

EFFECTS OF THE NUMBER OF RELAY ANTENNAS AND RELAY-POWER ON MIMO PRECODED TWO-WAY RELAYING

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Abstract

In this treatise a two-way Amplify and Forward (AF) relay-aided system is considered, which employs the so-called Arithmetic Sum of Average Bit Error Rate (ASABER) based MIMO precoding technique. The two-way AF relay system is comprised of the pair of transceiver nodes S_1 and S_2 , and the Relay Node (RN) R , where each node is equipped with N_1 , N_2 and N_r antennas, respectively. We study the effects of varying N_r for fixed values of N_1 and N_2 , and as well as the effects of having a fixed transmission power at the RN on the achievable ASABER performance. Based on our intensive simulation campaign, we infer that the attainable diversity order is increased approximately by $N_r - \min(N_1, N_2)$, whenever N_r assumes a value higher than $\min(N_1, N_2)$ for fixed N_1, N_2 values. However, this is observation is only valid for relay power $p_r \geq (p_1, p_2)$, where p_1 and p_2 are the transmit power constraints imposed on the sources S_1 and S_2 , respectively. We also observe that the ASABER MIMO precoder's BER curve exhibits an error floor for $p_r \leq (p_1, p_2)$.

Keywords:

Two-way Relay, Bidirectional Relay, Amplify and Forward, MIMO Precoding

1. INTRODUCTION

One of the challenging design objectives of next generation wireless communication systems is to support an increased data-rate right across the entire propagation cell. Relay-assisted wireless transmission schemes are capable of achieving this challenging goal and hence they have attracted substantial research efforts over the past decade. Diverse cooperative protocols have been proposed for exchanging information amongst the transmitter, receiver and relay nodes. The basic two-way cooperative communication system consists of two transceiver nodes (SN), namely S_1, S_2 and a relay node R . All the nodes are assumed to be half-duplex, since practical transceivers cannot transmit and receive simultaneously¹. Since most practical communications sessions are bi-directional, they can invoke two-way relaying protocols. In [1] a detailed spectral efficiency study of diverse cooperative protocols was provided.

The family of cooperative systems may be classified as Amplify and Forward (AF), Decode and Forward (DF) as well as Compress and Forward (CF) regimes, based on the specific technique used for processing the received signal at the RN.

This work was carried out under the IU-ATC project funded by the Department of Science and Technology (DST), Government of India and the UK EPSRC Digital Economy Programme.

¹This is because the transmitter's high transmit power typically results in third-, fifth- and seventh-order out-of-band emissions due to the power-amplifier's non-linear distortion, which would saturate/desensitize the receiver's Automatic Gain Control (AGC) scheme configured for receiving low-power signals.

AF relaying is the least computationally complex technique amongst them, since no synchronization, demodulation or channel decoding is necessitated - the received signal is simply amplified by the RN. However, the signal and noise are jointly amplified, hence AF relaying fails to improve the Signal-to-Noise Ratio (SNR). In this contribution we consider an AF MIMO two-way relay-aided system.

The performance of every communication link is degraded by channel impairments. The attainable system performance may be improved by exploiting the knowledge of the channel's unique, user-specific impulse response about to be experienced with the aid of transmit preprocessing techniques used at the transmitter. Naturally, the vital prerequisite of acquiring the required Channel Impulse Response (CIR) is its accurate estimation at the distant receiver, which then has to quantize and signal the CIR back to the transmitter. Linear precoding is one of the most appealing preprocessing techniques by virtue of its low implementational complexity.

In two-way relay-aided systems we have the freedom of designing precoders at S_1, S_2 and R . There are numerous methods available in the open literature, which discuss the design of linear precoders at the source and relay, which may invoke diverse optimization criteria, such as maximizing the achievable sum-rate, the Arithmetic Sum of Average Bit Error Rate (ASABER) criterion and the Arithmetic Sum of Average Mean Square Error (ASAMSE) criterion. In [2], [3], [4] and [5] optimal RN precoders were designed for maximizing the system's sum-rate, while in [6] an optimal RN precoder was designed for minimizing the Sum of the Mean Squared Error (SMSE) of the two-way AF system, which was equipped with multiple antennas at all three nodes. These methods may be referred to as being 'relay-only' precoders (ROP), since they only specify the MIMO precoder at the RN.

