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SIDE LOBE REDUCTION OF A UNIFORMLY EXCITED CONCENTRIC RING ARRAY ANTENNA USING EVOLUTIONARY ALGORITHMS

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Abstract
Reduction of sidelobe level in concentric ring arrays results in wide first null beamwidth (FNBW). Theauthors propose a pattern synthesis method based on modified Particle Swarm Optimization (PSO) algorithm and Differential Evolution (DE) algorithm to reduce sidelobe level while keeping the first null beamwidth (FNBW) fixed or variable. This is achieved by optimizing both ring spacing and number of elements in each ring of a concentric circular ring array of uniformly excited isotropic antennas. The first null beamwidth is attempted to be made equal to or less than that of a uniformly excited and 0.5 λ spaced concentric circular ring array of same number of elements and same number of rings. The comparative performance of modified Particle Swarm Optimization (PSO) algorithm and Differential Evolution (DE) algorithms based on this particular problem in terms of FNBW, sidelobe level and computational time is also studied.

Keywords:
Concentric Ring Array, Differential Evolution (DE) Algorithm, First Null Beam Width (FNBW), Modified Particle Swarm Optimization (PSO) Algorithm

1. INTRODUCTION

A circular ring array also known as concentric circular array (CCA) is a planar array that consists of one or more concentric rings, each having equally spaced array elements on its circumference. Its main attraction is the cylindrical symmetry of its radiation pattern and compact structure. However, in its simple form the array suffers from high side lobe problem. One of the important configurations regarding CCA is uniform concentric circular array (UCCA) where interelement spacing in each individual ring is kept almost half of the wavelength and all the elements in the array are uniformly excited. Generally low sidelobes in the array factor are obtained through optimum amplitude weights of the signals at each array element.

The radiation pattern function of a concentric ring array has been expressed by Stearns and Stewart [1] as a truncated Fourier-Bessel series and the non-uniform distribution of the rings has been approximated to a smaller number of equally spaced ones. N. Goto and D. K Cheng showed that for a Taylor weighted ring array the maximum allowable inter-element spacing should be about four-tenths of a wavelength, if high sidelobes are to be avoided [2].

L.Biller and G. Friedman used steepest descent iterative process to find out element weights and ring spacing to get lower side lobe levels and control over beam width [3]. D. Huebner reduced the sidelobe levels for small concentric ring array by adjusting the ring radii using optimization technique [4]. B. P. Kumar and G. R. Branner also proposed optimum ring radii for getting lower sidelobes [5]. M. Dessouky, H. Sharshar and Y. Albagory showed that the existence of central element in concentric circular array of smaller innermost ring reduced the sidelobe levels significantly while minor increase in the beamwidth [6]. Side lobe levels can be reduced by thinning the array [7-8]. The array is thinned by turning off selected elements from the uniform array. Sidelobe level can also be reduced by optimizing both radii of the rings and the number of elements in each ring of a concentric ring array.

Reduction in the sidelobe level also increases first null beam width (FNBW) significantly. In this paper we have reduced the sidelobe level significantly, keeping FNBW fixed by optimizing both ring spacing and number of elements in each ring. Here modified Particle Swarm Optimization (PSO) algorithm and Differential Evolution (DE) algorithm have been successfully applied as an evolutionary algorithm [9-10] to find out those optimum values. Keeping the fitness function same, the comparative performance of modified PSO [11-13] and DE [14-16] for this particular problem is being studied.

