

SVD-BASED TRANSMIT BEAMFORMING FOR VARIOUS MODULATIONS WITH CONVOLUTION ENCODING

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Abstract

This paper presents a new beamforming technique using singular value decomposition (SVD) for closed loop Multiple-input, multiple-output (MIMO) wireless systems with various modulation techniques such as BPSK, 16-QAM, 16-PSK, DPSK and PAM along with convolution encoder. The channel matrix is decomposed into a number of independent orthogonal modes of excitation, which refer to as eigenmodes of the channel. Transmit precoding is performed by multiplying the input symbols with unitary matrix to produce the transmit beamforming, and the precoded symbols are transmitted over Rayleigh fading channel. At the receiver, combining process is performed by using maximum ratio combiner (MRC), and the receiver shaping is performed to retrieve the original input symbols by multiplying the received signal with conjugate transpose of the unitary matrix. Furthermore, the expressions for average bit error rate (BER) for M-PSK and average BER for M-QAM are derived. The superiority of the proposed work is proved by simulation results and the proposed work is compared to the other beamforming methods.

Keywords:

MIMO, Transmit-Beamforming, Diversity, SVD, Rayleigh Fading Channel

1. INTRODUCTION

MIMO systems have been extensively studied over the last decade [1]. The capacity analysis of MIMO systems has shown significant gains over single-input single-output (SISO) systems [2] [3]. Depending on the channel condition, the first generation MIMO technique aims at achieving a higher data rate, such as spatial multiplexing [4], or a higher diversity, such as space-time coding [5]. These techniques do not require the knowledge of channel state information (CSI) at the transmitter. In [6] and [7], it is suggested that an additional performance gain can be extracted from multiple antennas in the presence of channel state information at the transmitter.

When perfect CSI is available at both ends, beamforming is used to maximize the SNR at the receiver. Beamforming separates the MIMO channel into parallel independent subchannels. When the subchannel with the largest gain is used for transmission, the technique is called single beamforming [8]. MIMO systems can also be used to enhance the throughput of wireless systems [9]. In the context of feedback systems, more than one subchannel can be used to improve the capacity. This technique is called multiple beamforming [8]. Most work on closed-loop MIMO systems has been carried out by performing SVD of the channel transfer matrix. Among linear precoders, the SVD-based beamforming technique combined with a proper power allocation method is shown to be optimum in terms of capacity [10] [11] [12]. Maximum ratio combining algorithm

was generalized for both transmit beamforming and receive combining in [13] where the concept of maximum ratio transmission (MRT) was presented. In order to maximize the received signal-to-noise ratio (SNR), the transmit beamforming and receive combining vectors need to be jointly designed by utilizing the SVD of the channel matrix [14] [15].

In this paper, At the transmitter we propose a transmit precoding approach by multiplying the matrix V to perform transmit beamforming and the precoded symbols are transmitted over the rayleigh fading channel by M_t transmitting antenna, at the receiver faded version of the transmitted symbols which are received by M_r receiving antenna and the MRC is used as a combiner and the receiver shaping is performed by multiplying the channel output with matrix U^H . Here we use the M-PSK, M-QAM, DPSK and PAM for modulating the transmit signals, the convolutional encoder and viterbi decoder is used for encode and decode the signal respectively. We derive the expression for an average BER for the M-PSK over the Rayleigh fading channel and also we derive the average BER for the M-QAM over the Rayleigh fading channel with MRC reception.

The organization of the paper is as follows: Section II presents a general description of SVD-based transmit beamforming system model and description about proposed SVD-based single and multiple beamforming. In Section III, we derive the expression for average BER for the M-PSK over the Rayleigh fading channel and also we derive the average BER for the M-QAM over the Rayleigh fading channel with maximum ratio combining reception. Section IV shows the simulation results and compares the results of various modulations. Finally, the paper is terminated with conclusions in Section V.

2. SYSTEM MODEL

In this section, we present a general description of transmitter and receiver sections of SVD-based transmit beamforming. We consider the MIMO system models with M_t transmit antennas and M_r receives antennas, as shown in Fig.1. Transmission is over a Rayleigh fading channel and both the transmitter and receiver are assumed to have perfect knowledge of the channel. The system consists of a beamforming processing at the transmitter and combining processing at the receiver. At the transmitter, the information bits $s = \{s_1, s_2, \dots, s_k\}$ are encoded with convolutional encoder and the encoded bits are modulated with either M-PSK, M-QAM, DPSK or PAM modulator which is used as symbol mapper, to yield the symbol vector of $\bar{x} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k\}$, where k is the number of transmitted symbols.