In [7], [8], [9], [10] and [11]² the joint design of the SN and RN precoders was advocated for the sake of maximizing the sum-rate of the two-way relaying system. The joint design of SN and RN precoders was used for minimizing the ASAMSE and the ASABER criteria of the two-way relaying system in [8] and [12]. The method discussed in [12] and [11] was shown to outperform other existing methods.

Our main goal is to study the effect of varying the number of antennas at relay on the attainable ASABER performance of the MIMO-aided precoded two-way AF system. Here the precoders used at the SN and RN are jointly designed based on the ASABER criterion, as in [12].

²This study is based on the collaboration of IIT Madras and of the University of Southampton, which was funded by the IU-ATC project.

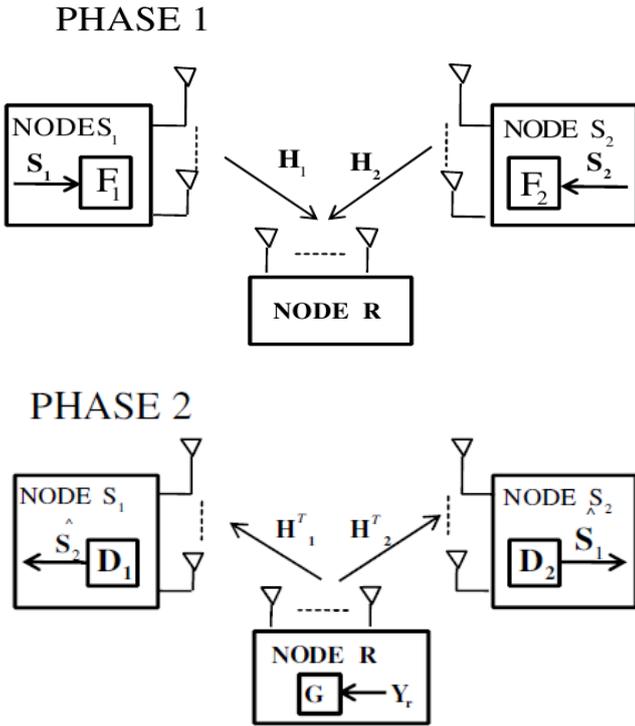


Fig.1. Two-way AF relay system

Let N_1 , N_2 and N_r denote the number of antennas at the two SNs and the RN node respectively and $L = \min(N_1, N_2)$ denote the number of parallel streams transmitted from S_1 to S_2 and vice versa. Based on our intensive simulation campaign, we will demonstrate that the approximate diversity order of the system is $(N_1 - L + 1) \times (N_2 - L + 1) + (N_r - L)$.

The outline of the paper is as follows. In Section II we detail our system model, while in Section III we formulate expressions for characterizing the scenarios associated with the assignment of different transmit powers to the RN for different number of antennas employed at the relay. Finally, in Section III we discuss our simulation results, before concluding in Section V.

2. SYSTEM MODEL

We consider a two-way AF relay-aided communication system comprised of the transceiver nodes S_1 , S_2 and RN R , as shown in Fig.1. The nodes S_1 , S_2 and R are equipped with N_1 , N_2 and N_r antennas, respectively. During the first communication phase the nodes S_1 and S_2 simultaneously transmit information to the RN R . In next phase of transmission the RN amplifies the received signal and broadcasts it to nodes S_1 and S_2 . The following common assumptions are exploited in this treatise:

1. The channels are flat fading and reciprocal.
2. Perfect channel state information³ (CSI) is assumed to be available for the channels spanning from $S_1 \rightarrow R$ and $S_2 \rightarrow R$ at all the three nodes.
3. The direct path between the SNs S_1 and S_2 is not exploited.

³In [13], [14] and [15] various channel estimation methods have been proposed for two-way relay channels.