2. THEORETICAL FORMULATION

The far field pattern of a concentric circular planar array shown in Fig.1 on the x – y plane with central element feeding can be defined as [6-7]:

$$E(\theta,\varphi) = 1 + \sum_{m=1}^{M}\sum_{n=1}^{N_m} I_m e^{j[k\, r_m \sin\theta \cos(\varphi - \varphi_{mn}) + \phi_m]} \quad (1)$$

Normalized absolute power pattern, P(θ,φ) in dB can be expressed as follows:

$$P(\theta,\varphi) = 10\log_{10}\left[\frac{|E(\theta,\varphi)|}{|E(\theta,\varphi)|_{max}}\right]^2 = 20\log_{10}\left[\frac{|E(\theta,\varphi)|}{|E(\theta,\varphi)|_{max}}\right] \quad (2)$$

Where M = Number of concentric rings, N_m = Number of isotropic elements in each ring, I_m = excitation amplitude of elements on m-th circular ring, d_m = interelement arc spacing of m-th circle, $r_m = N_m d_m/2\pi$, Radious of the m[th] ring, $\varphi_{mn}=2n\pi/N_m$, angular position of mn-th element with $1\le n \le N_m$, θ, φ = polar, azimuth angle, λ=wave length, k = wave number = 2π/λ, j=complex number, ϕ_m = excitation phase of elements on m-th ring. All the elements have same excitation phase of zero degree.

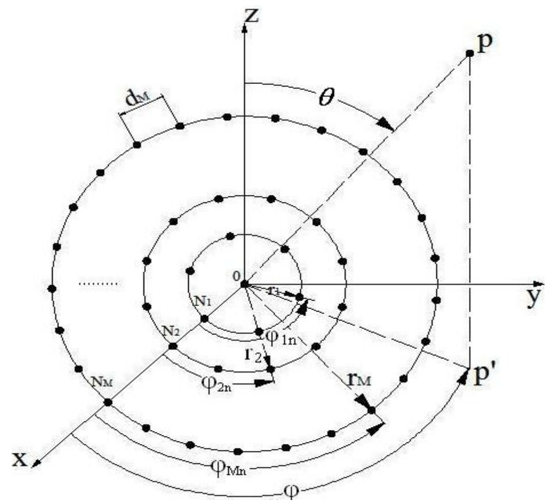

Fig.1 Concentric ring arrays of isotropic antennas in XY plane

3. METHOD OF SIDELOBE REDUCTION

Sidelobe level of a uniform concentric ring array can be reduced by optimizing both the ring spacing non-uniformly and the number of elements in each ring. Initially it is assumed that all the elements are uniformly excited, interelement spacing is 0.5λ and the ring spacing $r_m = m\lambda/2$. The number of elements in m-th ring of a concentric ring array can be expressed as:

$$N_m = \frac{2\pi r_m}{d_m} \tag{3}$$

Since the number of elements should be an integer so only the computed integer values of Eq. (3) are taken. The radii of the rings are varied by first assuming that all the rings are separated by a minimum distance of 0.5λ. After that, a non-uniform separation is included so that the new radius r_m becomes:

$$r_m = r_{m-1} + \frac{\lambda}{2} + \Delta_m \text{, (where } 0 \le \Delta m \le \lambda \text{)} \tag{4}$$

The radii of the rings and the number of elements in each ring are varied such that the interelement spacing d_m of m-th circle lied between $0.5\lambda \le \mathrm{dm} \le \lambda$.

The optimum values of r_m and N_m are individually computed by first using modified Particle Swarm Optimization algorithm (PSO) and then by Differential Evolution (DE) algorithm, while keeping the fitness function same for both the cases.

The fitness functions for this problem are:

$$Fitness1 = k_1 \, max\, SLL + k_2 \left(FNBW_o - FNBW_d \right)^2 H(T) \tag{5}$$

$$Fitness2 = max\, SLL \tag{6}$$

Where , *max SLL* is the value of maximum side lobe level, $FNBW_o$ and $FNBW_d$ are the obtained and desired values of first null beam width respectively, k_1, k_2 are weighting coefficients to control the relative importance given to each term of Eq. (5) and *H(T)* is a Heaviside step function is defined as:

$$T = \left(FNBW_o - FNBW_d \right) \tag{7}$$

$$H(T) = \begin{cases} 0, & if \ T < 0 \ , \\ 1, & if \ T \ge 0 \end{cases} \tag{8}$$

Eq. (6) is when a fixed value of FNBW was not taken into consideration. Eq. (5) and (6) are minimized individually by modified PSO and DE for optimal synthesis of array.