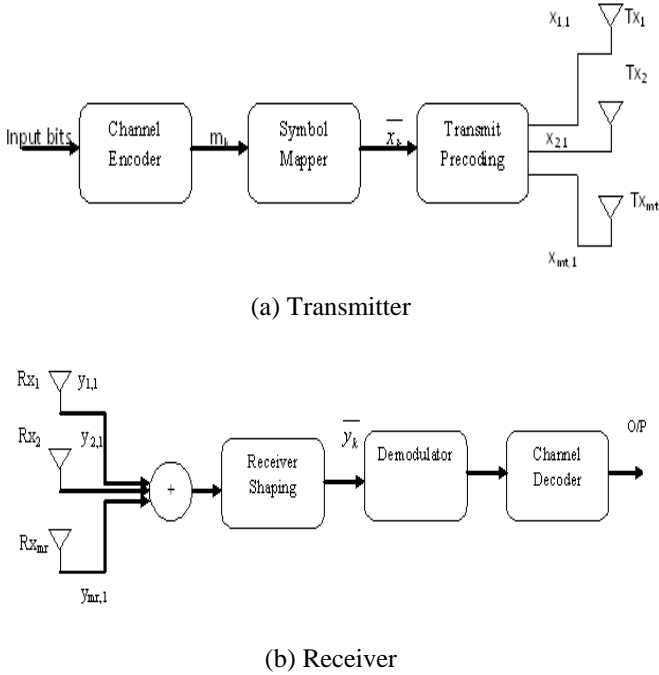


Fig. 1. Schematic diagram of a transmit beamforming and receive combining scheme with M_t transmit and M_r receive antennas

This data symbol s is applied to the transmit precoding and multiplied by matrix V to perform transmit beamforming, and the precoded symbols $x = \{x_1, x_2, \dots, x_k\}$ are transmitted over Rayleigh fading channel. We assume that the elements of the MIMO channel matrix are obtained from an independent and identical distribution (i.i.d) complex Gaussian distribution with zero mean and unit variance.

If x is the $1 \times M_t$ vector containing the symbols to be transmitted, H is the channel matrix of size $M_t \times M_r$ and n a vector of additive white gaussian noise (AWGN) on the receiving antenna of size $1 \times M_r$, the vector of received symbol can be expressed as,

$$y = Hx + n. \quad (1)$$

At the receiver, the transmitted symbol $y = \{y_1, y_2, \dots, y_k\}$ are received with addition of channel noise n and the combining process to be performed for receiving symbols with MRC, receiver shaping is performed at the receiver by multiplying the channel output y with to produce $\bar{y} = \{\bar{y}_1, \bar{y}_2, \dots, \bar{y}_k\}$ and finally the demapping and decoding operation is performed to produce the output.

2.1 SVD-BASED BEAMFORMING OVERVIEW

Beamforming is implemented by multiplying the symbols with appropriate beamforming vectors, both on transmitter and the receiver. In this paper we assume CSI is available at both the ends in such a case, the beamforming vectors are obtained via SVD of the channels then the SVD of channel can be written as,

$$H = U \Sigma V^H. \quad (2)$$

where, U and V are the two unitary matrices of size $M_t \times M_t$ and $M_r \times M_r$, respectively, and $(.)^H$ denotes the conjugate transpose and Σ is the $M_t \times M_r$ diagonal matrix with non-negative real

numbers on the diagonal, $\Sigma = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{M_r})$ where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{M_r} > 0$ are the singular values and by using SVD, the MIMO channel is divided into independent and parallel sub channels.

2.2 PROPOSED SVD-BASED SINGLE BEAMFORMING

Only one symbol is transmitted over the subchannel with the largest gain. The channel matrix H may be decomposed into a number of independent- orthogonal modes of excitation, which we will refer to as an eigenmodes of the channel.