Let $s_i \in \mathbb{C}^{L \times 1}$, $\forall i = 1, 2$ be the input information symbol vector at node S_i , where we have $L = \min(N_1, N_2)$. Let p_i be the transmit power available at node S_i . The symbol vector transmitted from node S_i is given by $x_i = F_i s_i$, $\forall i = 1, 2$. Here $F_i \in \mathbb{C}^{N_i \times L}$, $\forall i = 1, 2$ represents the linear precoder used at node S_i , where the design of F_i satisfies the following power constraint, $\text{tr}(\mathbb{E}[x_i x_i^H]) = \text{tr}(F_i F_i^H) \leq p_i$. (1)

During the first phase, the signal received at node R is given by,

$$y_r = \sum_{i=1}^2 H_i x_i + n_r, \quad (2)$$

where $H_i \in \mathbb{C}^{N_r \times N_i}$, $\forall i = 1, 2$ describes the channel spanning from node $S_i \rightarrow R$ and $n_r \in \mathbb{C}^{N_r \times 1}$ is the Additive White Gaussian Noise (AWGN) experienced at R , which is distributed as $\text{CN}(0, I_{N_r \times N_r})$. In the second phase, node R broadcasts the precoded signal x_r given by

$$x_r = G y_r, \quad (3)$$

where, $G \in \mathbb{C}^{N_r \times N_r}$ is the linear precoder matrix of the RN. Let p_r be the transmit power available at R , so that the design of G satisfies the following relay-power constraint,

$$\text{tr}(\mathbb{E}[x_r x_r^H]) = \text{tr} \left(G \left[\sum_{i=1}^2 H_i F_i F_i^H H_i^H + I \right] G^H \right) \leq p_r. \quad (4)$$

The signals received at the nodes S_1 and S_2 are given by,

$$y_1 = H_1^T G H_1 x_1 + H_1^T G H_2 x_2 + H_1^T G n_r + n_1 \quad (5)$$

$$y_2 = H_2^T G H_2 x_2 + H_2^T G H_1 x_1 + H_2^T G n_r + n_2, \quad (6)$$

where $n_i \in \mathbb{C}^{N_i \times 1}$, $\forall i = 1, 2$ is the AWGN vector satisfying $n_i \sim \text{CN}(0, I_{N_i \times N_i})$. Let \tilde{y}_1 and \tilde{y}_2 be the signals obtained after canceling the self interference from the received signals y_1 and y_2 respectively, yielding,

$$\tilde{y}_1 = H_1^T G H_2 x_2 + \tilde{n}_1 \quad (7)$$

$$\tilde{y}_2 = H_2^T G H_1 x_1 + \tilde{n}_2. \quad (8)$$

The effective additive noise at node S_i is given as, $\tilde{n}_i = H_i^T G n_r + n_i$. Let $R_{n,i}$ be the effective AWGN covariance matrix, which is given by,

$$R_{n,i} = \mathbb{E}[\tilde{n}_i \tilde{n}_i^H] = H_i^T G G^H H_i + I_{N_i \times N_i}. \quad (9)$$

Following noise whitening, the signals received at nodes S_1 and S_2 are given by,

$$\tilde{y}_{1,eff} = R_{n,1}^{-\frac{1}{2}} \tilde{y}_1 = H_{12,eff} F_2 s_2 + R_{n,2}^{-\frac{1}{2}} \tilde{n}_1 \quad (10)$$

$$\tilde{y}_{2,eff} = R_{n,2}^{-\frac{1}{2}} \tilde{y}_2 = H_{21,eff} F_1 s_1 + R_{n,1}^{-\frac{1}{2}} \tilde{n}_2, \quad (11)$$

where $H_{12,eff} = R_{n,1}^{-\frac{1}{2}} H_1^T G H_2$ and $H_{21,eff} = R_{n,2}^{-\frac{1}{2}} H_2^T G H_1$ are the effective channels spanning from $S_2 \rightarrow S_1$ and vice versa. The effective signals received at the nodes S_1 and S_2 , namely $\tilde{y}_{1,eff}$ and $\tilde{y}_{2,eff}$ are then processed by linear MMSE equalizers, in order to generate the estimates of s_2 and s_1 respectively, namely, $\hat{s}_1 = D_2 \tilde{y}_{2,eff}$, (12)

$$\hat{s}_2 = D_1 \tilde{y}_{1,eff}, \quad (13)$$

where \hat{s}_i is the estimated symbol vector of s_i and $D_i \in \mathbb{C}^{N_i \times L}$, $\forall i=1, 2$ is the equalizer's weight matrix used at node S_i .