4. MODIFIED PARTICLE SWARM OPTIMIZATION ALGORITHM

Particle Swarm Optimization (PSO) is a population based stochastic optimization tool inspired by social behavior of bird flock, fish school etc. as developed by Kennedy and Eberhart in 1995 [11]. In PSO, a member in the swarm, called a particle, represents a potential solution, which is a point in the search space. The global optimum is regarded as the location of food. Each particle has a fitness value and a velocity to adjust its flying direction according to the best experiences of the swarm in search for the global optimum in the D-dimensional solution space. The steps involved in modified PSO are given below:

Step 1: Initialize positions and associate velocity to all particles (potential solutions) in the population randomly in the D-dimension space.

Step 2: Evaluate the fitness value of all particles.

Step 3: Compare the personal best (*pbest*) of every particle with its current fitness value. If the current fitness value is better, then assign the current fitness value to *pbest* and assign the current coordinates to *pbest* coordinates.

Step 4: Determine the current best fitness value in the whole population and its coordinates. If the current best fitness value is better than global best (*gbest*), then assign the current best fitnessvalue to *gbest* and assign the current coordinates to *gbest* coordinates.

Step 5: Update velocity (V_{id}) and position (X_{id}) of the *d*-th dimension of the *i*-th particle using the following equations:

$$\begin{aligned} V_{id}^{t} &= w(t) * V_{id}^{t-1} + c_1(t) * rand1_{id}^{t} * \left(pbest_{id}^{t-1} - X_{id}^{t-1} \right) \\ &+ c_2(t) * \left(1 - rand1_{id}^{t} \right) * \left(gbest_{d}^{t-1} - X_{id}^{t-1} \right) \end{aligned} \tag{9}$$

$$\begin{aligned} &If \ \ V_{id}^{t} > V_{max}^{d} \ \ or \ \ V_{id}^{t} < V_{min}^{d}, \\ &then \ \ V_{id}^{t} = U(V_{min}^{d}, V_{max}^{d}) \end{aligned} \tag{10}$$

$$X_{id}^{t} = rand2_{id}^{t} * X_{id}^{t-1} + (1 - rand2_{id}^{t}) * V_{id}^{t} \tag{11}$$

c_1(t), c_2(t) = time-varying acceleration coefficients with c_1(t) decreasing linearly from 2.5 to 0.5 and c_2(t) increasing linearly from 0.5 to 2.5 over the full range of the search , *w(t)*=time-varying inertia weight changing randomly between U(0.4,0.9) with iterations, *rand*1, *rand*2 are uniform random numbers between 0 and 1, having different values in different dimension, *t* is the current generation number.

Eq. (10) has been introduced to clamp the velocity along each dimension to uniformly distributed random value between V_{min}^{d} and V_{max}^{d} if they try to cross the desired domain of interest.

These clipping techniques are sometimes necessary to prevent particles from explosion. The maximum velocity is set to the upper limit of the dynamic range of the search $(V_{max}^d = X_{max}^d)$ and the minimum velocity (V_{min}^d) is set to (X_{min}^d).

However, position-clipping technique is avoided in modified PSO algorithm. Moreover, the fitness function evaluations of errant particles (positions outside the domain of interest) are skipped to improve the speed of the algorithm

Step 6: Repeat steps 2-5 until a stop criterion is satisfied or a pre-specified number of iteration is completed, usually when there is no further update of best fitness value.

