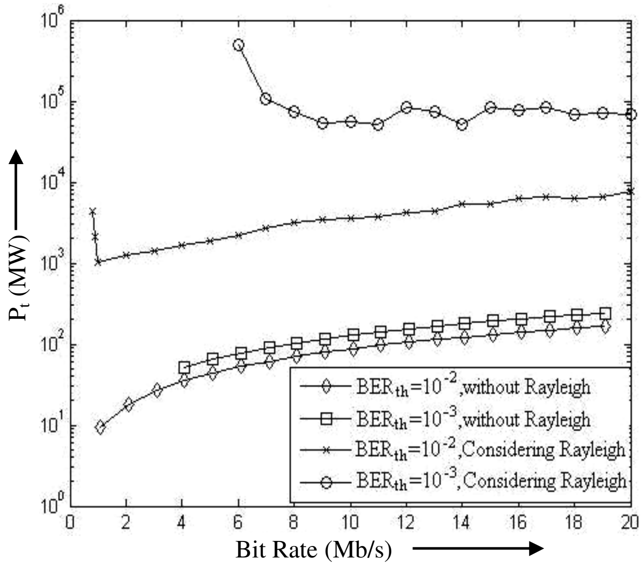

Fig.5. Optimal common transmit power in a network in presence of Rayleigh fading and without Rayleigh fading

In presence of Rayleigh fading the optimal common transmission power is very high than that of case without Rayleigh fading. For example, the required common optimal transmission power to obtain the $BER_{th} = 10^{-2}$ in presence of Rayleigh fading is around 3W at bit rate of 10 Mbps, where it is only around 0.1W for the case without Rayleigh fading with same BER threshold value and same bit rate as above. Referring to Fig. 5, the percentage of degradation in presence of Rayleigh fading may be computed. Here the required optimal transmission power in Rayleigh fading environment increases 30 times as compared to that of the case without Rayleigh fading for a bit rate of 10 Mb/s and BER_{th} at 10^{-2}.

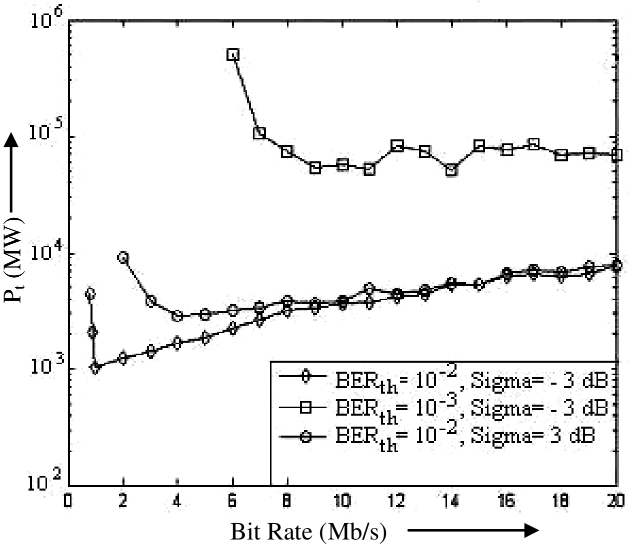

Fig.6. Optimal common transmit power for different values of variance in presence of Rayleigh fading

In Fig. 6, we compare the optimal common transmission power as a function of bit rate in the presence of Rayleigh fading for different values of variance. When variance of fading increases, it requires higher transmission power to maintain the same BER threshold value. From Fig. 6 it is observed that the optimal transmission power increases from 1.5 W to 10 W when the variance value varies from -3dB to 3dB for the bit rate of 2 Mb/s and BER_{th} of 10^{-2}.

4. CONCLUSION

In this paper, we have derived the optimal common transmit power for wireless sensor networks in Rayleigh fading environment and under several network conditions. Optimal common transmission power is the minimum power required to maintain the network connectivity satisfying a given BER threshold value. It is seen that in presence of Rayleigh fading the link BER and route BER performance degrades. It is also seen that increasing the bit rate improves the link and route BER performance of the wireless sensor networks. The performance of the network gradually deteriorates with the increase of fading severity. Optimal transmission power is seen to be significantly higher in Rayleigh fading environment as compared to path loss case. Optimal transmission power also increases with the severity of the Rayleigh fading to achieve the same BER threshold. There exist a critical bit rate below which a desired BER can not be achieved with any amount of transmit power. Critical bit rate increases from around 4 Mb/s to 6 Mb/s in presence of Rayleigh fading for a given BER threshold value of

10^{-3}. Critical bit rate also increases with the increase of fading severity.

For further minimization of the transmission power we may involve diversity combining technique. It can be further investigated using other MAC protocols. Moreover, Results can be studied in different channel environment also.

REFERENCES

[1] Sooksan Panichpapiboon, Gianluigi Ferrari, and Ozan K. Tonguz, "Optimal Transmit Power in Wireless Sensor Networks", IEEE Transaction on Mobile Computing, Vol. 5, No. 10, October 2006, pp. 1432-1447.

[2] C. Bettstetter and J. Zangl, "How to Achieve a Connected Ad Hoc Network with Homogeneous Range Assignment: An Analytical Study with Consideration of Border Effects", Proc. IEEE Int'l Workshop Mobile and Wireless Comm. Network, pp. 125-129, September. 2002.

[3] C.-C. Tseng and K.-C. Chen, "Power Efficient Topology Control in Wireless Ad Hoc Networks", Proc. IEEE Wireless Comm. and Networking Conf. (WCNC), Vol. 1, pp. 610-615, March 2004.

[4] S. Narayanaswamy, V. Kawadia, R.S. Sreenivas, and P.R. Kumar, "Power Control in Ad-Hoc Networks: Theory, Architecture, Algorithm and Implementation of the COMPOW Protocol", Proc. European Wireless 2002 Next Generation Wireless Networks: Technologies, Protocols, Services, and Applications, pp. 156-162, February. 2002.

[5] Q. Dai and J. Wu, "Computation of Minimal Uniform Transmission Power in Ad Hoc Wireless Networks", Proc. IEEE Int'l Conf. Distributed Computing Systems Workshops (ICDCS), pp. 680-684, May 2003.

[6] Bernard Sklar, "Rayleigh Fading Channels in Mobile Digital Communication System Part I: Characterization", IEEE Communications Magazine, July 1997, pp 90-100.

[7] C.E. Perkins, Ad Hoc Networking. Addison-Wesley, 2001.

[8] G. Ferrari and O.K. Tonguz, "Performance of Ad Hoc Wireless Networks with Aloha and PR-CSMA MAC Protocols", Proc. IEEE Global Telecomm. Conf. (GLOBECOM), pp. 2824-2829, December. 2003.

[9] S. Haykin, Communication Systems, fourth ed. John Wiley & Sons, 2001.

[10] T.S. Rappaport, Wireless Communications Principles and Practice, Upper Saddle River, N.J.: Prentice-Hall, 1996.

[11] M. C. Jeruchim, P. Balaban and K. S. Shanmugan, Simulation of Communication system, Kulwer Academic/Platinum Publishers, New York, 1992.

[12] Alberto Leon-Garcia, Probability and Random Pocesses for Electrical Engineering, Second Edition. Addison-Wesley Publishing Company.