3. EFFECT OF VARYING THE NUMBER OF ANTENNAS AT THE RELAY

Our main goal is now to study the effects of varying the number antennas at the relay on the achievable ASABER performance of the precoded MIMO-aided two-way AF relaying system considered under the following scenarios,

Case 1: Varying transmit power at the relay;

Case 2: Fixed transmit power at the relay.

Before we commence our related study, let us first introduce the MSE matrices of E_1 and E_2 at nodes S_1 and S_2 respectively as,

$$E_1(F_2, G, D_1) = E[(\hat{s}_2 - s_2)(\hat{s}_2 - s_2)^H] \\ = I_{L \times L} - D_1^H P_1 - P_1^H D_1 + D_1^H R_1 D_1, \quad (14)$$

$$E_2(F_1, G, D_2) = E[(\hat{s}_1 - s_1)(\hat{s}_1 - s_1)^H] \\ = I_{L \times L} - D_2^H P_2 - P_2^H D_2 + D_2^H R_2 D_2, \quad (15)$$

where P_1, P_2, R_1 and R_2 are defined as, follows:

$$P_1 = H_1^T G H_2 F_2, \quad (16)$$

$$P_2 = H_2^T G H_1 F_1, \quad (17)$$

$$R_1 = P_1 P_1^H + H_1^T G G^H H_1^* + I_{N_1 \times N_1}, \quad (18)$$

$$R_2 = P_2 P_2^H + H_2^T G G^H H_2^* + I_{N_2 \times N_2}. \quad (19)$$

We note that for fixed values of F_1, F_2 and G the trace $tr(E_i)$, $\forall i = 1, 2$ is a convex function with respect to D_i . Therefore, the optimal equalizer weight matrix D_i is obtained by solving the equation $\nabla_{D_i} tr(E_i) = 0$, which yields

$$D_{i,opt} = R_i^{-1} P_i, \quad \forall i = 1, 2. \quad (20)$$

After substituting $D_{i,opt}$ into Eq.(14) and Eq.(15), the MSE matrix E_1 and E_2 of node S_1 and S_2 may be written as,

$$\tilde{E}_1 = (I + F_2 R_{H,2} F_2^H)^{-1}, \quad (21)$$

$$\tilde{E}_2 = (I + F_1 R_{H,1} F_1^H)^{-1}, \quad (22)$$

respectively, where we have

$$R_{H,1} = H_1^H G^H H_2^* R_{n,2}^{-1} H_2^T G H_1, \quad (23)$$

$$R_{H,2} = H_2^H G^H H_1^* R_{n,1}^{-1} H_1^T G H_2. \quad (24)$$

Let us first define the relationship between the MSE, SINR and BER of the each individual stream as given in [16]. The MSE of i^{th} node and j^{th} parallel stream is defined as,

$$MSE_{i,j} = (\tilde{E}_i)_{j,j}. \quad (25)$$

Assuming that all the sub-streams use the same modulation constellation associated with Gray encoding, the corresponding BER and SINR are given by,

$$BER_{i,j} = \frac{1}{m} \left(\alpha_i Q \left(\beta_i \sqrt{SINR_{i,j}} \right) \right), \quad (26)$$

$$SINR_{i,j} = \frac{1}{MSE_{i,j}} - 1 \quad \forall i = 1, 2, \quad \forall j = 1, \dots, L, \quad (27)$$

where $m = \log_2 M$ is the number of bits per symbol and M is the size of the constellation used. The symbols α and β depend on the transmitted signal constellation and $Q(\cdot)$ is the Gaussian Q-

function defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$.