In this problem number of particles is taken 40 and the algorithm is run for 800 generations. The maximum number of generation is kept at a value where there is no further update of global best solutions

5. DIFFERENTIAL EVOLUTION ALGORITHM

Differential Evolution is a simple evolutionary algorithm introduced by Storn and Price [14-16]. Similar to GA[17], DE is also an algorithm based on population. DE algorithm is a stochastic optimization method minimizing an objective function that can model the problem's objectives while incorporating constraints. The algorithm mainly has three advantages; finding the true global minima regardless of the initial parameter value, fast convergence and using a few control parameters [14-16]. DE can be described as below [14-16]:

Step 1: **Initialization:**

The generation number is set to t=0 and a population of *NP* individuals are randomly initialized in the D-dimensional search space as, $P_t = \{\vec{X}_1(t), \ldots\ldots, \vec{X}_{NP}(t)\}$, where $\vec{X}_i(t) = [x_{i,1}(t), x_{i,2}(t), \ldots, x_{i,D}(t)]$ and each individuals are uniformly distributed in domain $[\vec{X}_{min}, \vec{X}_{max}]$.

Step 2: Evaluate the fitness:

Evaluate the fitness of each individual at current generation.

Step 3: Mutation:

Create donor vector $\vec{V}_i(t)$ corresponding to the i-th target vector $\vec{X}_i(t)$ for all the individuals at current generation using any one of the DE mutation scheme [14-16].

In this problem we have used the mutation strategy known as DE/best/1, expressed as:

$\vec{V}_i(t) = \vec{X}_{best}(t) + F.(\vec{X}_{r_1^i}(t) - \vec{X}_{r_2^i}(t))$ for i=1,2,...,*NP*

where, $\vec{X}_{best}$ is the best vector of the current population, $\vec{X}_{r_1^i}$ and $\vec{X}_{r_2^i}$ are randomly picked up vectors from the current generations, F is the scale factor, $F \in (0,1+)$, a positive real number that controls the rate at which the population evolves.

Step 4: Crossover:

Use any one of the crossover scheme in DE [14-16] to form the trial vector $\vec{U}_i(t)$. It is achieved by exchanging the components of the donor vector $\vec{V}_i(t)$ and the target vector $\vec{X}_i(t)$ with a crossover probability of $C_r\ (C_r \in [0,1])$, for all the individuals at current generation.

Step 5: Selection:

Select the best individuals for the next generation as follows:

$$\vec{X}_i(t+1) = \begin{Bmatrix} \vec{U}_i(t), if, f(\vec{U}_i(t)) \le f(\vec{X}_i(t)) \\ \vec{X}_i(t), if, f(\vec{U}_i(t)) > f(\vec{X}_i(t)) \end{Bmatrix}, for, i = 1,2,\ldots,NP$$

Compute $\vec{X}_{Gbest}(t)$ at current generation as follows:

From *NP* individuals of $\vec{X}_i(t+1)$, find out the individual for which $f(X_i(t+1)), for, i = 1,2,\ldots,NP$, becomes minimum (for minimization problem) and assign that vector to $\vec{X}_{Gbest}(t)$.

Here, $f(\vec{X})$ is the function to be minimized. Since the selection process employs a binary decision the population size remains fixed throughout generations.

Step 6:

Increase the iteration count 't' by one, and repeat steps 2-5 until the termination condition is satisfied. Return $\vec{X}_{Gbest}(t)$ as the result.

The termination condition can be defined:

i. When a fixed number of iteration t_{max}, with a suitably large value of t_{max}, depending upon the complexity of the objective function is reached.
ii. When best fitness of the population does not change appreciably over successive iterations.

Mutation demarcates one DE scheme from another. Each mutation strategy combines with either 'exponential' or 'binomial' type crossover and produce new working strategy. There are in total ten different working strategies of DE as suggested by Storn and Price [14-16].

In this problem we have used DE/best/1/exp strategy along with number of population (*NP*) =40 and crossover rate (C_r) = 0.7 and the termination condition was defined as t_{max}=800