The transmitter multiplies x with V before sending into the antennas and receiver multiplies the signals received on each antenna by the matrix U^H . Fig.2, present the transmitter precoding and the receiver shaping transform the MIMO channel into R_H (rank of H) parallel SISO channel with input \bar{x} and output \bar{y} , since the overall transmission relationship,

$$\bar{y} = U^H (H x + n), \quad (3)$$

$$= U^H (U \Sigma V^H x + n), \quad (4)$$

$$= U^H U \Sigma V V^H \bar{x} + U^H n, \quad (5)$$

$$\bar{y} = \Sigma \bar{x} + \bar{n}. \quad (6)$$

where, $\bar{n} = U^H n$, is the multiplication by a unitary matrix does not change the distribution of the noise (i.e.) n and \bar{n} they are identically distributed. The optimal vectors to be used at the transmitter side and receiver side are the first column of U and V corresponding to the largest singular value of H . Then, the received signal can be represented by

$$y = \bar{x} U_1^H H V_1 + U_1^H n, \quad (7)$$

$$y = \lambda_1 \bar{x} + \bar{n}, \quad (8)$$

where, λ_1 is the largest singular value of H . If the noise process is band limited with bandwidth B , the noise power of n is given by $BN_0 = 2\sigma_n^2$. The equalized signal y outlines that each transmitted symbol is weighted by its singular value which is shown from Eq.(9),

$$y_i = \lambda_{1_i} \bar{x}_i + \bar{n}_i. \quad (9)$$

where, $i=1, 2, \dots, M_t$ from Eq.(9) it is feasible to calculate each SISO SNR value shown by Eq.(10)

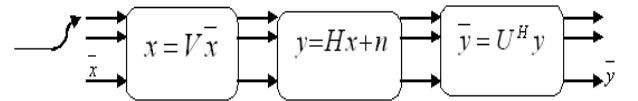


Fig.2. Schematic diagram of a transmit precoding and receive shaping scheme

$$SNR_i = \lambda_{1_i}^2 \frac{|x_i|^2}{2\sigma_n^2}. \quad (10)$$

Since for a reliable communication the singular values must be greater than zero, therefore the singular values are also treated

as MIMO processing gain, if the power of all symbols is normalized, so that the overall average power is given by,

$$\overline{P_s} = \frac{1}{M_t} \sum_{i=1}^{M_t} |x_i|^2 = 1. \quad (11)$$

2.3 PAIRWISE ERROR PROBABILITY (PEP) OF SBF

SBF achieves the diversity order of $M_t M_r$ for arbitrary M_t and M_r , we will show that by analyzing the PEP. SBF uses the subchannel with largest gain λ_1 , to transmit only one symbol. Assume that the symbol x is sent and \bar{x} is detected, by using the maximum-likelihood (ML) criterion, the PEP of x and \bar{x} , given CSI can be written as in [16]

$$P(x \rightarrow \bar{x} / H) = P\left(|y - \lambda_1 x^2 \geq y - \lambda_1 \bar{x}^2\right) \quad (12)$$

By simplifying Eq.(12), PEP can be bounded by

$$P(x \rightarrow \bar{x}) \leq \frac{1}{2} \left(\frac{d_{\min}^2}{4 N_0 M_t} + 1 \right)^{-M_t M_r}, \quad (13)$$

$$P(x \rightarrow \bar{x}) \approx \frac{1}{2} \left(\frac{d_{\min}^2 E_b}{4 M_t N_0} \right)^{-M_t M_r}. \quad (14)$$

for high SNR. Eq.(14) Shows that diversity order of SBF is $M_t M_r$.

2.4 SVD-BASED MBF

Multiple symbols are simultaneously sent over different parallel subchannel. The optimal vectors to be used as weights at the transmitter side and receiver side are the first S columns of U and V corresponding to the first S larger singular values of H , when the S subchannel for multiple beamforming becomes,

$$y_k = \frac{1}{\sqrt{S}} \lambda_i x_i + n_i. \quad (15)$$

Multiple beamforming achieves the diversity order of $(M_t - S + 1)(M_r - S + 1)$, for arbitrary M_t, M_r, S where S is the number of subchannel.