The ASABER performance measure is defined as follows:

$$f_2 = \frac{1}{2L} \left(\frac{1}{m} \sum_{i=1}^2 \sum_{j=1}^L \alpha_i Q \left(\beta_i \sqrt{\frac{1}{(\tilde{E}_i)_{j,j}} - 1} \right) \right). \quad (28)$$

The effective channels H_{12eff} and H_{21eff} of the ASABER MIMO precoded system may be diagonalized after an appropriate rotation of the input data symbols. The effective channel matrices H_{12eff} and H_{21eff} have $(N1 \times N2)$ and $(N2 \times N1)$ elements respectively, and their size is independent of N_r . Hence as N_r becomes higher than L , the total number of independently fading paths spanning from $S_1 \rightarrow R \rightarrow S_2$ and $S_2 \rightarrow R \rightarrow S_1$ is increased, which increases the achievable diversity order of the system. Based on our simulation campaign, we are able to infer the approximate diversity order in the form of

$$d = (N1 - L + 1) \times (N2 - L + 1) + (N_r - L). \quad (29)$$

We note however that Eq.(29) is only valid for $p_r \geq (p_1, p_2)$. When we have $p_r < (p_1, p_2)$, instead of amplifying the received signal, the RN reduces the signal power and then broadcasts it. As a result, the BER curve of the ASABER technique exhibits an error floor in this reduced power region. Hence, the above-mentioned diversity-order given by Eq.(29) becomes invalid in this case.

4. SIMULATION RESULTS

All the simulations we assumed having $N_1 = N_2 = 2$, $L = 2$ and the entries of the channel matrices H_1 and H_2 were independent and distributed as CN(0, 1). The entries of s_1 and s_2 assume one of the legitimate values from the QPSK constellation, where the bits are mapped to symbols using classic Gray coding. The BER was calculated using Eq.(27) for $\alpha = 1$ and $\beta = 2$, which again, corresponds to the QPSK constellation [16]. Table.1 lists the different precoding schemes used in this paper.

Table.1. List of different precoding schemes

Precoding Methods	
UP	<u>U</u> niform <u>P</u> ower Allocation
ROP	<u>R</u> elay <u>O</u> nly <u>P</u> recoding
JSRP	<u>J</u> oint <u>S</u> ource <u>R</u> elay <u>P</u> recoding
ROPM	<u>R</u> OP with <u>A</u> SAMSE as cost func.
ROPB	<u>R</u> OP with <u>A</u> SABER as cost func.
JAM	<u>J</u> SRP with <u>A</u> SAMSE as cost func.
JABM	<u>J</u> SRP with <u>A</u> SABER and <u>A</u> SAMSE as cost func.
JAB	<u>J</u> SRP with <u>A</u> SABER as cost func.

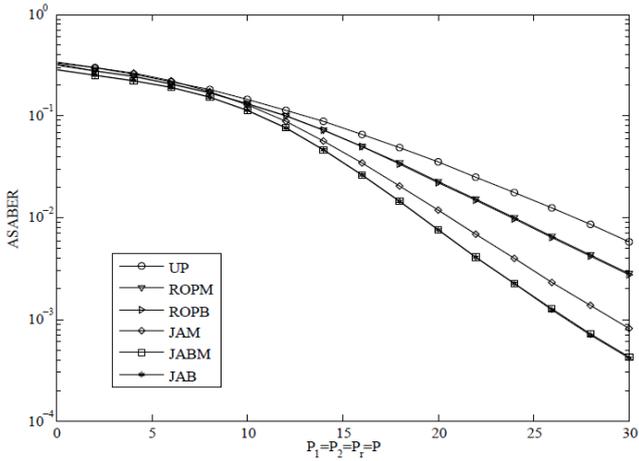


Fig.2. Performance comparison of different algorithms in terms of ASABER for $N_1 = 2, N_2 = 2$ and $N_r = 3$

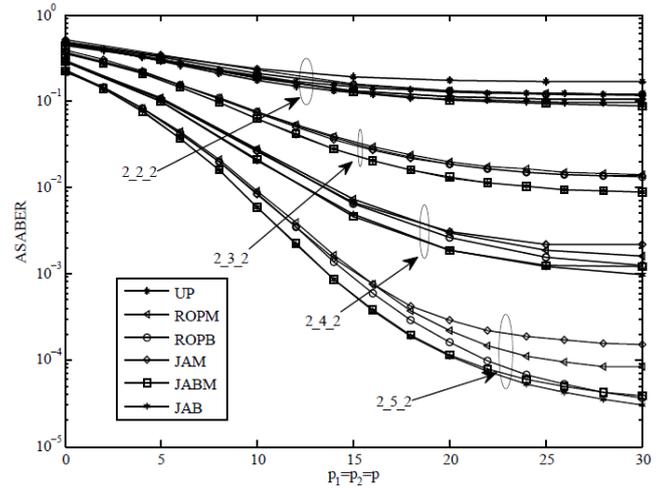


Fig.4. Comparison of effect of fixed power at RN i.e., $p_r = 15\text{dB}$, when varying $p_1 = p_2$ in terms of ASABER