6. SIMULATION RESULTS

For a nine ring concentric ring array of isotropic antennas [7] the initial radius of the rings are $r_m = m\lambda/2$ (m-th ring) and the interelement spacing in each ring is kept at $\lambda/2$. For this arrangement the total number of isotropic elements is 279. Uniform excitation and constant phase angle between the elements gives sidelobe level -17.4 dB [7] and FNBW 14.8 degree. The problem is to find out the optimum set of ring radii and the number of elements in each individual ring that would generate a pencil beam in the XZ plane keeping FNBW below or equal to that of a nine ring uniform concentric circular ring array. All these simulations are performed using a PC having Intel core2 processor with 3 GHz clock frequency, 2 GB of RAM and Microsoft windows XP 32 bit operating system. Table 1 shows that using modified PSO we were able to reduce the sidelobe level below -29 dB keeping FNBW fixed and reduce it below -31 dB without fixing FNBW . Fig.2 and Fig.3 shows the normalized power patterns of the optimized arrays in dB along with initial 9 ring uniform concentric ring array for fixed and variable FNBW cases computed using PSO. DE reduces the sidelobe level below -32 dB for fixed FNBW and below -33 dB without fixing FNBW. However DE requires slightly greater amount of time to give optimum result. Fig.4 and Fig.5 shows the normalized power patterns of the optimized arrays in dB along with initial 9 ring uniform concentric ring array for fixed and variable FNBW cases computed using DE.

Table 2 and Table 3 shows the rings radii and the number of elements in each ring for the optimized arrays of fixed and variable FNBW computed using modified PSO and DE. From Table 2, we can find the total number of elements in the optimized array including the central element. Using modified PSO for fixed FNBW case it is 242 or 86.74% of the initial 9-ring uniform concentric ring array and for variable FNBW case it is 238 or 85.30% of the initial 9-ring uniform concentric ring array. From Table 3, it can be seen that the total number of isotropic elements in the optimized array including central element, using DE for fixed FNBW case is 225 or 80.64% of the initial 9-ring uniform concentric ring array and for variable FNBW it is 198 or 70.97% of the initial 9-ring uniform concentric ring array. The maximum radius of the optimized array computed using DE for both the cases is lesser than that of the optimized arrays computed using modified PSO.

Table.1. Comparative performance of modified PSO and DE based on the fitness function in Eq. (5) and (6)

Parameters	**Modified PSO**		**DE**	
	Opt. Array (fixed FNBW)	Opt. Array (variable FNBW)	Opt. Array (fixed FNBW)	Opt. Array (variable FNBW)
Sidelobe level (dB)	-29.71	-31.82	-32.05	-33.24
FNBW (degree)	13.1	15.0	14.8	16.9
Computation time (min:sec)	66:26	51:28	80:67	78:92
Fitness Value	-29.71	-31.82	-32.05	-33.24

Table.2. Ring radii and number of elements in each ring computed by modified Particle Swarm Optimization (PSO) algorithm

Ring Number	**Parameters for fixed FNBW**		**Parameters for variable FNBW**	
	$r_m(\lambda)$	N_m	$r_m(\lambda)$	N_m
1	0.747	9	0.503	6
2	1.273	16	1.044	13
3	1.905	14	1.547	19
4	2.435	26	2.104	24
5	3.078	33	2.619	25
6	3.755	31	3.244	38
7	4.599	34	4.006	39
8	5.399	36	4.919	35
9	6.338	42	5.744	38

Table.3. Ring radii and number of elements in each ring computed by Differential Evolution (DE) algorithm

Ring Number	**Parameters for fixed FNBW**		**Parameters for variable FNBW**	
	$r_m(\lambda)$	N_m	$r_m(\lambda)$	N_m
1	0.556	6	0.501	6
2	1.056	12	1.002	12
3	1.559	18	1.503	18
4	2.182	27	2.016	21
5	2.809	25	2.527	26
6	3.309	30	3.064	23
7	4.045	34	3.588	28
8	4.861	33	4.309	29
9	5.810	39	5.219	34