2.5 PAIRWISE ERROR PROBABILITY (PEP) OF MBF

MBF achieves the diversity order of $(M_t - S + 1)(M_r - S + 1)$ for arbitrary M_t, M_r , and S . We will show that by analyzing the PEP. The performance is dominated by the weakest subchannel. Therefore, when S symbols are transmitted, PEP is bounded by

$$P(x \rightarrow \bar{x}) \leq E \left[\frac{1}{2} \exp \left(\frac{-\mu_s d_{\min}^2}{4 S N_0} \right) \right] \quad (16)$$

where μ_s is the S^{th} largest eigenvalue of HH^H , i.e., $\mu_s = \lambda_s^2$. The pdf for the S^{th} largest eigenvalue can be approximated as in [16].

$$\rho(\mu_s) \approx K \mu_s^{(M_t - S + 1)(M_r - S + 1)} \quad (17)$$

where K is constant. The PEP of MBF is written as

$$P(x \rightarrow \bar{x}) \leq \gamma \left(\frac{d_{\min}^2 E_b}{4 S M_t N_0} \right)^{-(M_t - S + 1)(M_r - S + 1)}. \quad (18)$$

where γ is constant, Eq.(18) shows that the diversity order of MBF is $(M_t - S + 1)(M_r - S + 1)$ when S subchannels are simultaneously used.

3. PERFORMANCE ANALYSIS

3.1 DERIVATION OF THE AVERAGE BER FOR THE M-PSK OVER THE RAYLEIGH FADING CHANNEL

The average BER for the M-PSK system in the presence of Rayleigh fading is considered in this section for fading channel, the conditional BER for M-PSK is given in [17],

$$P_b(E)_{M-PSK} = \frac{1}{\log_2 M} \operatorname{erfc} \left[\sqrt{\frac{\log_2(M) E_b}{N_0}} \sin \left(\frac{\pi}{M} \right) \right], \quad (19)$$

However in the presence of channel h , the effective bit energy to noise ratio is, $|h|^2 E_b / N_0$. So that bit error probability for a given value of h is,

$$P_{b|h}(E)_{M-PSK} = \frac{1}{\log_2 M} \operatorname{erfc} \left[\sqrt{\log_2(M) \gamma} \sin \left(\frac{\pi}{M} \right) \right] \quad (20)$$

where, $\gamma = |h|^2 E_b / N_0$ is the instantaneous SNR per bit of the received signal $\operatorname{erfc}(\cdot)$ is the complementary error function. In addition, the probability density function (PDF) of γ for Rayleigh fading channel is given by [17],

$$P(\gamma) = \frac{1}{\gamma} e^{-\gamma/\bar{\gamma}} \quad \gamma \geq 0 \quad (21)$$

where, $\bar{\gamma} = E_b / N_0$, the average BER can then be obtained using Eq.(20) and Eq.(21),

$$P_b(E)_{M-PSK} = \int_0^{\infty} \frac{1}{\log_2 M} \operatorname{erfc} \left[\sqrt{\log_2 M \gamma} \sin \left(\frac{\pi}{M} \right) \right] P(\gamma) d\gamma \quad (22)$$

In BPSK, the symbols $\{-1, +1\}$ are used for transmitting information, the average BER for the BPSK system in the presence of Rayleigh fading is given by,

$$P_b(E)_{BPSK} = \frac{1}{2\gamma} \left[\bar{\gamma} \operatorname{erfc}(\sqrt{\bar{\gamma}}) e^{-\frac{\bar{\gamma}}{\gamma}} + \bar{\gamma} \frac{\operatorname{erfn} \left(\sqrt{\frac{\bar{\gamma}}{\gamma+1}} \right)}{\sqrt{\gamma+1}} \sqrt{\bar{\gamma}} \right], \quad (23)$$

$$P_b(E)_{BPSK} = 0.5 \left[1 - \frac{\bar{\gamma}}{\gamma+1} \right], \quad (24)$$

$$P_b(E)_{BPSK} = 0.5 \left[1 - \frac{E_b/N_0}{\sqrt{(E_b/N_0)+1}} \right] \quad (25)$$

In 16-PSK the alphabets $\left\{ 1, e^{\frac{j2\pi}{M}}, e^{\frac{j4\pi}{M}}, \dots, e^{\frac{j2\pi(M-1)}{M}} \right\}$ is used,

where $M=16$, the bit error rate for 16-PSK is given as

$$P_b(E)_{16-PSK} = \frac{1}{4} \operatorname{erfc} \left[\sqrt{\frac{4E_b}{N_0}} \sin \left(\frac{\pi}{16} \right) \right] \quad (26)$$