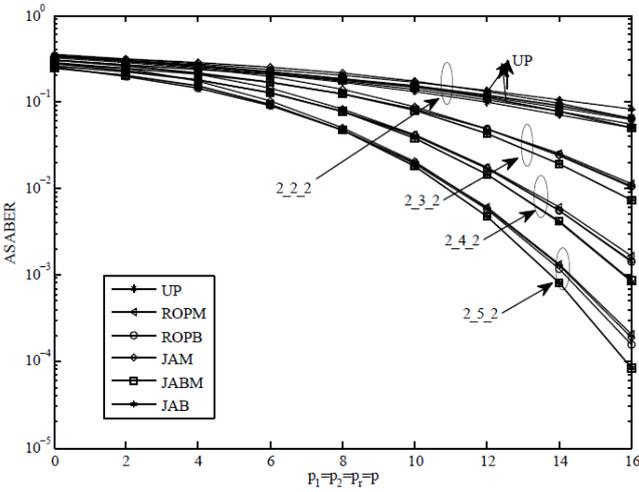


Fig.3. Comparison of effect of varying N_r from 2 – 5, when $N_1 = N_2 = 2$ in terms of ASABER measure

Fig.2. shows the ASABER scheme’s ABER performance in comparison to a range of existing algorithms⁴. Fig.3. quantifies the effects of varying N_r from 2 to 5 on the ASABER scheme’s ABER performance. The following observation can be made from Fig.3,

- As N_r increases, the attainable diversity order of the UP algorithm fails to improve;
- When N_r is increased by 1, the diversity order the JAM, JAB and JABM schemes is increased by 2.
- The performance of the UP algorithm recorded for the 2-3-2 system is similar to that of the JAB and JABM system using the 2-2-2 configuration.

Fig.4. characterizes the attainable performance of the ASABER scheme, when the RN’s power was fixed to $p_r = 15\text{dB}$, while $p_1=p_2$ were varied from 0 – 30dB and number of antennas at the RN was varied from 2 to 5 i.e., we had $N_r \rightarrow 2$ to 5.

- As the number of antennas is increased, the JAM technique starts to perform more poorly than the ROPM and ROPB arrangements.
- The diversity order equation remains valid until we have $p_1 = p_2 \leq p_r$. However, once p_1 and p_2 are increased beyond p_r , the ASABER curve exhibits an “error floor”.
- The JAB and JABM techniques attain a similar performance and they outperform all the other methods.

The effect of varying N_r from 2 to 5 in Case 1 and Case 2 may be summarized as follows:

- The attainable diversity order of the two-way AF relay system was increased in line with $N_r - L$ for each increment in N_r , when N_1 and N_2 were fixed.
- Case1 outperformed Case2 as N_r was increased from 2 to 5.
- The JAB and JABM techniques outperformed all the other methods in all scenarios.

5. CONCLUSIONS

In this paper we have studied the quantitative effects of having a fixed relay power and a variable number of relay antennas on the achievable diversity gain and error rate performance of the system. As N_r became higher than the number of parallel transmission streams L , the diversity order was increased by $(N_r - L)$ for $p_1 = p_2 = p_r$. This result in Fig.3 is valid up to SNR of 20 dB and diversity expression in Eq.(29) does not hold for $\text{SNR} > 20\text{dB}$. This needs to be further investigated. However, the ASABER as seen in Fig.4 the ABER curve exhibited an error-floor for $p_1 = p_2 = p$ and $p_r < p$. Therefore maximum diversity gain for given N_r at relay is a function of relative values of p_1, p_2 and p_r . This interplay between diversity gain and power constraint, especially for higher SNR, needs further study.

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⁴For the detailed analysis of all the algorithms considered please refer to [12].

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