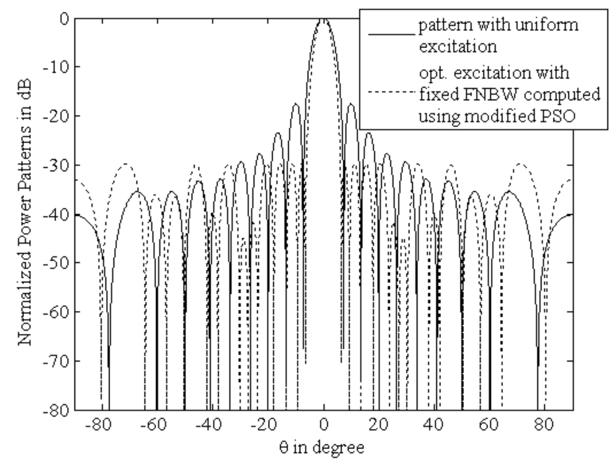

Fig.2. Normalized absolute power patterns in dB for uniform concentric ring array and optimized array with fixed FNBW using modified PSO

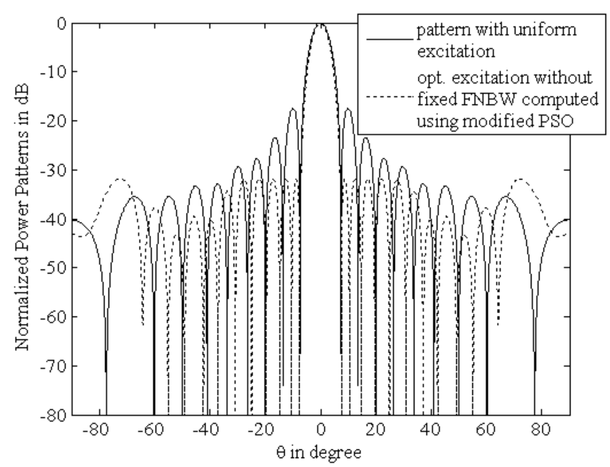

Fig.3. Normalized absolute power patterns in dB for uniform concentric ring array and optimized array without fixing FNBW using modified PSO

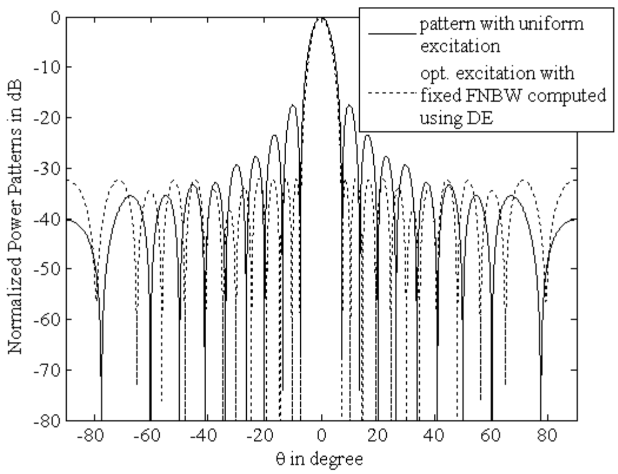

Fig.4. Normalized absolute power patterns in dB for uniform Concentric ring array and optimized array with fixed FNBW using DE

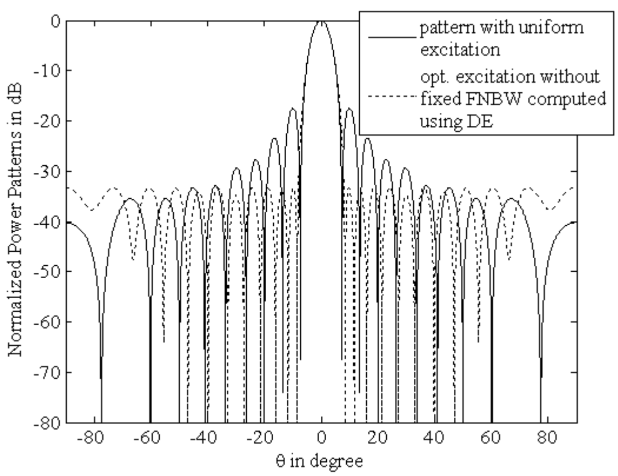

Fig.5. Normalized absolute power patterns in dB for uniform concentric ring array and optimized array without fixed FNBW using DE

7. CONCLUSIONS

It has been shown that by optimizing radii of the rings and the number of elements in each individual ring it is possible to reduce the sidelobe level of a concentric ring array significantly. Here modified Particle Swarm Optimization algorithm and Differential Evolution (DE) algorithm have been effectively used as a global optimization algorithm to find out optimum set of r_m and N_m. From the result it can be inferred that the performance of Differential evolution (DE) algorithm in this problem is better than that of modified Particle Swarm Optimization (PSO) algorithm in terms of sidelobe level, number of elements in the array, and compact structure although DE requires slightly greater amounts of time to give optimum results. Here both the algorithms satisfy the desired array characteristics and give satisfactory results.