3.2 DERIVATION OF THE AVERAGE SER FOR THE M-QAM OVER THE RAYLEIGH FADING CHANNEL

Let us consider square M-QAM signals whose constellation size is given by $M=2^k$ with k even. The probability of symbol error for M-QAM as in [18],

$$P_b(E)_{M-QAM} = 4 \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right) \times \mathcal{Q} \left(\sqrt{\frac{3E_b}{N_0(M-1)}} - 4 \left(\frac{\sqrt{M}-1}{\sqrt{M}} \right)^2 \mathcal{Q}^2 \left(\sqrt{\frac{3E_b}{N_0(M-1)}} \right) \right), \quad (27)$$

where, $\mathcal{Q}(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(\frac{-x^2}{2 \sin^2 \theta}\right) d\theta$. is the Gaussian \mathcal{Q} -function. The conditional SER for M-QAM with L -branch MRC receiver is given by Eq.(27) as,

$$P_b(E/\{\gamma_l\}_{l=1}^L)_{M-QAM} = 4 \left(1 - \frac{1}{\sqrt{M}} \right) \mathcal{Q} \left(\sqrt{2 g_{QAM} \gamma_l} \right) - 4 \left(1 - \frac{1}{\sqrt{M}} \right)^2 \mathcal{Q}^2 \left(\sqrt{2 g_{QAM} \gamma_l} \right), \quad (28)$$

where $\gamma_l =$ average SNR of l^{th} branch and $\gamma_L = \sum_{l=1}^L \gamma_l$ is the total conditional SNR per symbol at the output the MRC and $g_{QAM} = 3/2[(M-1)]$, by averaging Eq.(27) over the rayleigh pdf and using a standard integral involving the function $\mathcal{Q}^2(\cdot)$, we obtain the performance of M-QAM over L i.i.d Rayleigh fading channel as,

$$P_b(E)_{M-QAM} = 4 \left(1 - \frac{1}{\sqrt{M}} \right) \left(\frac{1-\mu c}{2} \right)^L \sum_{l=0}^{L-N} \binom{L-1+l}{l} \left(\frac{1+\mu c}{2} \right)^l - 4 \left(1 - \frac{1}{\sqrt{m}} \right)^2 \left(\frac{1-\frac{\mu c}{4}}{\pi} (R_b - R_a) \right) \quad (29)$$

where,

$$R_b = \left[\frac{\pi}{2} - \tan^{-1} \mu c \right] \sum_{l=0}^{L-1} \frac{\binom{2l}{l}}{4(1+g_{QAM}\bar{\gamma})^l} \quad (30)$$

$$R_a = \sin \left(\tan^{-1} \mu c \right) \times \sum_{l=1}^{L-1} \sum_{i=1}^l \frac{Til}{(1+g_{QAM}\bar{\gamma})^l} \left[\cos \left(\tan^{-1} \mu c \right) \right]^2 (l-i) + 1 \quad (31)$$

$$\mu c = \sqrt{\frac{g_{QAM}\bar{\gamma}}{1+g_{QAM}\bar{\gamma}}} \quad (32)$$

and

$$T_{il} = \frac{\binom{2l}{l}}{\binom{2(l-i)}{l-i} 4^i (2(l-i)+1)} \quad (33)$$

The Q-function can be expressed in terms of the complementary error function (erfc) as in [19],

$$\mathcal{Q}(x) = \frac{1}{2} \operatorname{erfc} \left(\frac{x}{\sqrt{2}} \right) \quad (34)$$

By relating the Eqs.(29), (34) and substitute $M=16$, we can get the bit error rate for 16-QAM as,

$$P_b(E)_{16-QAM} = \frac{3}{8} \operatorname{erfc} \left(\sqrt{\frac{4E_b}{10N_0}} \right) \quad (35)$$

4. SIMULATION RESULTS

In this section, we present the simulation results to demonstrate the BER of SVD-based single beamforming over the Rayleigh fading channel. Here, we use six types of modulation techniques, such as BPSK, 16-QAM, 16-PSK, 64-QAM, 2-DPSK and 4-PAM. In that we compare the BER performance of both modulation for single and multiple beamforming, with number of transmit and receive antennas modulations but it produce the low raw data rate than other modulations.