REFERENCES

[1] C. Stearns and A. Stewart, 1965, “An investigation of concentric ring antennas with low sidelobes”, *IEEE Transactions on Antennas Propagation,* Vol. 13, No. 6, pp.856–863.

[2] N. Goto and D. K. Cheng, 1970, “On the synthesis of concentric-ring arrays”, *IEEE Proc.*, Vol. 58, No. 5, pp.839–840.

[3] L. Biller and G. Friedman, 1973, “Optimization of radiation patterns for an array of concentric ring sources”, *IEEE Trans., Audio and Electroacoustics*, Vol. 21, No. 1, pp.57–61.

[4] D. Huebner, 1978, “Design and optimization of small concentric ring arrays,” *IEEE Proc., Antennas Propagation Int. Symp.*, Vol. 16, pp. 455–458.

[5] B. P. Kumar and G. R. Branner, 1999, “Design of low sidelobe circular ring array by element radius optimazation,” Orlando, *IEEE Proc., Antennas Propagation Int. Symp.*, pp. 2032–2035.

[6] Dessouky, M., H. Sharshar, and Y. Albagory, 2006, “Efficient sidelobe reduction technique for small-sized concentric circular array,” Progress In Electromagnetics Research, PIER 65, 187–200.

[7] Haupt. R. L., 2008, “Optimized Element Spacing for Low Sidelobe Concentric Ring Arrays,” *IEEE Trans., Antennas and Propag.*, Vol. 56, No. 1,pp.266-268.

[8] Haupt. R. L., 2008, “Thinned concentric ring arrays,” San Diego, *IEE proc., Antennas and Propagation Int. Symp.*, pp.1-4.

[9] David B. Fogel., 2006, *Evolutionary computation: toward a new philosophy of machine intelligence*, 3rd Edition, John Wiley and Sons.

[10] Thomas Baeck, D. B. Fogel, and Z Michalewicz., 1997, *Handbook of Evolutionary Computation,* Taylor & Francis, New York.

[11] J. Kennedy and R. C. Eberhart, 1995, “Particle swarm optimization,” *IEEE Proc., Int. Conf. Neural Networks*, Vol.4, pp. 1942–1948.

[12] D. W. Boeringer, and D. H. Werner, 2004, “Particle swarm optimization versus genetic algorithms for phased array synthesis,” *IEEE Trans., Antennas and Propagation*, Vol.52, No. 3, pp.771–779.

[13] N. Jin and Y. Rahmat-Samii, 2007, “Advances in particle swarm optimization for antenna designs: Real-number, binary, single- objective and multiobjective implementations,” *IEEE Trans., Antennas Propag.*, Vol. 55, no. 3, pp.556-567.

[14] R. Storn, and K. Price, 1997, “Differential evolution—A simple and efficient heuristic for global optimization over continuous spaces,” *Journal of Global Optimization*, Vol.11, No. 4, pp.341-359.

[15] Kenneth V. Price, Rainer M. Storn, Jouni A. Lampinen, 2005, *Differential evolution: a practical approach to global optimization,* Springer.

[16] Swagatam Das, Ajith Abraham and Amit Konar, 2009, *Metaheuristic Clustering,* Springer.

[17] K.F. Man, K.S. Tang, S. Kwong, 1996, “Genetic Algorithms: Concepts and Applications,” *IEEE Trans. Industrial Electronics*, vol.43, no. pp 519 – 533.