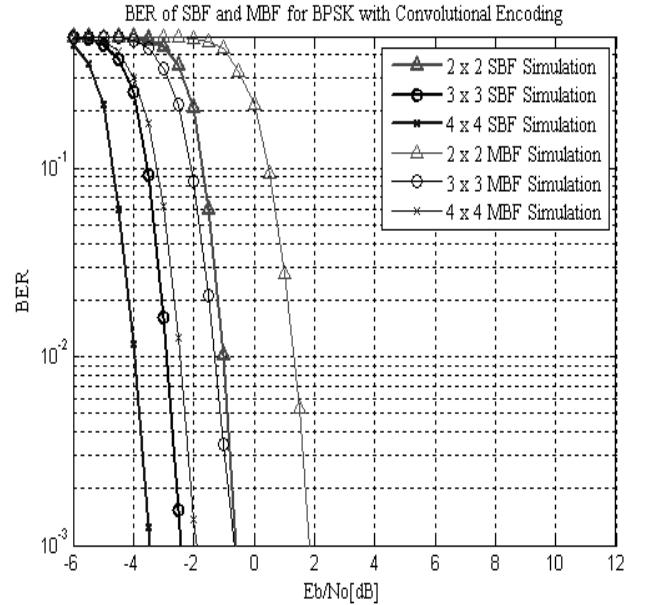


Fig.3. BER of SVD-based SBF and MBF for BPSK with $M_t = M_r = 2, 3, 4$

In Fig.3, we present the BER performance of the SVD-based single and multiple beamforming with respect to E_b/N_0 in dB with $M_t = M_r = 2, 3, 4$ and BPSK, and also compare the BER performance of SBF and MBF. This Fig. shows that the BER of the SVD-based single and multiple beamforming is reduced with the increasing number of transmit and receive antennas and MBF exhibit higher error rate than SBF, however MBF produce higher raw data-rate than SBF. Figs. 4,5,6,7 and 8 displays and compare the BER performance of the SVD-based single and

multiple beamforming with respect to E_b/N_0 in dB with $M_t = M_r = 2, 3, 4$ and 16-QAM, 16-PSK, 64-QAM, 2-DPSK and 4-PAM respectively, and these figures deliver the same result as Fig.3.

Fig.9 shows BER comparison for 2x2 single beamforming systems with modulations. This shows that the 16-PSK exhibit higher error rate than all other modulations, however it deliver higher raw data-rate than other modulations. 2-DPSK produce lower error rate than 16-PSK but higher than other modulations. 16-QAM and 4-PAM are moreover delivers the same amount of error rate. BPSK exhibit the lower error rate than other.

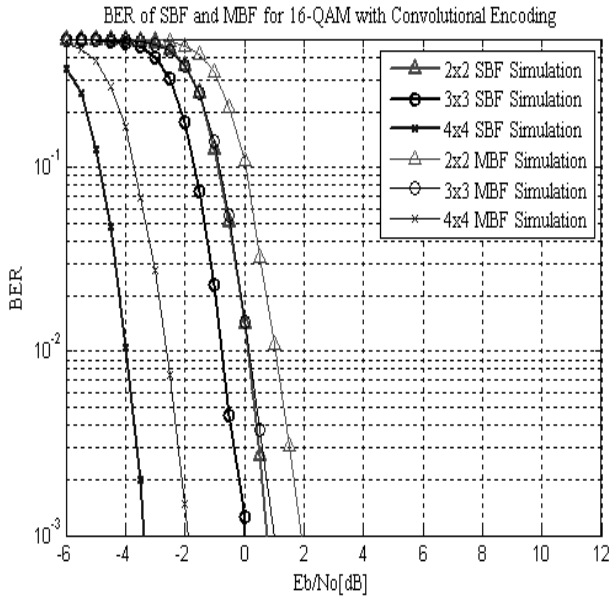


Fig.4. BER of SVD-based SBF and MBF for 16-QAM with $M_t = M_r = 2, 3, 4$

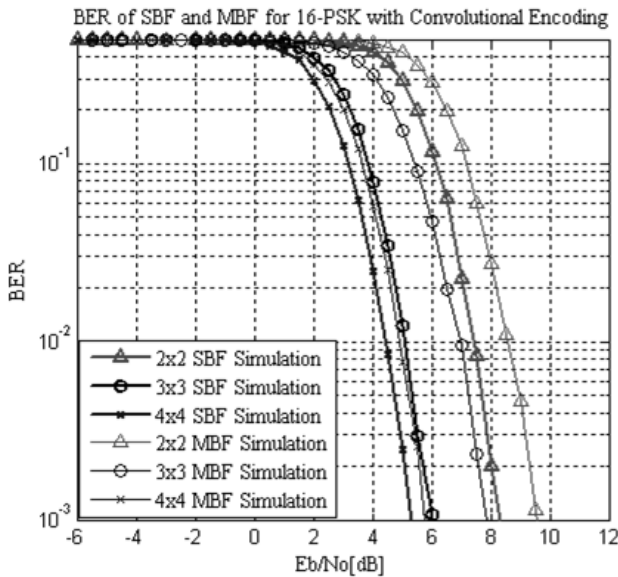


Fig.5. BER of SVD-based SBF and MBF for 16-PSK with $M_t = M_r = 2, 3, 4$

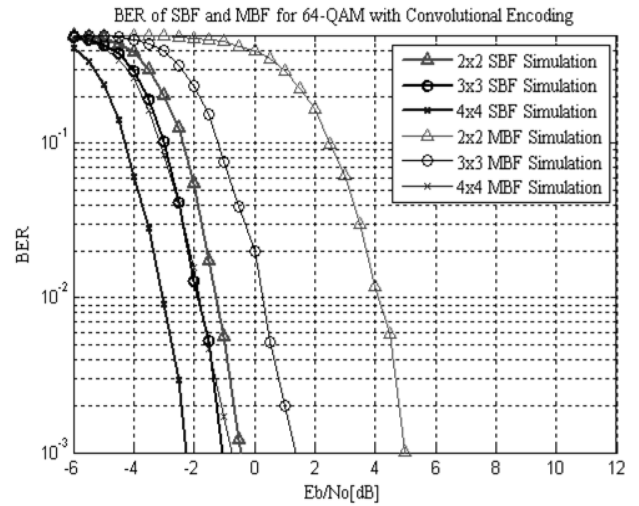


Fig.6. BER of SVD-based SBF and MBF for 64-QAM with $M_t = M_r = 2, 3, 4$

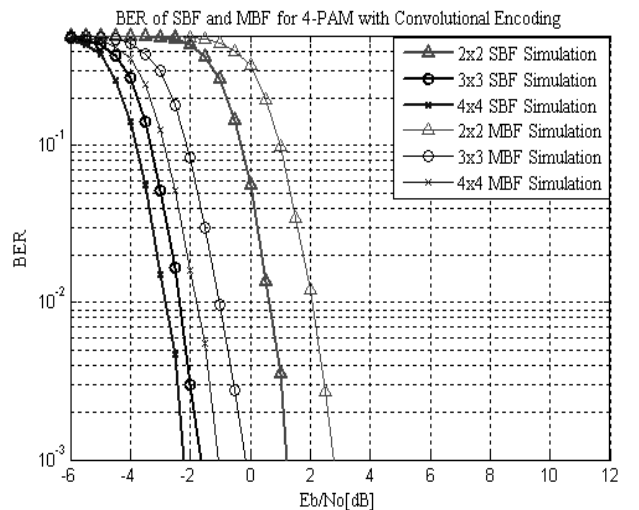


Fig.7. BER of SVD-based SBF and MBF for 2-DPSK with $M_t = M_r = 2, 3, 4$

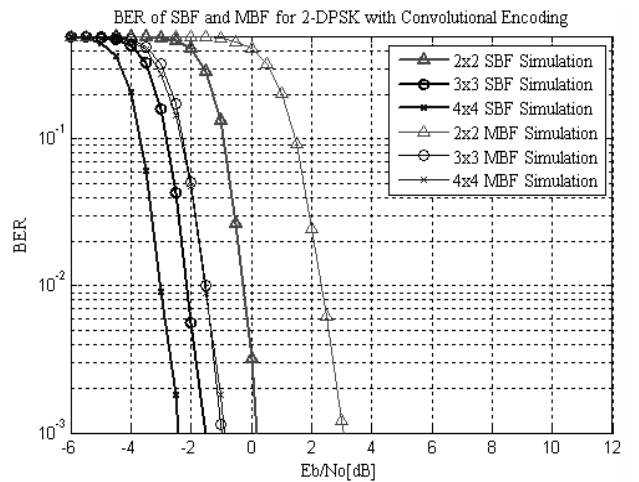


Fig.8. BER of SVD-based SBF and MBF for 4-PAM with $M_t = M_r = 2, 3, 4$

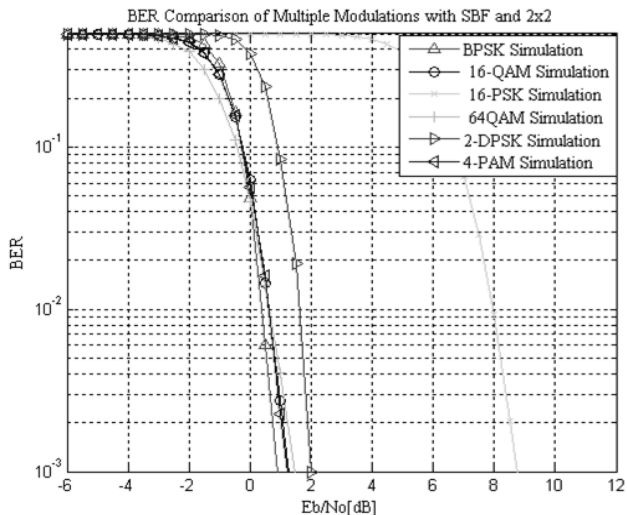


Fig.9. BER comparison of the modulations for SVD-based single beamforming with $M_t = M_r = 2$

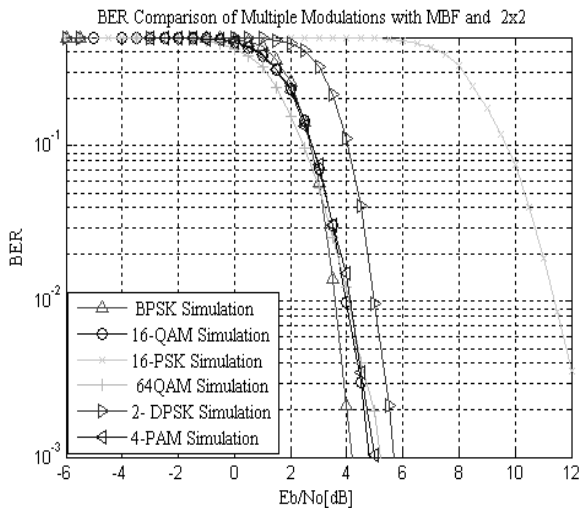


Fig.10. BER comparison of the modulations for SVD-based multiple beamforming with $M_t = M_r = 2$

Fig.10 shows bit-error-rate performance comparison of BPSK, 16-QAM, 16-PSK, 64-QAM, 2-DPSK, 4-PAM for SVD-based multiple beamforming with 2×2 . It shows that BER of the SVD-based multiple beamforming is reduced with the increasing number of transmit and receive antennas and also produce the same result as Fig.9. When we compare the Fig.9 and Fig.10, single beamforming delivers lower error rate than multiple beamforming in all type of modulations but multiple beamforming produce the high raw data rate than single beamforming in all type of modulation.

5. CONCLUSION

Transmit-beamforming scheme for MIMO system based on SVD-based single and multiple beamforming with BPSK, 16-QAM, 16-PSK, 64-QAM, 2-DPSK and 4-PAM modulations is presented in this paper. And expression for average BER for M-PSK over the Rayleigh fading channel when SVD-based

beamforming is used for channel decomposition and BER for M-QAM with MRC at the receiver over the Rayleigh fading channel also derived. Comparison of the BER performance is made for different type of modulations with various number of transmits and receives antennas for SBF and MBF. The proposed SVD-based beamforming schemes yield the better performance when compare to the other beamforming methods [20] with little bit of system complexity, when the increased number of transmit and receive antennas [20]. The simulation shows that the BER performance of SVD-based beamforming with convolution encoding is reduced with the increased number of transmit and receive antennas and, the 16-PSK exhibit higher error rate than other modulations however 16-PSK delivers higher raw data-rate than other modulations for various combination of transmit and receive antennas. BPSK delivers lower error rate than other modulations at same time it produce low raw data rate than other modulations. This work can be extended by using a new beamforming structure based on Transmit-MRC [20] which is combined with adaptive bit loading and power allocation for reducing complexity in SVD-based beamforming.

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